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# Statistical Physics and Thermodynamics

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Vol. 2: Exercises and Thermodynamic Tables



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# Statistical Physics and Thermodynamics

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ULisboa School, Shanghai University textbook

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Exercises

**1**

## On the use of Artificial Intelligence tools for Thermodynamics problem solving

The deployment of artificial intelligence tools to support learning processes is a perspective that is both promising and carries some risks, especially if the student is not fully aware of the extent of the capabilities of the various tools available on the World Wide Web. One of the most critical aspects is the potential for failure at providing meaningful answers by these virtual assistants, or put in simpler terms: Can artificial intelligence tools be safely used to support the process of solving thermodynamics exercises by students?

In this section, a simple exercise calculating the energy lost due to the condensation of one liter of steam was submitted to ChatGPT. This exercise was submitted via the link <https://talkai.info/chat/> on March 25, 2025. The algorithm response is reproduced and critically analyzed in this document.

The student is invited to copy the question verbatim and present it to ChatGPT (or other AI tools) to verify that the answer below is indeed the one proposed by the algorithm.

### Exercise 1.0.1



You

How much heat does a liter of vapor at atmospheric pressure and an initial temperature of 120 °C loses when the temperature is decreased to 80 °C?

### Answer 1.0.1

#### ChatGPT Solution



ChatGPT

To calculate the heat lost by a liter of vapor (in this case, we'll assume it's water vapor) when it cools from 120 °C to 80 °C, we can use the formula for heat transfer:

$$Q = m \cdot c \cdot \Delta T$$

Where:

- ▶  $Q$  is the heat lost (in joules).
- ▶  $m$  is the mass of the vapor (in kilograms).
- ▶  $c$  is the specific heat capacity of the vapor (in J/(kg °C)).
- ▶  $\Delta T$  is the change in temperature (in °C).

1. **Determine the mass of the vapor:** Since we have 1 liter of vapor and the density of water vapor at 120 °C is approximately 0.6 kg/m<sup>3</sup>, we can find the mass.

The density of water vapor can vary, but for this calculation, we'll use a rough estimate:

$$m = \text{density} \times \text{volume} = 0.6 \text{ kg/m}^3 \times 0.001 \text{ m}^3 = 0.0006 \text{ kg}$$

However, this density is quite low. The density of water vapor at 120 °C is closer to 0.6 kg/m<sup>3</sup>, which translates to:

$$m = 0.6 \text{ kg/m}^3 \times 1 \text{ L} = 0.6 \text{ kg}$$

2. **Determine the specific heat capacity of water vapor:** The specific heat capacity of water vapor is about 1.996 kJ/(kg °C), or 1996 J/(kg °C).
3. **Calculate the change in temperature:**

$$\Delta T = T_{\text{final}} - T_{\text{initial}} = 80 \text{ °C} - 120 \text{ °C} = -40 \text{ °C}$$

## 4. Calculate the heat lost:

$$Q = m \cdot c \cdot \Delta T$$

$$Q = 0.6 \text{ kg} \cdot 1996 \text{ J/(kg}^\circ\text{C)} \cdot (-40^\circ\text{C)}$$

$$Q = 0.6 \cdot 1996 \cdot (-40)$$

$$Q = -47\,904 \text{ J}$$

5. **Interpret the result:** The negative sign indicates heat is lost. Therefore, the heat lost by 1 liter of water vapor when it cools from  $120^\circ\text{C}$  to  $80^\circ\text{C}$  is approximately 47 904 J or 47.9 kJ.

## Answer 1.0.1

## Critical correction of the proposed solution

A first step is understanding that vapor initially at  $120^\circ\text{C}$  condensing to  $80^\circ\text{C}$  at atmospheric pressure will undergo a phase change to a liquid. Therefore the properties for the process, which we label as  $1 \rightarrow 2$  will be taken from the superheated vapor tables for state (1) and from the compressed liquid tables for state (2). We accordingly collate the necessary data from the corresponding tables (see Chapter C.2) and we report:

$$h_1 = 2716.5 \text{ kJ/kg}$$

$$= 48.938 \text{ kJ/mol}$$

$$v_1 = 1769.4 \text{ L/kg}$$

$$h_2 = 335.06 \text{ kJ/kg}$$

$$= 6.0361 \text{ kJ/mol}$$

$$v_2 = 1.0290 \text{ L/kg}$$

With this supporting data, we may now critically assess the proposed solution (our comments in red):



ChatGPT

To calculate the heat lost by a liter of vapor (in this case, we'll assume it's water vapor) when it cools from  $120^\circ\text{C}$  to  $80^\circ\text{C}$ , we can use the formula for heat transfer:

$$Q = m \cdot c \cdot \Delta T$$

Where:

- ▶  $Q$  is the heat lost (in joules).
- ▶  $m$  is the mass of the vapor (in kilograms).
- ▶  $c$  is the specific heat capacity of the vapor (in  $\text{J}/(\text{kg}^\circ\text{C})$ ).
- ▶  $\Delta T$  is the change in temperature (in  $^\circ\text{C}$ ).

**(This is wrong since vapor at atmospheric pressure will condense to liquid water at a temperature of  $100^\circ\text{C}$ ). Therefore, at  $80^\circ\text{C}$  the water will be in its liquid state. The heat transfer formula above can only be used when there is no transition of phase.)**

1. **Determine the mass of the vapor:** Since we have 1 liter of vapor and the density of water vapor at  $120^\circ\text{C}$  is approximately  $0.6 \text{ kg/m}^3$ , we can find the mass.  
The density of water vapor can vary, but for this calculation, we'll use a rough estimate:

$$m = \text{density} \times \text{volume} = 0.6 \text{ kg/m}^3 \times 0.001 \text{ m}^3 = 0.0006 \text{ kg}$$

However, this density is quite low. The density of water vapor at  $120^\circ\text{C}$  is closer to  $0.6 \text{ kg/m}^3$ , which

translates to:

$$m = 0.6 \text{ kg/m}^3 \times 1 \text{ L} = 0.6 \text{ kg}$$

**(What is this I don't even)**

2. **Determine the specific heat capacity of water vapor:** The specific heat capacity of water vapor is about  $1.996 \text{ kJ}/(\text{kg } ^\circ\text{C})$ , or  $1996 \text{ J}/(\text{kg } ^\circ\text{C})$ .

**(No idea where this value is coming from but it is wrong, the specific heat capacity of water vapor at  $120^\circ\text{C}$  is  $2716.5 \text{ kJ/kg}$ )**

3. **Calculate the change in temperature:**

$$\Delta T = T_{\text{final}} - T_{\text{initial}} = 80^\circ\text{C} - 120^\circ\text{C} = -40^\circ\text{C}$$

**(Again this ignores that there is a phase change at  $T = 100^\circ\text{C}$ )**

4. **Calculate the heat lost:**

$$Q = m \cdot c \cdot \Delta T$$

$$Q = 0.6 \text{ kg} \cdot 1996 \text{ J}/(\text{kg } ^\circ\text{C}) \cdot (-40^\circ\text{C})$$

$$Q = 0.6 \cdot 1996 \cdot (-40)$$

$$Q = -47904 \text{ J}$$

**(Incomplete equation, wrong value for the vapor mass, wrong value for the initial heat capacity of vapor, all garbage)**

5. **Interpret the result:** The negative sign indicates heat is lost. Therefore, the heat lost by 1 liter of water vapor when it cools from  $120^\circ\text{C}$  to  $80^\circ\text{C}$  is approximately  $47904 \text{ J}$  or  $47.9 \text{ kJ}$ .

#### **Solution with the correct values:**

Using the thermodynamic property tables, we determine the specific volume of steam at  $p = 1 \text{ atm}$  and  $T = 120^\circ\text{C}$  to be  $v_1 = 1769.4 \text{ L/kg}$  therefore  $1 \text{ L}$  of steam will weigh  $1/1769.4 = 0.57 \times 10^{-3} \text{ kg}$ . Considering the enthalpies we reported at the beginning for states 1 (superheated vapor) and 2 (compressed liquid) we may now straightforwardly calculate the lost heat in terms of mass times difference between initial and final enthalpies:

$$\begin{aligned} Q &= m \cdot (h_1 - h_2) \\ &= 0.57 \times 10^{-3} \text{ kg} \cdot (2716.5 \text{ kJ/kg} - 335.06 \text{ kJ/kg}) \\ &= 1.346 \text{ kJ} \end{aligned}$$

Therefore, one liter of vapor at atmospheric pressure and  $T = 120^\circ\text{C}$  loses approximately  $1.35 \text{ kJ}$  of heat when the temperature is decreased to  $80^\circ\text{C}$ , leading to its condensation into liquid water.

#### **ChatGPT incurred in two critical errors:**

- it is unaware of the concept of latent heat (for phase changes) and,
- it is not capable of accurately determine the mass of one liter of superheated vapor at  $T = 120^\circ\text{C}$ .

**OK... maybe this is just ChatGPT being bad at solving this specific problem, lets check out how other AI tools solve this problem:**

**Answer 1.0.1****GPT-4o Solution**

To determine the heat lost by a liter of vapor when its temperature decreases from 120 °C to 80 °C, you can use the specific heat capacity of the vapor. The heat loss ( $Q$ ) can be calculated using the formula  $Q = m \cdot c \cdot \Delta T$ , where  $m$  is the mass of the vapor,  $c$  is the specific heat capacity, and  $\Delta T$  is the change in temperature.

For water vapor, the specific heat capacity is approximately 1.996 kJ/(kg °C). Since 1 liter of water vapor has a mass of about 1 kg (**Comment: this is wrong, this is the mass for liquid water, steam is three orders of magnitude lighter**), the calculation would be:

$$\Delta T = 120\text{ °C} - 80\text{ °C} = 40\text{ K} \qquad Q = 1\text{ kg} \times 1.996\text{ kJ}/(\text{kg °C}) \times 40\text{ K} = 79.84\text{ kJ} \qquad (1.1)$$

Thus, the vapor loses approximately 79.84 kJ of heat.

**Answer 1.0.1****Deepseek-V3 Solution**

To determine how much heat a liter of water vapor loses when its temperature decreases from 120 °C to 80 °C at **atmospheric pressure**, we can follow these steps:

1. **Determine the Mass of the Vapor** At atmospheric pressure (1 atm), water vapor behaves approximately as an ideal gas. Using the **ideal gas law**:

$$PV = nRT$$

Where:

- ▶  $P = 1\text{ atm}$
- ▶  $V = 1\text{ L}$
- ▶  $R = 0.0821\text{ L atm}/(\text{K mol})$
- ▶  $T = 120\text{ °C} = 393.15\text{ K}$

Solve for  $n$  (number of moles):

$$n = \frac{PV}{RT} = \frac{(1\text{ atm})(1\text{ L})}{(0.0821\text{ L atm}/(\text{K mol}))(393.15\text{ K})} \approx 0.031\text{ mol}$$

The **molar mass of water (H<sub>2</sub>O)** is 18 g/mol, so the mass  $m$  is:

$$m = n \times \text{molar mass} = 0.031\text{ mol} \times 18\text{ g/mol}, \approx 0.558\text{ g}$$

**(Comment: Deepseek correctly calculated the mass of the vapor!)**

2. **Calculate the Heat Loss** The heat lost ( $Q$ ) when the vapor cools can be calculated using the **specific heat capacity of water vapor at constant pressure ( $c_p$ )**, which is approximately 1.996 J/(g °C), or 1.996 kJ/(kg °C)

The temperature change ( $\Delta T$ ) is:

$$\Delta T = 120\text{ °C} - 80\text{ °C} = 40\text{ °C (or 40 K)}$$

Now, compute the heat loss:

$$Q = m \cdot c_p \cdot \Delta T$$

Convert mass to kg for consistency with  $c_p$ :

$$m = 0.558 \text{ g} = 0.000558 \text{ kg}$$

$$Q = (0.000558 \text{ kg}) \times (1.996 \text{ kJ}/(\text{kg K})) \times (40 \text{ K})$$

$$Q \approx 0.0446 \text{ kJ} = 44.6 \text{ J}$$

### 3. Final Answer

The **heat lost** by the vapor is approximately:

$$\boxed{44.6 \text{ J}}$$

**(Comment: Deepseek also failed at taking into account latent heat for phase changes)**

Summarizing the answers we have:

**Table 1.1:** Proposed Solutions to the problem

AI tool	Answer
ChatGPT	47 904 J
GPT-4o	79 840 J
Deepseek-V3	44.6 J
Correct Answer	1346 J

We may easily conclude that using IA tools for problem-solving exercises in the discipline of Thermodynamics is, in the current state-of-the-art, a very risky proposition!

## 1.1 Week 1

### 1.1.1 Exercises

**Note:** This series of exercises is different from the following ones as they mostly aim at revising basic concepts that will be useful for the study of the discipline of Statistical Physics and Thermodynamics. For some of those, the theoretical background has not been introduced yet but will be briefly outlined here.

#### Differentials and Integrals

##### Exercise 1.1.1

Solve the following integrals and differentials:

$$\text{a) } \int \frac{a}{x^b} dx, \quad b > 1 \quad \text{b) } \int \frac{a}{x^b} dx, \quad b = 1 \quad \text{c) } \frac{\partial [\ln(x^2 + y^2)]}{\partial x} \quad \text{d) } \frac{\partial [\cos(xy)]}{\partial y}$$

#### Units

##### Exercise 1.1.2

A thunderstorm cloud has an electric potential of  $1 \times 10^7$  V compared to the ground, and discharges through a lightning which lasts 0.2 s, with an electrical current of  $1 \times 10^3$  A. If all the electrical power of the lightning is converted into thermal energy, what is the amount of heat released into the atmosphere?

##### Exercise 1.1.3

On average, every house in Portugal spends 12.96 GJ of electric energy per year (2017 values). Calculate each home average daily power consumption in kW h.

##### Exercise 1.1.4

What is the power of an air conditioner with a power of 8000 BTU/h for cooling and 14 000 BTU/h for heating?

Note: The unit energy *British Thermal Unit* has the following conversion: 1 BTU=1055.055 J.

#### Pressure and Force

##### Exercise 1.1.5

A person weighing 80 kg has a footprint of  $450 \text{ cm}^2$ . Determine the pressure this person exerts on the ground if

- he/she is standing on both feet;
- he/she is leaning in on foot.



##### Exercise 1.1.6

The same 80 kg person with a footprint of  $450 \text{ cm}^2$  wishes to walk on snow, however snow doesn't stand pressures above 0.5 kPa. Determine the minimum area of snowshoes necessary (area per shoe) so that this person may walk over the snow without sinking.



**Exercise 1.1.7** Earth's gravity field attracts and retains Earth's atmosphere, creating a surface pressure large enough to support a 76 cm high mercury column.

(Note: This is the barometer principle, and the basis for the pressure unit system of mmHg.)

- a) The pressure which corresponds to a 76 cm high mercury column with a base area of  $1 \text{ cm}^2$  is designated as 1 atmosphere (1 atm). How many Pascal correspond to 1 atm? And how many  $\text{kgf} \cdot \text{m}^{-2}$ ?

(Note: Kilogram-force (kgf) is an unit defined as the force exerted by 1 kg mass subject to a given gravity. When converting 1 kgf to N the obtained value is numerically identical to the local gravity.)

- b) Neglecting the variation of the the gravity force as a function of height, obtain an estimate for the overall mass of Earth's atmosphere  $M_{\text{atm}}$ .

(Note: Consider Earth's radius as  $R_T = 6400 \text{ km}$ , the density of Mercury as  $\rho_{\text{Hg}} = 13.6 \text{ g/cm}^3$  and the molecular mass of air as approximately  $29 \text{ g/mol}$ .)

### Ideal Gas Law

The Ideal Gas Law ( $pV = nRT$ ) will only be introduced and conceptualized in Chapter 3 of the theory book. The exercises below only require the application of the relationship to correlate pressure, temperature and volume, and do not require any deep knowledge on the topic.

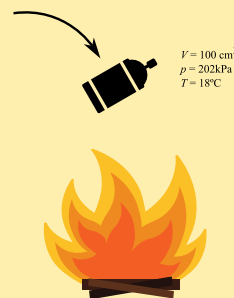
#### Exercise 1.1.8

A container in the shape of a cube 7.5 cm on each edge contains air (with equivalent molar mass  $28.96 \text{ g/mol}$ ) at atmospheric pressure and temperature 300 K. Find

- the mass of the gas,
- the gravitational force exerted on it, and
- the force it exerts on each face of the cube.
- Why does such a small sample exert such a great force?

#### Exercise 1.1.9

A spray can contains a gas at twice the atmospheric pressure (202 kPa), within a volume of  $100 \text{ cm}^3$  at a temperature of  $18^\circ\text{C}$ . The can is thrown into a fire (never do this, its dangerous). Once the temperature inside the can reaches  $205^\circ\text{C}$ , what is the pressure inside the can? Assume that the can retains its original volume.



#### Exercise 1.1.10

At 25 m below the surface of the sea, where the temperature is  $5^\circ\text{C}$ , a diver exhales an air bubble having a volume of  $1.0 \text{ cm}^3$ . If the surface temperature of the sea is  $22^\circ\text{C}$ , what is the volume of the bubble just before it breaks the surface?



## Energy

Once again, the concept of heat capacity and energy conservation will only be fully introduced in Chapter 4 of Volume 1, however it is already used for the problem solving exercises below, again, in a very simplified fashion.

### Exercise 1.1.11

How long does a 2 kW electrical water heater takes to heat 1 L of water from 20 °C to 60 °C, knowing that for heating 1 L of water by 1 °C you need to deliver 4.184 kJ of energy.

### Exercise 1.1.12

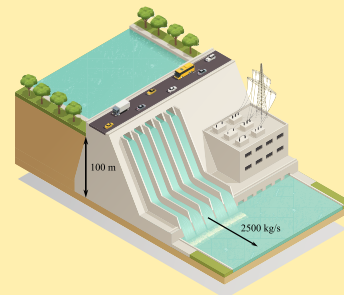
The highest waterfall in the world is the Salto del Angel in Venezuela. Its longest single falls has a height of 807 m. If water at the top of the falls is at 15 °C, what is the maximum temperature of the water at the bottom of the falls? Assume all the kinetic energy of the water as it reaches the bottom goes into raising its temperature.

Note: For heating 1 L of water by 1 °C you need to deliver 4.184 kJ of energy.



### Exercise 1.1.13

A turbine–hydraulic generator setup is used to generate electric power 100 m below the surface of a large water dam, ensuring a continuous energy flux of 2500 kg/s water. Determine the maximum electric power generation potential of this hydroelectric central.



## Formulae

### Constants

$$\begin{aligned}
 1 \text{ eV} &= 1.602 \times 10^{-19} \text{ J} \\
 k_B &= 1.381 \times 10^{-23} \text{ m}^2/(\text{kg s}^2 \text{ K}) \text{ or J/K} \\
 R &= 8.314 \text{ J}/(\text{mol K}) \\
 N_A &= 6.022 \times 10^{23} / \text{mol} \\
 P_{atm} &= 101.325 \text{ kPa} \\
 T_{amb} &= 300 \text{ K} \\
 T(\text{K}) &= 273.15 + T(^{\circ}\text{C}) \\
 g &= 9.81 \text{ m/s}^2
 \end{aligned}$$

### Formulas

**Mechanical kinetic energy:**  $K = \frac{1}{2}mv^2$

**Gravitic potential energy:**  $U = mgh$

**Average kinetic energy:**  $K = \frac{3}{2}k_bT$

**Impulsion force:**  $I = \rho g V_{moved}$

**Ideal gas equation:**  $PV = nRT$

**Specific Heat Capacity:**  $Q = mc\Delta T$

**Latent Heat:**  $Q = m\lambda$

## 1.1.2 Solutions to Exercises

### Answer 1.1.1

- a) We use the general integration rule:  $F(x) = \int_a^x f(t)dt = \frac{ax^{-b+1}}{-b+1} + C$
- b) Particular case for  $b = 1$ :  $F(x) = \int_a^x f(t)dt = a \ln|x| + C$
- c) We apply the chain rule:  $\frac{\partial [\ln(x^2 + y^2)]}{\partial x} = \frac{1}{x^2 + y^2} \frac{\partial}{\partial x} (x^2 + y^2) = \frac{2x}{x^2 + y^2}$
- d) We apply the chain rule:  $\frac{\partial [\cos(xy)]}{\partial y} = -\sin(xy) \frac{\partial}{\partial y} (xy) = -\sin(xy)x$

### Answer 1.1.2

$$\begin{aligned}\frac{dE}{dt} &= P = V \cdot I \\ E &= \int_{t_i}^{t_f} V \cdot I dt = V \cdot I \Delta t \\ E &= 10^7 \cdot 10^3 \cdot 0.2 = 2 \times 10^9 \text{ J}\end{aligned}$$

### Answer 1.1.3

$$1 \text{ W} = 1 \text{ J/s}; \quad 1 \text{ kWh} = 3600 \times 10^3 \text{ J} \Leftrightarrow 1 \text{ J} = 0.277 \times 10^{-6} \text{ kWh}.$$

$$12 \text{ GJ/yr} = \frac{12 \times 10^9}{365} [\text{J/d}] \times 0.277 \times 10^{-6} [\text{kWh/J}] = \frac{12 \times 10^9 \times 0.277 \times 10^{-6}}{365} [\text{kWh/d}] = 9.1 [\text{kWh/d}].$$

### Answer 1.1.4

To calculate the power in kW, we need to convert BTU  $\leftrightarrow$  J and h  $\leftrightarrow$  s.  $1 \text{ BTU} = 1055.055 \text{ J}$ .

- a) Cooling:  $8000 \text{ BTU/h} = \frac{8000 \times 1055.055 [\text{J}]}{3600 [\text{s}]} = 2.34 \times 10^3 \text{ W} = 2.34 \text{ kW}$
- b) Heating:  $14000 \text{ BTU/h} = \frac{14000 \times 1055.055 [\text{J}]}{3600 [\text{s}]} = 4.10 \times 10^3 \text{ W} = 4.10 \text{ kW}$

### Answer 1.1.5

Pressure=Force/Area, Gravitational force  $F_g = mg$ .

The person exerts a force over both feet or a single foot area:

$$\begin{aligned}\blacktriangleright p &= F_g/A = \frac{80 [\text{kg}] \cdot 9.81 [\text{m/s}^2]}{2 \cdot 450 \times 10^{-4} [\text{m}^2]} = 8720 \text{ Pa} \quad (\text{both feet}). \\ \blacktriangleright p &= F_g/A = \frac{80 [\text{kg}] \cdot 9.81 [\text{m/s}^2]}{1 \cdot 450 \times 10^{-4} [\text{m}^2]} = 17440 \text{ Pa} \quad (\text{single foot}).\end{aligned}$$

### Answer 1.1.6

Pressure=Force/Area, Gravitational force  $F_g = mg$ .

Area=Force/Maximum pressure

$A = \frac{80 \text{ [kg]} \cdot 9.81 \text{ [m/s}^2\text{]}}{500 \text{ [Pa]}} = 1.5696 \text{ m}^2$  of total area for both shoes,  $0.785 \text{ m}^2$  per shoe.  
The person needs to wear “snow rackets/snowshoes”.

**Answer 1.1.7**

a)  $1 \text{ Pa} = 1 \text{ N/m}^2$

$$\rho_{\text{Hg}} = 13.6 \text{ g/cm}^3 \Leftrightarrow \rho_{\text{Hg}} V_{\text{Hg}} = m_{\text{Hg}}.$$

$$m_{\text{Hg}} = 13.6 \times 76 = 1033.6 \text{ g} = 1.0336 \text{ kg}$$

$$F_g = m_{\text{Hg}} \times g = 1.0336 \times 9.81 \approx 10.13 \text{ N}$$

Therefore the pressure  $p$  exerted over a column of Hg is:

$$p = \frac{F_g}{A} = \frac{10.13 \text{ N}}{1 \text{ cm}^2} = 1.013 \times 10^5 \text{ N/m}^2 = 1.013 \times 10^5 \text{ Pa} = 1 \text{ atm}$$

$$1 \text{ kgf} = 9.81 \text{ N, therefore } p = \frac{1.013 \times 10^5}{9.81} \frac{\text{kgf}}{\text{m}^2} \approx 1.034 \times 10^4 \frac{\text{kgf}}{\text{m}^2}$$

b) For each  $\text{m}^2$ , the atmosphere exerts  $1.013 \times 10^5 \text{ N}$  of force. Earth has a total area  $A_T = 4\pi R_T^2 \approx 5.147 \times 10^{14} \text{ m}^2$ . Therefore the total force that the atmosphere exerts on Earth is  $F_{\text{total}} = A_T \times p = 5.214 \times 10^{19} \text{ N}$  such that  $F_{\text{total}} = M_{\text{atm}} \times g \Leftrightarrow M_{\text{atm}} \approx 5.32 \times 10^{18} \text{ kg}$ .

**Answer 1.1.8**

a) We know the gas is at  $T = 300 \text{ K}$  and  $p = 1 \text{ atm} = 101\,325 \text{ Pa}$ . We determine the volume of the cube  $V = 0.075^3 \text{ m}^3$  and we use the ideal gas law to determine the number of moles:

$$pV = nRT \Rightarrow n = \frac{pV}{RT} = \frac{101325 \times 0.075^3}{300 \times 8.31447} = 0.0171 \text{ mol}$$

We now may determine the air mass using the molar mass:

$$m = 28.96 \text{ g/mol} \cdot 0.017 \text{ mol} = 0.496 \text{ g}$$

b) The gravitational force  $F_g = mg = 0.496 \times 10^{-3} \cdot 9.81 = 4.8687 \times 10^{-3} \text{ N}$

c) We may also determine pressure  $p$  through the following relationship:

$$F = p \times A \rightarrow F = 101325 \times 0.075^2 \Leftrightarrow F = 569.95 \text{ N}$$

d) Although the force exerted by air pressure on the cube is way above the gravitational force, this force is also in equilibrium with the pressure force exerted on the cube exterior, assuming the exterior is also at atmospheric pressure. Therefore the resulting force is  $F = 0$ .

**Answer 1.1.9**

Assuming the gas is ideal:

$$pV = nRT$$

As the spray can remains closed, then  $V$  and  $n$  remain constant. We then may conclude that:

$$\frac{p}{T} = \text{Const.} \Rightarrow \frac{p_1}{T_1} = \frac{p_2}{T_2}$$

Using this relationship we may determine  $p_2$ :

$$p_2 = \frac{p_1}{T_1} T_2 = 2 \times 101325 \times \frac{(205 + 273.15)}{(18 + 273.15)} \Leftrightarrow p_2 = 331.74 \text{ kPa}$$

**Answer 1.1.10**

Initially, the bubble is at a depth  $h = 25$  m with a temperature  $T_1 = 5^\circ\text{C}$ , volume  $V_1 = 1.0\text{ cm}^3$  and pressure:

$$p_1 = p_0 + \rho gh = 3.54 \times 10^5 \text{ Pa}$$

When the bubble hits the sea surface, the depth is zero, the temperature is  $T_2 = 22^\circ\text{C}$ , and the pressure  $p_2$  is equal to the atmospheric pressure  $p_0$ . As the number of moles of air inside the bubble is constant, the final bubble volume  $V_2$  may be determined considering the pressure  $p$  and temperature  $T$  shifts, just like we did for solving the previous exercise:

$$\frac{p_1 V_1}{T_1} = nR = \frac{p_2 V_2}{T_2} \Rightarrow V_2 = \frac{p_1 V_1 T_2}{T_1 p_2}$$

Substituting yields:

$$V_2 = \frac{p_1 V_1 T_2}{T_1 p_2} = 3.71 \text{ cm}^3$$

**Answer 1.1.11**

Determining the time required for the boiler to heat the water requires firstly determining the energy (heat) required to heat 1 L (equivalent to 1 kg) of water from  $20^\circ\text{C}$  to  $60^\circ\text{C}$ :

$$Q = mc\Delta T = 1 \times (60 - 20) \times 4.184 = 167.36 \text{ kJ}$$

Knowing the necessary energy allows the determination of the required time, accounting for the boiler power (2 kW):

$$t = \frac{Q}{P} = \frac{167.36 \text{ kJ}}{2 \text{ kW}} \approx 84 \text{ s}$$

**Answer 1.1.12**

First, we need to calculate the potential gravitational energy  $E_p$  which is converted into heat as the water descends a height of  $h$ :

$$E_p = mgh = m[\text{kg}] \times 9.81 [\text{m/s}^2] \times 807 [\text{m}] = 7916m [\text{J}]$$

Now, we know that the necessary energy for raising the temperature by  $1^\circ\text{C}$  is  $c = 4.184 \text{ kJ}$  (or  $4184 \text{ J}$ ). We may calculate the temperature variation  $\Delta T$  using:

$$\Delta T = \frac{E_p}{mc} = \frac{m \times 7916.67}{m \times 4184} = 1.89^\circ\text{C}$$

As such, the maximum temperature of the water in the bottom of the waterfall will be:

$$T_{\text{final}} = T_{\text{initial}} + \Delta T = 15 + 1.89 = 16.89^\circ\text{C}$$

The maximum water temperature is approximately  $16.9^\circ\text{C}$ .

**Answer 1.1.13**

The potential energy per kilogram is:

$$\frac{E_p}{m} = \frac{mgh}{m} = \frac{m[\text{kg}] \times 9.81 [\text{m/s}^2] \times 100 [\text{m}]}{m[\text{kg}]} = 0.981 [\text{kJ/kg}]$$

(considering  $[\text{J}] = [\text{kgm}^2/\text{s}^2]$ )

Knowing that the maximum mass flux is  $\dot{m}_{max} = 2500 \text{ kg/s}$  we have:

$$W_{max} = \dot{m}_{max} \frac{E_p}{m} = 2500 [\text{kg/s}] \times 0.981 [\text{kJ/kg}] = 2453 [\text{kW}]$$

## 1.2 Week 2

### 1.2.1 Exercises

#### Exercise 1.2.1

The world's power consumption is currently about 13 TW and growing! (1 TW =  $10 \times 10^{12}$  W.) Burning one ton of crude oil (which is nearly seven barrels worth) produces about 42 GJ (1 GJ =  $1 \times 10^9$  J). If the world's total power needs were to come from burning oil (a large fraction currently does), how much oil would be burning per second?

#### Exercise 1.2.2

The probability for a microstate  $r$  to be populated is given by equation

$$P(\text{microstate } r) = \frac{\exp(-E_r/k_B T)}{\sum_i \exp(-E_i/k_B T)}. \quad (1.2)$$

Check that the probability in the equation above is normalized, so that the sum of all possible probabilities is one ( $P_{all} = 1$ ).

#### Exercise 1.2.3

Estimate  $k_B T$  at room temperature, and convert this energy into electronvolts (eV). Using this result, answer the following:

1. Would you expect hydrogen atoms to be ionized at room temperature (The binding energy of an electron in a hydrogen atom is 13.6 eV.)
2. Would you expect the rotational energy levels of diatomic molecules to be excited at room temperature? (It costs about  $1 \times 10^{-4}$  eV to promote such a system to an excited rotational energy level.)

#### Exercise 1.2.4

Calculate the rms (root mean square) speed of hydrogen ( $H_2$ ), helium (He) and oxygen ( $O_2$ ) at room temperature. [The atomic masses of H, He, and O are 1, 4, and 16 respectively.] Compare these speeds with the escape velocity on the surface of (i) the Earth, (ii) the Sun<sup>a</sup>.

Note: The escape speed at a distance  $d$  from the center of a spherically symmetric primary body (such as a star or a planet) with mass  $M$  is given by the formula

$$v_e = \sqrt{\frac{2GM}{r}},$$

where  $G = 6.67 \times 10^{-11} \text{ m}^3/(\text{kg s}^2)$ . The masses  $M$  and radius  $r$  for the Sun and Earth are, respectively:

- $M = 1.988 \times 10^{30} \text{ kg}$ ;  $r = 6.96 \times 10^8 \text{ km}$  for the Sun
- $M = 5.972 \times 10^{24} \text{ kg}$ ;  $r = 6371 \text{ km}$  for Earth.

Why aren't there any noticeable concentrations of  $H_2$  and He in Earth's atmosphere, unlike  $N_2$  and  $O_2$ , which are abundant on Earth?

Hints:

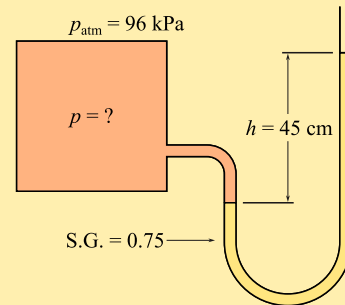
1. Use the proper SI values (metre, kilogram, second) for the calculations
2. Think about the overall molecule velocity distribution and not just the more probable speed
3. Think about the number of particle collisions per second (assume  $7 \times 10^9$  collisions/s for Earth, and  $510 \times 10^9$  for the Sun's surface), and the probability for having a speed higher than the escape speed

at the scale of Earth's age (billions of years).

<sup>a</sup> The Sun has a surface temperature  $T = 5,800\text{K}$ , but we will still take the room temperature value for this exercise, which will not be largely different since  $T$  is inside a square root;  $\sqrt{T} = 300 = 11.32$  and  $\sqrt{T} = 5800 = 76.16$

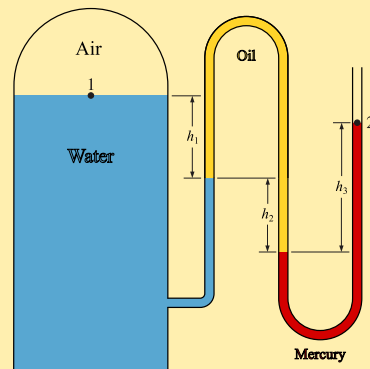
### Exercise 1.2.5

A manometer is used to measure the pressure in a tank. The fluid used has a specific gravity of 0.75 (The density of a fluid with a specific gravity value  $X$  is  $X$  multiplied by the density of water of  $1000\text{ kg/m}^3$ ), and the manometer column height is 45 cm, as shown in the figure. If the local atmospheric pressure is 96 kPa, determine the absolute pressure within the tank.



### Exercise 1.2.6

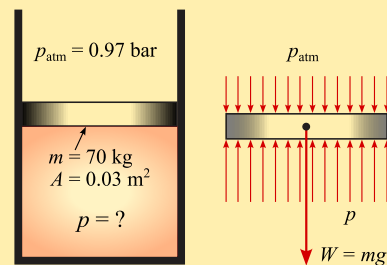
The water in a tank is pressurized by air, and the pressure is measured by a multifluid manometer as shown in the figure below. The tank is located on a mountain at an altitude of 1400 m where the atmospheric pressure is 85.6 kPa. Determine the air pressure in the tank if  $h_1 = 0.2\text{ m}$ ,  $h_2 = 0.3\text{ m}$ , and  $h_3 = 0.45\text{ m}$ . Take the densities of water, oil, and mercury to be  $1000\text{ kg/m}^3$ ,  $850\text{ kg/m}^3$ , and  $13,600\text{ kg/m}^3$ , respectively.



### Exercise 1.2.7

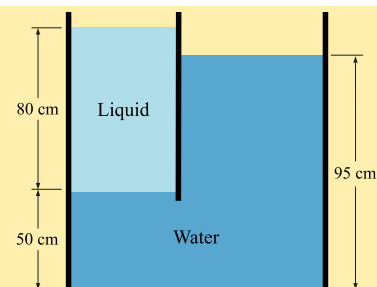
The piston of a vertical piston-cylinder device containing a gas has a mass of 70 kg and a cross-sectional area of  $0.03\text{ m}^2$ , as shown in the figure below. The local atmospheric pressure is 0.97 bar, and the gravitational acceleration is  $9.81\text{ m/s}^2$ .

1. Determine the pressure inside the cylinder
2. If some heat is transferred to the gas and its volume is doubled, do you expect the pressure inside the cylinder to change?



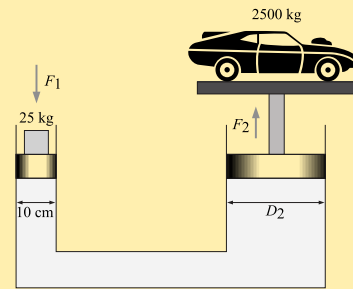
### Exercise 1.2.8

The top part of a water tank is divided into two compartments, as shown in the figure below. Now a fluid with an unknown density is poured into one side, and the water level rises a certain amount on the other side to compensate for this effect. Based on the final fluid heights shown on the figure, determine the density of the fluid added. Assume the liquid and water do not mix.

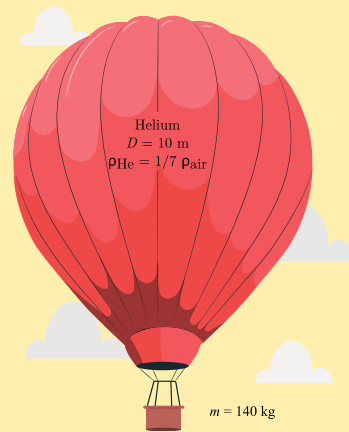


**Exercise 1.2.9**

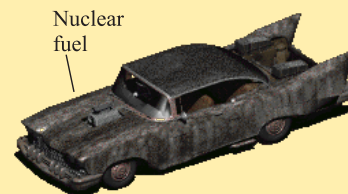
A hydraulic lift is to be used to lift an automobile of 2500 kg weight by putting a 25 kg weight on a piston with a diameter of 10 cm. Determine the diameter of the piston on which the automobile is to be placed.

**Exercise 1.2.10**

Balloons are often filled with helium gas because it weighs only about 1/7th of what air weighs under identical conditions. The buoyancy force, which can be expressed as  $F_b = \rho_{\text{air}} g V_{\text{balloon}}$ , will push the balloon upward. If the balloon has a diameter of 10 m and carries two people, 70 kg each, determine the acceleration of the balloon when it is first released. Assume the density of air is  $\rho = 1.16 \text{ kg/m}^3$ , and neglect the weight of the ropes and the cage.

**Exercise 1.2.11**

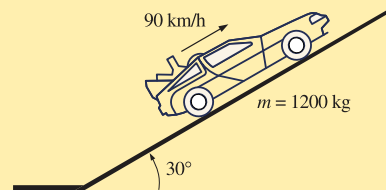
An average automobile consumes about 5 L of gasoline a day, and the capacity of the fuel tank of an automobile is about 50 L. Therefore an automobile needs to be refueled once every 10 days. Also the density of gasoline ranges from 0.68 to 0.78 kg/L, and its lower heating value is about 44 000 kJ/kg (that is, 44 000 kJ of heat is released when 1 kg of gasoline is completely burned). Suppose all the problems associated with the radioactivity and waste disposal of nuclear fuels are resolved, and an automobile is to be powered by U-235. If a new automobile comes equipped with 0.2 kg of nuclear fuel U-235, determine if this automobile will ever need refueling under average driving conditions. Assume that half (50%) of the U-235 fuel can be transmuted through fission, and that the heat release of this reaction is  $6.73 \times 10^{10} \text{ kJ/kg}$ .



**Exercise 1.2.12**

Consider a 1200 kg automobile cruising steadily on a level road at 90 km/h. Now the automobile starts climbing a hill that is sloped  $30^\circ$  from the horizontal (see figure below). If the velocity of the automobile is to remain constant during climbing, determine the additional power that must be delivered by the engine.

Note: Power (in W) corresponds to work (in J) divided by time (in s).

**Exercise 1.2.13**

Determine the necessary power for accelerating a 900 kg automobile from 100 to 150 km/h in 10 s on a perfectly horizontal road. Ignore the losses from friction, air drag, and rolling resistance.

Note: Power (in W) corresponds to work (in J) divided by time (in s).

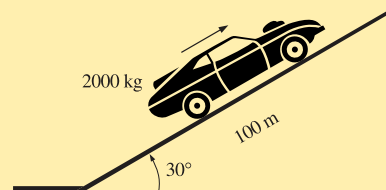
**Exercise 1.2.14**

Determine the power required for a 2000 kg automobile to climb a 100 m-long uphill road with a slope of  $30^\circ$  (from horizontal) in 10 s at

1. constant velocity
2. from rest to a final velocity of 30 m/s
3. from 35 m/s to a final velocity of 5 m/s

Disregard friction, air drag, and rolling resistance.

Note: Power (in W) corresponds to work (in J) divided by time (in s).



## Formulae

### Constants

$1 \text{ eV} = 1.602 \times 10^{-19} \text{ J}$   
 $k_B = 1.381 \times 10^{-23} \text{ m}^2/(\text{kg s}^2 \text{ K})$  or J/K  
 $R = 8.314 \text{ J}/(\text{mol K})$   
 $N_A = 6.022 \times 10^{23}/\text{mol}$   
 $P_{atm} = 101.325 \text{ kPa}$   
 $T_{amb} = 300 \text{ K}$   
 $T(\text{K}) = 273.15 + T(^{\circ}\text{C})$   
 $g = 9.81 \text{ m/s}^2$

### Formulas

**Mechanical kinetic energy:**  $K = \frac{1}{2}mv^2$   
**Gravitic potential energy:**  $U = mgh$   
**Average kinetic energy:**  $K = \frac{3}{2}k_bT$   
**Impulsion force:**  $I = \rho g V_{moved}$

**Specific Heat Capacity:**  $Q = mc\Delta T$

**Latent Heat:**  $Q = m\lambda$

### 1.2.2 Solutions to Exercises

#### Answer 1.2.1

$$13 \text{ TW} = 13 \times 10^{12} \text{ W}$$

$$42 \text{ GJ/Ton} = 42 \times 10^9 \text{ J/Ton de crude}$$

Therefore the crude oil consumption rate may be calculated as:

$$\dot{m}_{\text{crude}} = \frac{13 \times 10^{12} \text{ J/s}}{42 \times 10^9 \text{ J/Ton}} \approx 300 \text{ Ton/s}$$

#### Answer 1.2.2

If the probability for a microstate  $r$  is given by:

$$P(\text{micro } r) = \frac{\exp(-E_r/k_B T)}{\sum_i \exp(-E_i/k_B T)}$$

Then, the probability for all possible microstates is:

$$P(\text{all}) = P(R = 1) + P(R = 2) + P(R = 3) + \dots + P(R = i)$$

We may accordingly write:

$$P(\text{all}) = \frac{\sum_i \exp(-E_i/k_B T)}{\sum_i \exp(-E_i/k_B T)} = 1$$

Therefore the probability is properly normalized.

#### Answer 1.2.3

$$k_B T = 1.381 \times 10^{-23} \times 300 \text{ J}$$

$$1 \text{ eV} = 1.602 \times 10^{-19} \text{ J}$$

$$k_B T \approx 0.025 \text{ eV}$$

##### 1. Average energy for an atom/molecule at 300K

The average energy for an atom/molecule is given by:

$$\langle E \rangle = \frac{N}{2} k_B T \quad [\text{with } N = 3 \text{ for an hydrogen atom, or } N = 5 \text{ for a molecule}]$$

Then,

$$\langle E \rangle = \frac{N}{2} \times 0.025 \text{ eV} \ll 13.6 \text{ eV}$$

The average energy for an hydrogen atom ( $N=3$ ) is very inferior to the energy needed for ionizing this atom, say in a particle collision.

**Answer: NO.**

##### 2. Rotational levels excitation

For rotational levels excitation, we have:

$$\langle E \rangle = \frac{N}{2} \times 0.025 \text{ eV} > 1 \times 10^{-4} \text{ eV} \quad \text{with } N = 5 \text{ (diatomic molecule, H}_2\text{)}$$

The average energy of a molecule is superior to the energy needed to excite rotation.

**Answer: YES.**

#### Answer 1.2.4

The average quadratic speed of molecules is given by:

$$v_{\text{rms}} = \sqrt{\frac{3k_B T}{m}} = \begin{cases} 4.9 \text{ km/s} & \text{for H}_2 \\ 1.4 \text{ km/s} & \text{for He} \\ 0.48 \text{ km/s} & \text{for O}_2 \end{cases}$$

The escape velocity for a heavenly body is:

$$v_{\text{esc}} = \sqrt{\frac{2GM}{R}} = \begin{cases} 11.2 \text{ km/s} & \text{for Earth} \\ 618 \text{ km/s} & \text{for the Sun} \end{cases}$$

It is much more likely, by several orders of magnitude, that a light molecule such as H<sub>2</sub>, He, will reach a velocity above Earth's escape velocity (The student may for example apply the Maxwell distribution for an energy equivalent to 11.2 km/s and verify the corresponding velocity distribution functions for H<sub>2</sub>, He versus O<sub>2</sub>).

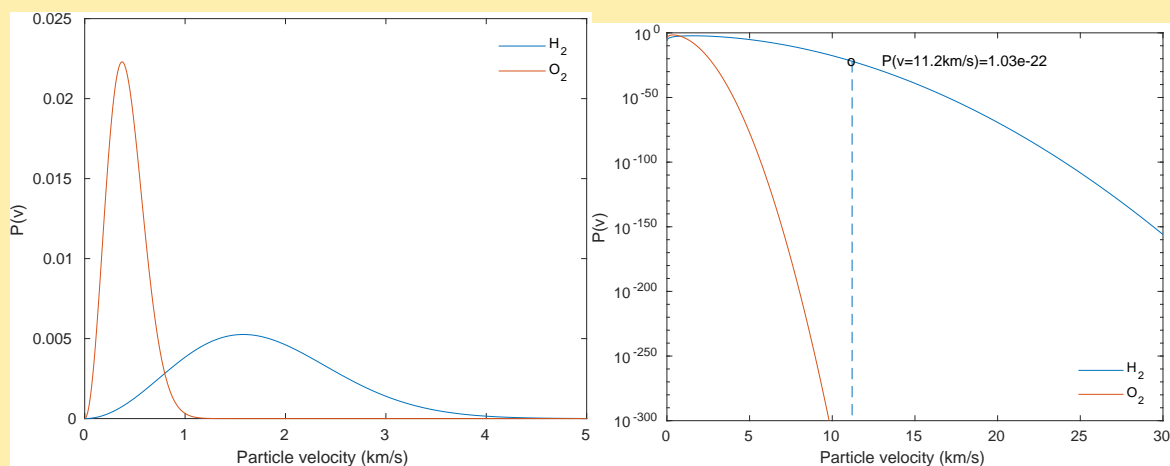
Although the probability is low for a single event, there are billions of collisions per second, which means that occasionally lighter molecules will exit the atmosphere, whereas heavier molecules, such as O<sub>2</sub>, won't be able to escape.

For timescales lasting billions of years, these occasional events build up and ultimately lead to the loss of light elements, such as H<sub>2</sub> or He, from Earth's atmosphere.

For the Sun, the gravitational force is much larger, and the probability for any molecule to escape, even the lighter ones, is essentially zero.

#### Extended comments to the exercise

Let us plot the Maxwell distribution as calculated for the masses of H<sub>2</sub> and O<sub>2</sub>, at a  $T = 300 \text{ K}$  temperature:



Velocity distribution function for H<sub>2</sub> and O<sub>2</sub> molecules. Left: linear  $y$  scale; Right: logarithmic  $y$  scale.

We find out that the probability of an H<sub>2</sub> molecule having a velocity  $v = 11.2 \text{ km/s}$  is  $P = 1.03 \times 10^{-22}$ . For the O<sub>2</sub> molecule, the probability  $P$  is below the underflow capabilities of the software used (Matlab), and we only know that  $P < 1 \times 10^{-300}$ .

Now the problem mentioned that the average number of collisions between molecules in room air ( $T = 300\text{ K}$ ) is around  $C_s = 7 \times 10^9$  collisions/s.

Let us now calculate how many collisions a molecule has experienced since the birth of Earth (around 4.5 Billion years):

$$C = 7 \times 10^9 \times 60 \times 60 \times 24 \times 365.25 \times 4.5 \times 10^9 = 9.94 \times 10^{26} \quad (1.3)$$

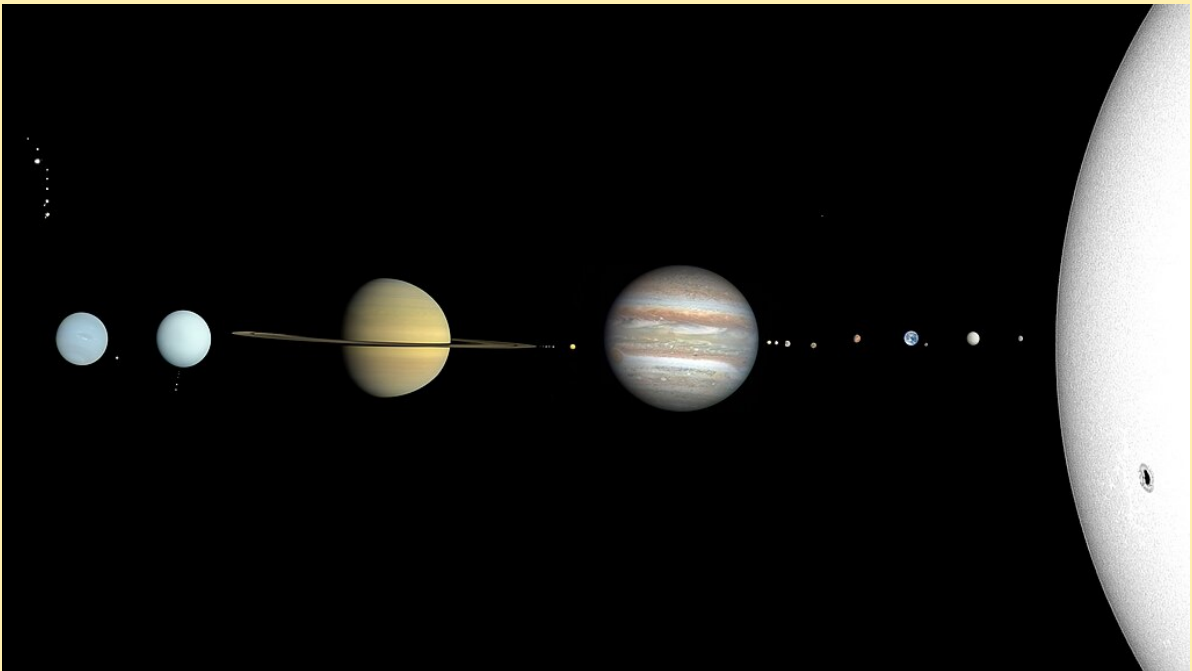
(we multiply  $C_s$  by seconds, minutes, hours, days, and years)

If an  $\text{H}_2$  molecule at the edge of the atmosphere has a probability  $P = 1.03 \times 10^{-22}$  of reaching an escape velocity  $v \geq 11.2\text{ km/s}$ , then over the whole age of the Earth, the probability will be (approximately):

$$P \times C = 1.03 \times 10^{-22} \times 9.94 \times 10^{26} > 1 \quad (1.4)$$

So the particle will have escaped the Earth a long time ago... That is not the case of a particle of  $\text{O}_2$  where  $P < 1 \times 10^{-300}$ , and the above calculation will give essentially an escape probability of zero for the heavier  $\text{O}_2$  molecule.

This is why the smallest planets in the Solar system are rocky planets with an atmosphere comprised of heavy elements only ( $\text{N}_2\text{--O}_2$  for Earth;  $\text{CO}_2\text{--N}_2$  for Venus and Mars; and  $\text{N}_2\text{--CH}_4$  for Titan, in this case a large satellite of Saturn), since they lost long time ago (or did not had to start with) lighter elements such as  $\text{H}_2$  and He. Heavier planets with larger gravity pull will have an atmosphere mostly composed of  $\text{H}_2$  and He: These are the Gas and Ice Giant planets Jupiter (escape velocity  $v_e = 60.20\text{ km/s}$ ); Saturn ( $v_e = 36.09\text{ km/s}$ ); Uranus ( $v_e = 21.38\text{ km/s}$ ) and Neptune ( $v_e = 23.56\text{ km/s}$ ).



True color picture of the Solar system. Sizes to scale. Distances not to scale.

**Answer 1.2.5**

The absolute pressure is given by:

$$p_{\text{absolute}} = p_{\text{atmospheric}} + \rho g h,$$

where:

$$\rho = SG \times \rho_{\text{water}} = 0.75 \times 1000 = 750 \text{ kg/m}^3$$

$$h = 45 \text{ cm} = 0.45 \text{ m}, \quad g = 9.81 \text{ m/s}^2$$

We substitute in the equation:

$$p_{\text{absolute}} = 96 \times 10^3 + 3318.38 = 99\,318.38 \text{ Pa}$$

Converting to kPa:

$$p_{\text{absolute}} = 99.32 \text{ kPa}$$

Therefore the absolute pressure inside the container is 99.32 kPa.

**Answer 1.2.6**

The pressure inside the container  $p_1$  may be determined through the following equation:

$$p_1 + \rho_{\text{water}} g h_1 + \rho_{\text{oil}} g h_2 - \rho_{\text{mercury}} g h_3 = p_{\text{atm}}$$

Rearranging for  $p_1$ :

$$p_1 = p_{\text{atm}} + g (\rho_{\text{mercury}} h_3 - \rho_{\text{water}} h_1 - \rho_{\text{oil}} h_2)$$

Substituting:

$$p_1 = 85.6 \text{ kPa} + 9.81 \text{ m/s}^2 [(13\,600 \text{ kg/m}^3)(0.45 \text{ m}) - (1000 \text{ kg/m}^3)(0.2 \text{ m}) - (850 \text{ kg/m}^3)(0.3 \text{ m})]$$

We calculate the terms inside the parenthesis:

$$p_1 = 85.6 \text{ kPa} + 9.81 \text{ m/s}^2 [6120 - 200 - 255] \text{ kg/m}^2$$

$$p_1 = 85.6 \text{ kPa} + 9.81 \times 5665 \text{ Pa}$$

$$p_1 = 85.6 \text{ kPa} + 55\,527.15 \text{ Pa}$$

$$p_1 = 85.6 \text{ kPa} + 55.53 \text{ kPa}$$

$$p_1 = 141.13 \text{ kPa}$$

Therefore, the pressure inside the container is:

$$p_1 \approx 141.1 \text{ kPa}$$

**Answer 1.2.7**

The pressure inside the cylinder may be calculated considering the balance of forces in the piston:

$$p_{\text{int}} \cdot A = p_{\text{atm}} \cdot A + m \cdot g$$

Rearranging for  $p_{\text{int}}$ :

$$p_{\text{int}} = p_{\text{atm}} + \frac{m \cdot g}{A}$$

Substituting by the values:

$$p_{\text{int}} = (0.97 \times 10^5) + \frac{70 \times 9.81}{0.03}$$

$$p_{\text{int}} = 97000 + 22870 = 119\,870 \text{ Pa}$$

and converting to bar:

$$p_{\text{int}} = 1.1987 \text{ bar}$$

Therefore, the pressure inside the cylinder is approximately 1.20 bar.

Since the pressure applied in the piston is still the sum of the pressure from the weight and atmospheric pressure, the pressure will remain identical.

### Answer 1.2.8

We know that the pressure on the bottom of both compartments should be identical owing to hydrostatic equilibrium. We use the equation for hydrostatic pressure  $P = \rho gh$ , where  $\rho$  is the fluid density,  $g$  is the gravitational acceleration, and  $h$  is the fluid column height.

In equilibrium:

$$\rho_1 gh_1 = \rho_2 gh_2$$

Cancelling the term  $g$  on both sides:

$$\rho_1 h_1 = \rho_2 h_2$$

Solving for  $\rho_2$ :

$$\rho_2 = \frac{\rho_1 h_1}{h_2}$$

and substituting for the known variables:

$$\rho_2 = \frac{1000 \text{ kg/m}^3 \times 0.45 \text{ m}}{0.8 \text{ m}} = 562.5 \text{ kg/m}^3$$

Therefore the density of the unknown fluid is:  $\rho_2 = 562.5 \text{ kg/m}^3$ .

### Answer 1.2.9

We know that the pressure in both pistons should be identical, owing to the Pascal principle. Pressure  $p$  is given by:

$$p = \frac{F_1}{A_1} = \frac{F_2}{A_2}$$

where  $F_1$  is the force on the small piston and  $F_2$  is the force on the large piston. The force  $F$  is determined according to  $F = m \cdot g$ , with  $g = 9.81 \text{ m/s}^2$ .

**Step 1:** We calculate the forces:

$$F_1 = 25 \text{ kg} \times 9.81 \text{ m/s}^2 = 245.25 \text{ N}$$

$$F_2 = 2500 \text{ kg} \times 9.81 \text{ m/s}^2 = 24\,525 \text{ N}$$

**Step 2:** The area for the small piston is:

$$A_1 = \frac{\pi d_1^2}{4} = \frac{\pi(0.10 \text{ m})^2}{4} = 0.007854 \text{ m}^2$$

**Step 3:** The area for the large piston is:

$$A_2 = \frac{F_2 A_1}{F_1} = \frac{24525 \times 0.007854}{245.25} = 0.7854 \text{ m}^2$$

**Step 4:** The diameter for the large piston is:

$$A_2 = \frac{\pi d_2^2}{4} \Rightarrow d_2 = \sqrt{\frac{4A_2}{\pi}} = \sqrt{\frac{4 \times 0.7854}{\pi}} = 1.0 \text{ m}$$

Accordingly, the diameter for the large piston is  $d_2 = 1.0 \text{ m}$ .

#### Answer 1.2.10

The impulsion force may be obtained through the formula:

$$F_I = \rho_{\text{air}} g V_{\text{balloon}}$$

where  $\rho_{\text{air}} = 1.16 \text{ kg/m}^3$ ,  $g = 9.81 \text{ m/s}^2$  and the balloon volume is given by:

$$V_{\text{balloon}} = \frac{4}{3} \pi r^3 = \frac{4}{3} \pi \left(\frac{d}{2}\right)^3 = 523.6 \text{ m}^3$$

Therefore the impulsion force is:

$$F_I = 1.16 \times 9.81 \times 523.6 = 5961.4 \text{ N}$$

The force for Helium weight is:

$$F_{\text{He}} = \rho_{\text{He}} \cdot V_{\text{balloon}} \cdot g = 0.1657 \times 523.6 \times 9.81 = 850.81 \text{ N}$$

The force for the weight of two persons is:

$$F_{\text{people}} = 2 \times 70 \times 9.81 = 1373.4 \text{ N}$$

The total downward force is:

$$F_{\text{total}} = 850.81 + 1373.4 = 2224.21 \text{ N}$$

The acceleration of the balloon is given by:

$$a = \frac{F_I - F_{\text{total}}}{m_{\text{total}}} = \frac{5961.4 - 2224.21}{226.72} = 16.48 \text{ m/s}^2$$

Therefore, the acceleration of the balloon is  $16.48 \text{ m/s}^2$ .

#### Answer 1.2.11

The energy generated by the daily consumption of gasoline may be calculated from its rate of consumption and its heat capacity. Assuming the average density of gasoline to be  $0.69 \text{ kg/L}$ , the daily consumption of gasoline is:

$$m_{\text{gasoline}} = 5 \text{ L/day} \times 0.69 \text{ kg/L} = 3.45 \text{ kg/day}$$

The daily energy generated by the burning of oil is:

$$E_{\text{gasoline}} = m_{\text{gasoline}} \times \text{Heat capacity} = 3.45 \text{ kg/day} \times 44\,000 \text{ kJ/kg} = 151\,800 \text{ kJ/day}$$

Concurrently, the total energy generated by the fission of Uranium-235 is:

$$E_{\text{U-235}} = \frac{1}{2} \times 0.2 \text{ kg} \times 6.73 \times 10^{10} \text{ kJ/kg} = 1.346 \times 10^{10} \text{ kJ}$$

The number of days the automobile will be able to function with 0.2 kg of U-235 is:

$$\text{Days} = \frac{E_{\text{U-235}}}{E_{\text{gasoline}}} = \frac{1.346 \times 10^{10} \text{ kJ}}{151800 \text{ kJ/day}} = 88666 \text{ days}$$

Converting days into years:

$$\text{Years} = \frac{88666}{365} \approx 121 \text{ years}$$

Accordingly, the automobile may operate approximately during 121 years without needing refueling.

### Answer 1.2.12

The necessary force to overcome gravity during the climb is:

$$F = m \cdot g \cdot \sin(\theta)$$

Substituting:

$$F = 1200 \times 9.81 \times \sin(30^\circ) = 1200 \times 9.81 \times 0.5 = 5886 \text{ N}$$

The automobile velocity is:

$$v = \frac{90 \text{ km/h} \times 1000}{3600} = 25 \text{ m/s}$$

The additional required power is:

$$P = F \times v = 5886 \text{ N} \times 25 \text{ m/s} = 147150 \text{ W} = 147.15 \text{ kW}$$

Therefore the additional power provided by the engine to maintain constant velocity during the climb is 147.15 kW.

### Answer 1.2.13

Firstly we convert velocities from km/h to m/s:

$$v_i = \frac{100 \times 1000}{3600} = 27.78 \text{ m/s}, \quad v_f = \frac{150 \times 1000}{3600} = 41.67 \text{ m/s}$$

The initial and final kinetic energies are given by:

$$E_{c,i} = \frac{1}{2} \times 900 \times (27.78)^2 = 347222.22 \text{ J}$$

$$E_{c,f} = \frac{1}{2} \times 900 \times (41.67)^2 = 781250.00 \text{ J}$$

The work performed to accelerate the automobile is the difference between the final and initial kinetic energies:

$$W = E_{c,f} - E_{c,i} = 781250.00 - 347222.22 = 434027.78 \text{ J}$$

The power required to perform this work in 10 s is:

$$P = \frac{W}{t} = \frac{434027.78}{10} = 43402.78 \text{ W} = 43.40 \text{ kW}$$

Therefore the required power to accelerate the automobile is 43.40 kW.

**Answer 1.2.14****1. Constant Velocity**

The force exerted over the inclined path is:

$$F = m \cdot g \cdot \sin(\theta) = 2000 \times 9.81 \times \sin(30^\circ) = 9810 \text{ N}$$

The constant velocity is:

$$v = \frac{d}{t} = \frac{100}{10} = 10 \text{ m/s}$$

The necessary power is:

$$P = F \times v = 9810 \times 10 = 98\,100 \text{ W} = 98.1 \text{ kW}$$

**2. From rest to the final speed of 30 m/s**

Potential energy gained:

$$W_{\text{pot}} = m \cdot g \cdot d \cdot \sin(\theta) = 2000 \times 9.81 \times 100 \times 0.5 = 981\,000 \text{ J}$$

Final kinetic energy:

$$E_{c,f} = \frac{1}{2}mv_f^2 = \frac{1}{2} \times 2000 \times (30)^2 = 900\,000 \text{ J}$$

Total necessary work:

$$W_{\text{total}} = W_{\text{pot}} + E_{c,f} = 981\,000 + 900\,000 = 1\,881\,000 \text{ J}$$

Necessary power:

$$P = \frac{W_{\text{total}}}{t} = \frac{1\,881\,000}{10} = 188\,100 \text{ W} = 188.1 \text{ kW}$$

**3. From 35 m/s to the final velocity of 5 m/s**

Initial kinetic energy:

$$E_{c,i} = \frac{1}{2}mv_i^2 = \frac{1}{2} \times 2000 \times (35)^2 = 1\,225\,000 \text{ J}$$

Final kinetic energy:

$$E_{c,f} = \frac{1}{2}mv_f^2 = \frac{1}{2} \times 2000 \times (5)^2 = 25\,000 \text{ J}$$

Total necessary work:

$$W_{\text{total}} = (E_{c,f} - E_{c,i}) + W_{\text{pot}} = (25\,000 - 1\,225\,000) + 981\,000 = -219\,000 \text{ J}$$

Necessary power:

$$P = \frac{W_{\text{total}}}{t} = \frac{-219\,000}{10} = -21\,900 \text{ W} = -21.9 \text{ kW}$$

(We need to brake the car)

## 1.3 Week 3

### 1.3.1 Exercises

#### Exercise 1.3.1

What is the kinetic energy in electron-volts (eV) at the ambient temperature (300 K) of a molecule: (a) of  $N_2$ , (b) of  $O_2$ , (c) of CO and (d) of  $CO_2$ ?

[Hint: Do you need four separate calculations?]

#### Exercise 1.3.2

Estimate the number of molecules in an isothermal atmosphere as a function of altitude.

[Hint: Consider the potential energy of a molecule as a function of altitude and consider that all molecules follow a Boltzmann distribution]

#### Exercise 1.3.3

A “Ultra High Vacuum” (UHV) chamber is used to simulate the pressure at the surface of the Moon ( $1 \times 10^{-11}$  mbar). The chamber has a volume of 1000 L.

What is the number of moles of gas inside the chamber? Consider  $T = 300$  K.

#### Exercise 1.3.4

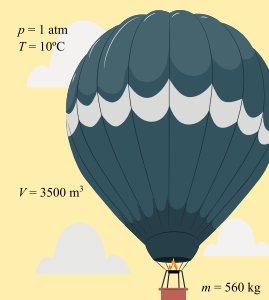
In a hot day with an outside temperature  $T = 33^\circ\text{C}$ , a driver verifies the pressure of its tires before the beginning of a trip and obtains the value of  $p = 29$  psi. At the end of the trip, the driver checks again the tires pressure and obtains  $p = 34$  psi. What is the temperature inside the tires at the end of the trip?

[Note: 1 psi (pound per square inch) = 6896.6 Pa]

#### Exercise 1.3.5

A hot air balloon with  $3500\text{ m}^3$  volume has an overall mass of 560 kg (balloon, cradle, passengers, and cargo). The ambient temperature and pressure are, respectively  $10^\circ\text{C}$  and 1 atm. At which temperature should the balloon air be brought so that it may take flight?

[Note: The density of dry air at a temperature of  $10^\circ\text{C}$  is  $1.25\text{ kg/m}^3$ , and the dry air effective mass is  $29\text{ g/mol}$ .]



#### Exercise 1.3.6

The length of the “25th April” bridge over the Tagus river in Lisbon, Portugal is about 2 km. If the yearly thermal amplitude is  $39^\circ$  (with typical minimum values of  $-1^\circ$  and maximum values of  $38^\circ$ ), what is the length variation whisthanded by the steel beams that sustain the driving lanes?

[Note: Steel linear expansion coefficient:  $\alpha = 1.27 \times 10^{-5}/^\circ\text{C}$ ]



### Exercise 1.3.7

The surface of a highway is built with 10 m long concrete plates. The highway was built when the average air temperature was  $10^\circ$ . If we assume that the temperature of concrete in a hot day may reach  $50^\circ$ , what is the necessary gap between contiguous plates?

[Note: Concrete linear expansion coefficient:  $\alpha = 1.2 \times 10^{-5} / ^\circ\text{C}$ ]



### Exercise 1.3.8

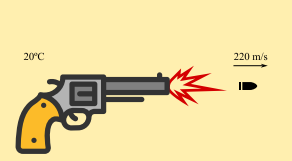
If we admit that all the potential energy of water is transformed in internal energy when water hits the ground, to which height should 10 g of water fall so that its temperature increases by  $0.5^\circ$ ? and for 100 g of water?

[Note: Specific heat of water:  $1 \text{ J/g}^\circ\text{C}$ .  $1 \text{ cal} = 4.184 \text{ J}$ ]

### Exercise 1.3.9

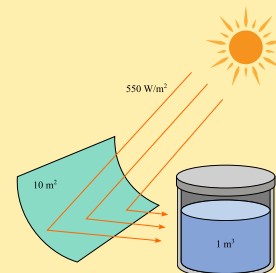
A cowboy shoots a lead bullet, which ends up stuck in a wooden wall. The bullet velocity immediately before impact is  $220 \text{ m/s}$ . Assuming that all the kinetic energy is spent to warm the bullet, calculate the final temperature of the bullet, assuming that the initial temperature is  $20^\circ\text{C}$ . Repeat the calculation for a silver bullet.

Note: consider the specific heat of lead to be  $128 \text{ J/(kg K)}$  and the specific heat of silver to be  $234 \text{ J/(kg K)}$ . The specific heat is the amount of energy that you need to raise the temperature of 1 kg of mass by 1 K.



### Exercise 1.3.10

A Solar collector with  $10 \text{ m}^2$  deflects Solar radiation towards  $1 \text{ m}^3$  of water. Knowing that Solar radiation has an intensity of  $550 \text{ W/m}^2$ , how long will it take to heat water from  $20^\circ$  to  $70^\circ$ ?

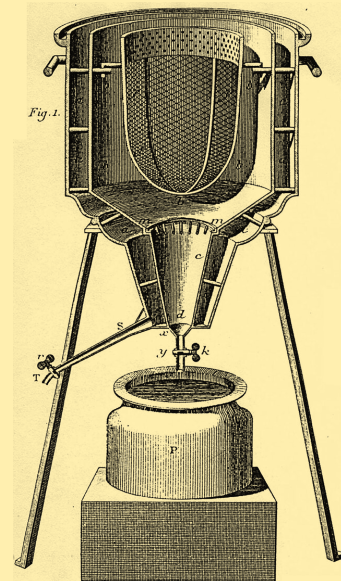


**Exercise 1.3.11**

A granite bloc of 44 g is kept during a certain amount of time in boiling water. Next, the bloc is immersed in a calorimeter of copper with mass 105 g, which contains 205 g of water at 25 °C. The equilibrium temperature is 28 °C. What is the specific heat of the metal?

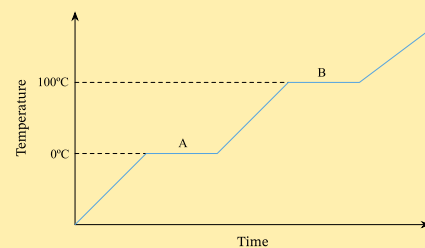
[Note: Specific heat of copper: 0.39 J/(g °C)]

[Historical note: This is the principle of the **Lavoisier–Laplace calorimeter** who was first devised in the winter of 1782–83, by *Antoine Lavoisier* and *Pierre–Simon Laplace*, to determine the heat involved in various chemical changes. These experiments mark the foundation of thermochemistry. The original calorimeter used ice instead of water, measuring the mass of melted ice dripping to a container and the corresponding (known) latent heat for the fusion of ice to determine the heat capacity of a substance.]

**Exercise 1.3.12**

The figure below represents the graphic of the temperature for a sample 2.3 kg of water as a function of time, in an experiment where water is uniformly heated. The heat source has a constant output of 3 kW. To how much time do the A and B steps correspond to?

[Note: Latent heat of fusion for ice: 333 kJ/kg  
Latent heat of vaporization for water: 2255 kJ/kg]

**Exercise 1.3.13**

We want to heat 0.2 kg of water from 20 °C to 50 °C. To do so, we add water vapor originally at 135 °C. The water is placed in a glass container with 0.1 kg of mass and a specific heat  $C = 837 \text{ J}/(\text{kg } ^\circ\text{C})$ . How much amount of water vapor (in g) is necessary to do so?

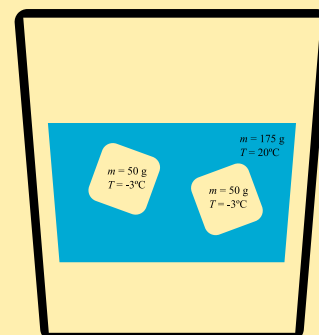
[Nota: Specific heat of liquid water: 4186 J/kg  
latent heat of vaporization for water: 2255 kJ/kg  
Specific heat of water vapor: 2100 J/kg]

**Exercise 1.3.14**

Two ice cubes of 50 g each are placed inside a glass with 175 g of water, initially at 20 °C. Ice has been removed from a freezer preset to −3 °C.

- What is the final temperature, when equilibrium is reached?
- Calculate the final temperature in case the same mass of water at 1 °C is added instead if ice.

[Note: Specific heat of water: 4186 J/(kg °C)  
Specific heat of ice: 2093 J/(kg °C)  
Latent heat of fusion for ice: 333 kJ/kg]



**Exercise 1.3.15**

Calculate air pressure for a temperature of 0 °C and a volume of 0.1 L, considering the air as an Ideal or van der Waals gas, either assuming i)  $n = 1$  mol; or ii)  $p = 1$  atm for the ideal gas.

[Note: Approximate air as  $N_2$ , assume the constants  $a = 1.35 \text{ L}^2 \text{ atm/mol}^2$  and  $b = 0.039 \text{ L/mol}$ , and use the appropriate units for  $R^a$ :  $R = 0.0821 \text{ L atm/(K mol)} = 8.314 \text{ J/(mol K)} \times \frac{1000}{101325} \text{ L atm/m}^3 \text{ Pa}$ ]

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<sup>a</sup> [J]=[Pa m<sup>3</sup>]

## Formulae

### Constants

$1\text{eV} = 1.602 \times 10^{-19} \text{ J}$   
 $k_B = 1.381 \times 10^{-23} \text{ m}^2/(\text{kg s}^2 \text{ K})$  or J/K  
 $R = 8.314 \text{ J/(mol K)}$   
 $N_A = 6.022 \times 10^{23}/\text{mol}$   
 $P_{atm} = 101.325 \text{ kPa}$   
 $T_{amb} = 300 \text{ K}$   
 $T(\text{K}) = 273.15 + T(^{\circ}\text{C})$

### Formulas

**Mechanical kinetic energy:**  $K = \frac{1}{2}mv^2$   
**Average kinetic energy:**  $K = \frac{3}{2}k_B T$   
**Impulsion force:**  $I = \rho g V_{moved}$   
**Ideal gas equation:**  $PV = nRT$   
**van der Waals gas equation:**  $\left(P + a \frac{n^2}{V^2}\right)(V - nb) = nRT$   
**Linear expansion:**  $\Delta L = L_0 \alpha \Delta T$   
**Area expansion:**  $\Delta A = A_0 2\alpha \Delta T$   
**Volume expansion:**  $\Delta V = V_0 3\alpha \Delta T$

**Specific Heat Capacity:**  $Q = mc\Delta T$

**Latent Heat:**  $Q = m\lambda$

### 1.3.2 Solutions to Exercises

#### Answer 1.3.1

We calculate the average molecular kinetic energy at room temperature ( $T=300$  K) according to:

$$\langle E_k \rangle = \frac{N}{2} k_B T$$

where  $k_B$  is the Boltzmann constant and  $N = 5$  for molecules at  $T=300$  K.

We calculate the average kinetic energy:

$$\langle E_k \rangle = \frac{5}{2} \times 8.617 \times 10^{-5} \text{ eV/K} \times 300 \text{ K}$$

$$\langle E_k \rangle = \frac{5}{2} \times 0.025851 \text{ eV}$$

$$\langle E_k \rangle \approx 0.0646 \text{ eV}$$

This average kinetic energy is the same for all the molecules, regardless of species ( $N_2$ ,  $O_2$ ,  $CO$ ,  $CO_2$ ), since it only depends on temperature. Therefore for all molecules:  $\langle E_k \rangle \approx 0.0646 \text{ eV}$

#### Answer 1.3.2

Assuming **constant temperature** and a gravitational potential in the shape of  $U = mgh$ , we may use Boltzmann equation which states:

$$p(E) \propto e^{-\frac{E}{k_B T}}$$

Substituting the energy  $E$  by the gravitational potential energy, we have:

$$p(h) \propto e^{-\frac{mgh}{k_B T}}$$

The, the number of molecules as a function of altitude is:

$$N(h) = N_0 e^{-\frac{mgh}{k_B T}}$$

Where  $N_0$  is the normalization factor.

#### Answer 1.3.3

We consider ideal gas state equation:

$$pV = nRT$$

The number of moles  $n$  is:

$$n = \frac{pV}{RT} = \frac{(10^{-11} \times 10^2) \times (1000 \times 10^{-3})}{8.314 \times 300} = 4 \times 10^{-13} \text{ mol}$$

#### Answer 1.3.4

We consider the ideal gas law:

$$\frac{p_1}{T_1} = \frac{p_2}{T_2}$$

Solving for  $T_2$ :

$$T_2 = T_1 \frac{p_2}{p_1}$$

We know the initial temperature  $T_1 = 33^\circ\text{C} = 306 \text{ K}$ ,  $p_1 = 29 \text{ psi}$  and  $p_2 = 34 \text{ psi}$ .

Substituting by the numerical values:

$$T_2 = 306 \times \frac{34}{29} = 359 \text{ K}$$

Converting to Celsius:

$$T_2 = 359 - 273 = 86^\circ \text{C}$$

### Answer 1.3.5

Given variables:

$$V = 3500 \text{ m}^3, \quad m_b = 560 \text{ kg}, \quad T_0 = 10^\circ \text{C}, \quad P_0 = 1 \text{ atm}, \\ \rho = 1.25 \text{ kg/m}^3, \quad M = 29 \text{ g/mol}, \quad R = 8.3143 \text{ J/(mol K)}$$

**Impulse:**

$$I = (\rho V)g = 1.25 \times 3500 \times 9.81 = 42\,918.75 \text{ N}$$

**Balloon total weight:**

$$F_g = (m_b + m'_a)g$$

where  $m'_a$  is the hot air mass inside the balloon.

For the balloon to rise, the gravitational force needs to be inferior to the impulse force:

$$F_g < I$$

$$(560 + m'_a)g < 42\,918.75 \quad \Rightarrow \quad m'_a < \frac{42\,918.75}{9.81} - 560 = 3815 \text{ kg}$$

**Air mass inside the balloon:**

The corresponding number of moles inside the balloon is:

$$n = \frac{m'_a}{M} = \frac{3815}{29 \times 10^{-3}} = 1.31 \times 10^5 \text{ mol}$$

**Pressure inside the balloon:**

As the balloon is open-ended on the bottom, the pressure inside the balloon is equal to the outside pressure:

$$P_0 V = nRT$$

$$101\,325 \times 3500 = 1.31 \times 10^5 \times 8.3143 \times T$$

Solving for  $T$ :

$$T = \frac{101\,325 \times 3500}{1.31 \times 10^5 \times 8.3143} = 325.6 \text{ K}$$

**Air temperature inside the balloon:**

Converting from Kelvin to Celsius:

$$T = 325.6 \text{ K} = 325.6 - 273.15 = 52.45^\circ \text{C}$$

Therefore, the air temperature inside the balloon should be  $52.45^\circ \text{C}$  for the balloon to rise up.

**Answer 1.3.6**

The length variation owing to the change of temperature is:

$$\Delta L = L_0 \alpha \Delta T$$

Substituting:

$$\Delta L = 2000 \times 1.27 \times 10^{-5} \times 39 \approx 0.99 \text{ m}$$

**Answer 1.3.7**

The necessary gap between adjoining plates, owing to thermal expansion, is given by:

$$\Delta L = L_0 \alpha \Delta T$$

Substituting:

$$\Delta L = 10 \times 1.2 \times 10^{-5} \times 40 = 0.48 \text{ cm}$$

Therefore, the necessary gap is approximately 0.48 cm.

**Answer 1.3.8**

Calculating the necessary height  $h$  for the water temperature to increase owing to the converted potential energy we use:

$$E_p = Q$$

where:

$$mgh = mc\Delta T$$

Solving for  $h$ :

$$h = \frac{c\Delta T}{g}$$

with

$$1 \text{ cal}/(\text{g } ^\circ\text{C}) = 4.186 \text{ J}/(\text{g } ^\circ\text{C}) = 4.186 \times 10^3 \text{ J}/(\text{kg K})$$

Substituting:

$$h = \frac{4.184^3 \times 0.5}{9.81} \approx 214 \text{ m}$$

Accordingly, the necessary height is approximately 214 m for any mass of water.

**Answer 1.3.9**

We apply the following formulas to determine post-impact temperature of the bullet:

1. **Kinetic Energy:**

$$E_k = \frac{1}{2}mv^2$$

2. **Temperature increase:**

$$\Delta T = \frac{E_k}{mc}$$

3. **Final temperature:**

$$T_f = T_i + \Delta T$$

For a lead bullet:

- ▶ Velocity  $v = 220 \text{ m/s}$ ,
- ▶ Specific heat  $c_{\text{Pb}} = 128 \text{ J}/(\text{kg K})$ ,
- ▶ Initial temperature  $T_i = 20 ^\circ\text{C}$ .

$$E_k = \frac{1}{2} \times 1 \times 220^2 = 24\,200 \text{ J}$$

$$\Delta T_{\text{Pb}} = \frac{24200}{128} \approx 189.06 \text{ K}$$

$$T_f = 20 + 189.06 \approx 209.06 \text{ }^\circ\text{C}$$

For a silver bullet:

- Specific heat  $c_{\text{Ag}} = 234 \text{ J/(kg K)}$ .

$$\Delta T_{\text{Ag}} = \frac{24200}{234} \approx 103.42 \text{ K}$$

$$T_f = 20 + 103.42 \approx 123.42 \text{ }^\circ\text{C}$$

### Answer 1.3.10

We calculate the necessary time to heat the water:

1. **Necessary energy:**

$$Q = mc\Delta T$$

$$Q = 1000 \times 4184 \times 50 = 209.2 \times 10^6 \text{ J}$$

2. **Collector total power:**

$$P = I \times A$$

$$P = 550 \times 10 = 5500 \text{ W}$$

3. **Necessary time:**

$$t = \frac{Q}{P} = \frac{209.2 \times 10^6}{5500} \approx 38\,036 \text{ s} \approx 10 \text{ h } 34 \text{ min}$$

### Answer 1.3.11

To find the specific heat of granite, we apply the heat conservation principle.

1. **Heat given by the granite:**

$$Q_m = m_m \cdot c_m \cdot (T_{m,i} - T_f)$$

2. **Heat received by the copper and water:**

For copper:

$$Q_c = m_c \cdot c_c \cdot (T_f - T_i)$$

For water:

$$Q_a = m_a \cdot c_a \cdot (T_f - T_i)$$

Total heat gained:

$$Q_{\text{total}} = Q_c + Q_a$$

3. **Equaling the gained and lost heat:**

$$m_m \cdot c_m \cdot (T_{m,i} - T_f) = Q_{\text{total}}$$

$$44 \cdot c_m \cdot 78 = (105 \cdot 0.39 \cdot -3) + (205 \cdot 4.186 \cdot -3)$$

$$44 \cdot c_m \cdot 78 = 122.85 + 2572.2$$

$$c_m = \frac{2693.5}{44 \cdot 78} \approx 0.785 \text{ J/(g } ^\circ\text{C)}$$

### Answer 1.3.12

Finding the corresponding times to the A and B plateaus:

#### 1. Plateau A (water fusion):

$$Q_1 = m \cdot \Delta_f = 2.3 \cdot 333000 = 765\,900 \text{ kJ}$$

$$t_A = \frac{Q_1}{P} = \frac{765900}{3000} = 255.3 \text{ s}$$

#### 2. Plateau A (water boiling):

$$Q_2 = m \cdot \Delta_{va} = 2.3 \cdot 2255000 = 5\,186\,500 \text{ J}$$

$$t_B = \frac{Q_2}{P} = \frac{5186500}{3000} = 1728.83 \text{ s}$$

### Answer 1.3.13

For heating 0.2 kg of water from 20 °C to 50 °C, we need to determine the necessary water vapor mass.

#### 1. Energy required to heat the water:

$$Q_w = m_w \cdot c_w \cdot (T_{w,f} - T_{w,i})$$

$$Q_w = 0.2 \cdot 4184 \cdot (50 - 20) = 25\,104 \text{ J}$$

#### 2. Energy required to heat the vessel:

$$Q_r = m_r \cdot C \cdot (T_{w,f} - T_{w,i})$$

$$Q_r = 0.1 \cdot 837 \cdot (50 - 20) = 2511 \text{ J}$$

#### 3. Total Energy required:

$$Q_{total} = Q_w + Q_r = 25104 + 2511 = 27\,615 \text{ J}$$

#### 4. Calculate the necessary water vapor mass:

Vapor cools from 135 °C to 100 °C, experiences transition to liquid phase at 100 °C and cools from 100 °C to 50 °C:

$$Q_v = m_v \cdot c_v \cdot (T_v - 100) + m_v \cdot \lambda_{v,l} + m_w \cdot (100 - T_{w,f})$$

$$27615 = m_v \cdot [2100 \cdot (135 - 100) + 2255 + 4186 \cdot (100 - 50)]$$

$$m_v = \frac{27615}{2464760.25} = 0.01088 \text{ kg} \approx 11 \text{ g}$$

**Answer 1.3.14**

a) **Final temperature with ice at  $-3^\circ\text{C}$**

1) **Energy for heating ice from  $-3^\circ\text{C}$  to  $0^\circ\text{C}$ :**

$$Q_1 = m_g \cdot c_g \cdot \Delta T = 2 \times 0.05 \cdot 2093 \cdot (0 - (-3)) = 627.9 \text{ J}$$

2) **Energy for fusing ice:**

$$Q_2 = m_g \cdot L_f = 2 \times 0.05 \cdot 333000 = 33300 \text{ J}$$

3) **Total energy released by the ice:**

$$Q_g = Q_1 + Q_2 = 627.9 + 33300 = 33927.9 \text{ J}$$

4) **Energy released by the water as it cools down to  $T = 0^\circ\text{C}$ :**

$$Q_a = m_a \cdot c_a \cdot (T_{a,i} - 0) = 0.175 \cdot 4186 \cdot 20 = 14718.5 \text{ J}$$

Since  $Q_g$  is higher than  $Q_a$ , the final temperature is  $T = 0^\circ\text{C}$ .

b) **Final temperature with water at initial temperature  $T = 1^\circ\text{C}$**

1) **Energy released by the water at  $T = 20^\circ\text{C}$ :**

$$Q_{20} = m_a \cdot c_a \cdot (T_{final} - T_{a,i})$$

2) **Energy absorbed by the water at  $T = 1^\circ\text{C}$ :**

$$Q_1 = m_{ad} \cdot c_a \cdot (T_{final} - T_{ad})$$

3) **Sum of energies:**

$$0.175 \cdot 4186 \cdot (T_{final} - 20) + 2 \times 0.05 \cdot 4186 \cdot (T_{final} - 1) = 0$$

Simplifying:

$$0.175T_{final} - 3.5 + 2 \times 0.05T_{final} - 2 \times 0.05 = 0$$

$$0.275T_{final} - 3.6 = 0$$

$$T_{final} = \frac{3.6}{0.275} \approx 13.1^\circ\text{C}$$

**Answer 1.3.15**

a) 1) **Ideal gas**

The ideal gas equation is given by:

$$pV = nRT$$

Solving for  $p$ :

$$p_{ideal} = \frac{nRT}{V} = \frac{1 \cdot 0.0821 \cdot 273.15}{0.1} = 224.4 \text{ atm}$$

Converting to Pa:

$$p = 224.4 \times 1.013 \times 10^5 = 227.51 \cdot 10^5 \text{ Pa}$$

2) **van der Waals gas**

The van der Waals equation is written as:

$$\left(p + \frac{an^2}{V^2}\right)(V - nb) = nRT$$

Substituting and solving:

$$\left(p + \frac{1.35}{0.1^2}\right)(0.1 - 0.039) = 1 \cdot 0.0821 \cdot 273.15$$

$$(p + 135) \cdot 0.061 = 22.44$$

$$p + 135 = \frac{22.44}{0.061} = 367.63$$

$$p = 367.63 - 135 = 232.63 \text{ atm}$$

Converting to Pa:

$$p_{vdw} = 232.63 \times 1.013 \times 10^5 = 235.51 \cdot 10^5 \text{ Pa}$$

b) 1) **Ideal gas**

for 1 atm in an ideal gas:

$$V = \frac{nRT}{P} = 22.4139 \text{ L}$$

2) **van der Waals gas**

Using this volume and substituting in the van der Waals equation:

$$\left(p + \frac{1.35 \cdot (1)^2}{(22.4139)^2}\right)(22.4139 - 1 \cdot 0.039) = 1 \cdot 0.0821 \cdot 273.15$$

$$\frac{1.35}{(22.4139)^2} = 0.00268 \text{ atm}$$

$$V - nb = 22.4139 - 0.039 = 22.3749 \text{ L}$$

$$nRT = 22.44 \text{ atm L}$$

$$(p + 0.00268) \cdot 22.3749 = 22.44$$

$$p + 0.00268 = \frac{22.44}{22.3749} \approx 1.0028$$

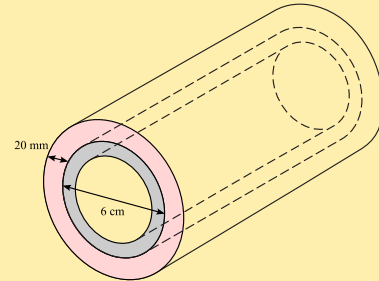
$$p = 1.0028 - 0.00268 \approx 0.997 \text{ atm}$$

## 1.4 Week 4

### 1.4.1 Exercises

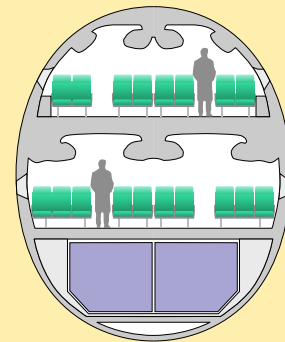
#### Exercise 1.4.1

A heating pipe with 6 cm diameter where water flows at  $80^\circ\text{C}$  is wrapped by an elastomeric insulator with thermal conductivity  $0.044\text{ W}/(\text{m }^\circ\text{C})$  and with 20 mm thickness. If the outside temperature is  $5^\circ$  what is the power transferred to the outside per unit length of the tube?



#### Exercise 1.4.2

A commercial jet aircraft (Airbus A-380) has an approximately cylindrical shape with 50 m length and about 3 m inside radius. The walls are insulated with a 7 cm thickness material with thermal conductivity  $k = 4 \times 10^{-5}\text{ cal}/(\text{cm K})$ . The temperature inside is to be maintained at  $25^\circ\text{C}$  where as the external temperature is  $-40^\circ\text{C}$ . What is the necessary heating rate for ensuring the internal cabin temperature?



#### Exercise 1.4.3

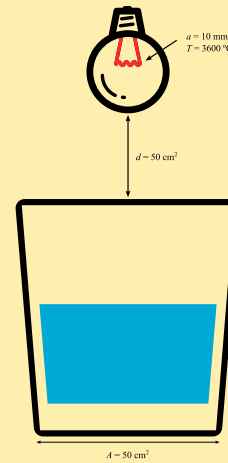
In a room with a temperature of about  $25^\circ\text{C}$ , the surface skin temperature of a person (with an overall area of about  $1.5\text{ m}^2$ ), without clothes, and at rest, is  $33^\circ\text{C}$ . The emissivity for the visible electromagnetic frequencies varies as a function of skin color. However, most of the radiation emitted by a human body occurs in the infrared region of the electromagnetic spectrum where the body acts as a blackbody ( $\varepsilon = 1$ ).

1. Calculate the heat loss from radiation transfer. Note that a person loses heat through radiation at a rate depending from the body temperature, but also absorbs radiation from the room at its ambient temperature.
2. Knowing that heat loss by conduction is negligible, and that convective heating losses are, in such conditions, about 50% of the overall heat losses, how many calories does a person needs to ingest daily just to ensure its metabolism in such conditions? You may assume that the person remains at rest all day and does not wish to gain weight.

**Exercise 1.4.4**

A glass of water is held below a functioning lamp, whose filament has a temperature  $T = 3200^\circ\text{C}$ . The distance from the lamp is 50 cm. The filament area is  $10\text{ mm}^2$ . The glass is cylindrical and has a base area of  $50\text{ cm}^2$ .

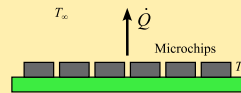
1. Determine the power emitted by the lamp's filament (consider the filament as a blackbody).
2. What is the wavelength corresponding to the maximum emitted light intensity?
3. How long does it take before water starts boiling if the initial temperature is  $T_{\text{water}} = 20^\circ\text{C}$  and the water volume inside the glass is  $V_{\text{water}} = 120\text{ mL}$ ? Consider that the water absorbs all the radiation that impinges it and disregard any energy losses from the water.

**Exercise 1.4.5**

A  $15\text{ cm} \times 10\text{ cm}$  circuit board has on its surface 100 logical chips closely spaced apart, each one dissipating  $0.03\text{ W}$  on a  $40^\circ\text{C}$  ambient temperature. Heat transfer from the back of the plate is negligible.

If the heat transfer coefficient on the surface of the plate is  $10\text{ W}/(\text{m}^2\text{ }^\circ\text{C})$ , determine:

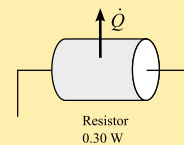
1. The heat flux on the surface of a circuit plate, in  $\text{W}/\text{m}^2$ .
2. The surface temperature of the chips.
3. The thermal resistance between the plate circuit surface and the cooling medium, in  $^\circ\text{C}/\text{W}$ .

**Exercise 1.4.6**

An element of a cylindrical resistance in a circuit board dissipates  $0.30\text{ W}$  of power on a medium at  $50^\circ\text{C}$ . The resistance has  $1.7\text{ cm}$  length and a diameter of  $0.3\text{ cm}$ .

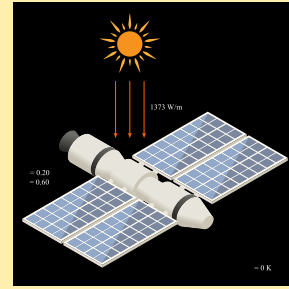
Assuming that heat is uniformly transferred from all surfaces, determine:

1. The heat quantity that the resistance dissipates during a 24 h period.
2. The heat flux on the resistance surface, in  $\text{W}/\text{m}^2$ .
3. The surface temperature of the resistance for a combined convection and radiation heat transfer coefficient of  $9\text{ W}/(\text{m}^2\text{ }^\circ\text{C})$



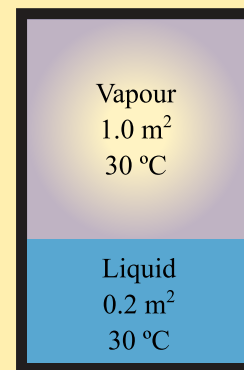
**Exercise 1.4.7**

Solar radiation impinges on the outside of a roughly cylindrical spacecraft a rate of  $1373 \text{ W/m}^2$ . The surface has an absorptivity  $\alpha_s = 0.20$  for solar radiation and an emissivity  $\epsilon = 0.60$  at ambient temperature. The exterior surface radiates heat to space, which may be considered to be at absolute zero ( $T = 0 \text{ K}$ ). Determine the surface equilibrium temperature, assuming that heat gains and losses are balanced (e.g. the system is in steady-state).

**Exercise 1.4.8**

A closed vessel contains  $0.2 \text{ m}^3$  of saturated liquid of R-134a coolant and  $1.0 \text{ m}^3$  of saturated vapor of R-134a, in equilibrium at  $30^\circ\text{C}$ . Determine the vapor percentage in terms of mass.

**Note:** Refer to the thermodynamic tables in appendix C.3.

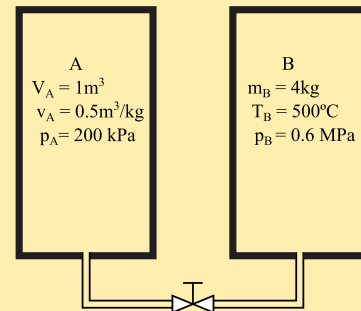
**Exercise 1.4.9**

Determine water pressure at  $200^\circ\text{C}$  with a specific volume  $v = 0.37 \text{ m}^3/\text{kg}$ .

**Note:** Refer to the thermodynamic tables in appendix C.2.

**Exercise 1.4.10**

Tank A of total volume  $V_A = 1 \text{ m}^3$  contains water at  $200 \text{ kPa}$ , with the specific volume  $v = 0.5 \text{ m}^3/\text{kg}$ . Tank B contains  $4 \text{ kg}$  of water at  $0.6 \text{ MPa}$  and  $500^\circ\text{C}$ . The two tanks are connected by a valve which is opened, bringing the two tanks to an uniform state. Find the final specific volume.

**Exercise 1.4.11**

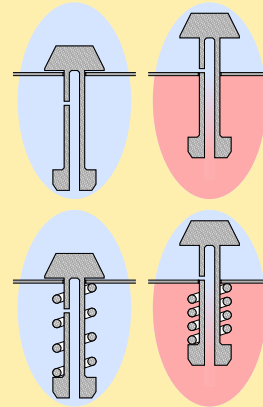
A closed vessel contains a mixture of liquid/vapor water at  $100^\circ\text{C}$ , with the liquid volume being  $1/10$ th of the vapor volume. It is heated until the pressure reaches  $1.0 \text{ MPa}$ . Find the final temperature. Has the final state more or less vapor than the initial state?

**Exercise 1.4.12**

Pressure cookers allow food to be cooked faster than at normal pressure, as the high pressure inside the cooker limits boiling and creates higher temperatures not possible at lower pressures. To avoid catastrophic explosions, these cookers have a valve for releasing pressure once it reaches a given value. First generation cookers do so through a so-called petcock valve, which is designed to raise once the steam pressure over a small opening is high enough to lift the petcock weight. Modern pressure cookers add an adjustable spring that allows adjusting the release pressure.

Consider a simple 1<sup>st</sup> generation pressure cooker, with a lid screwed on tight. The cooker has a 5 mm<sup>2</sup> opening covered with a petcock that can be lifted to let the steam escape. Assuming that the atmospheric pressure is 101.3 kPa, and that the cooker is designed to operate at an absolute pressure of 200 kPa, determine:

1. The weight of the petcock;
2. The cooking temperature.



Standard (top) and spring-loaded (bottom) petcock

**Exercise 1.4.13**

In a sink, we add 3 L of water at 75 °C to 1 kg of aluminium pots, 1 kg of steel cutlery, and 1 kg of glass, all at 25°. What is the final temperature, neglecting heat losses and work?

**Note:** Consider:

$\nu_{\text{H}_2\text{O}} = 0.001\,023 \text{ m}^3/\text{kg}$ ;  $c_{\text{H}_2\text{O}} = 4.184 \text{ kJ}/(\text{kg K})$ ;  $c_{\text{aluminium}} = 0.90 \text{ kJ}/(\text{kg K})$ ;  $c_{\text{steel}} = 0.46 \text{ kJ}/(\text{kg K})$ ;  $c_{\text{glass}} = 0.80 \text{ kJ}/(\text{kg K})$ .

**Exercise 1.4.14**

Consider 5 kg of liquid water at 25 °C and 100 kPa. We add 20 kJ of energy at constant pressure.

Determine:

1. The final temperature of the water if the energy is converted in heat.
2. The final velocity of the water if the energy is applied as a constant horizontal force.
3. The final altitude if the water is raised vertically using this energy.

**Note:** Consider  $c_{\text{H}_2\text{O}} = 4.184 \text{ kJ}/(\text{kg K})$ .

**Exercise 1.4.15**

A passive solar house that is losing heat to the outdoors at an average rate of 50 000 kJ/h is maintained at 20 °C at all times during a winter night for 10 h. The house is to be heated by 50 glass containers each containing 20 L of water that is heated to 70 °C during the day by absorbing solar energy. A thermostat-controlled 15 kW back-up electric resistance heater turns on whenever necessary to keep the house at 20 °C.

1. How long did the electric heating system run that night?
2. How long would the electric heater run that night if the house incorporated no solar heating?

**Note:** Consider the density and specific heat of water at room temperature as  $\rho = 1 \text{ kg/L}$  and  $c_{\text{H}_2\text{O}} = 4.184 \text{ kJ}/(\text{kg } ^\circ\text{C})$ .

## Formulae

### Constants

$$k_B = 1.381 \times 10^{-23} \text{ m}^2/(\text{kg s}^2 \text{ K}) \text{ or } \text{J/K}$$

$$R = 8.314 \text{ J}/(\text{mol K})$$

$$N_A = 6.022 \times 10^{23}/\text{mol}$$

$$P_{\text{atm}} = 101.325 \text{ kPa}$$

$$T_{\text{amb}} = 300 \text{ K}$$

$$T(\text{K}) = 273.15 + T(^{\circ}\text{C})$$

$$\sigma = 5.67 \times 10^{-8} \text{ W/m}^2/\text{K}^4$$

### Formulas

**Mechanical kinetic energy:**  $K = \frac{1}{2}mv^2$

**Work:**  $W = \int F dx = \int P dV$

**Average kinetic energy:**  $K = \frac{3}{2}k_B T$

**Ideal gas equation:**  $PV = nRT$

**Conduction heat transfer:**  $\frac{dQ}{dt} = -kA \frac{dT}{dx}$

**Convection heat transfer:**  $\frac{dQ}{dt} = hA\Delta T$

**Stefan–Boltzmann Law:**  $P = \epsilon \sigma AT^4$

**Power:**  $P[\text{W}] = I[\text{W/m}^2] \times A[\text{m}^2]$

**Wien's Law**  $\lambda_{\text{max}} T = b$ , with  $b = 2.898 \times 10^{-3} \text{ m K}$

**Thermal resistance:**  $R_t = \frac{1}{R_s h}$ , with  $h$  convective coefficient

**Specific Heat Capacity:**  $Q = mc\Delta T$

**Latent Heat:**  $Q = m\lambda$

**Specific Heats:**  $C_p = C_v + R$

**Specific Volume:**  $v = V/m$

**First Law of Thermodynamics:**  $\Delta U = Q - W$

**Internal Energy of a Gas:**  $U = \frac{N}{2}nRT$ ,  $N$ : degrees freedom.

### 1.4.2 Solutions to Exercises

#### Answer 1.4.1

Given the tube with internal radius  $R_1 = 3 \text{ cm} = 0.03 \text{ m}$ , and insulation thickness  $\Delta R = 20 \text{ mm} = 0.02 \text{ m}$ , the transmitted power per unit length  $L$  is given by:

$$\frac{1}{L} \frac{dQ}{dt} = \frac{k \cdot 2\pi(T_1 - T_2)}{\ln\left(\frac{R_1 + \Delta R}{R_1}\right)}$$

Where:

- ▶  $T_1 = 80^\circ\text{C}$ : Internal temperature.
- ▶  $T_2 = 5^\circ\text{C}$ : External temperature.
- ▶  $k = 0.044 \text{ W}/(\text{m}^\circ\text{C})$ : Thermal conductivity of the insulator.

Substituting:

$$\frac{1}{L} \frac{dQ}{dt} = \frac{0.044 \times 2 \times \pi \times (80 - 5)}{\ln\left(\frac{0.03+0.02}{0.03}\right)}$$

We have:

$$\frac{1}{L} \frac{dQ}{dt} = 40.59 \text{ W/m}$$

The power transferred per unit length is approximately  $40.6 \text{ W/m}$ .

#### Answer 1.4.2

For a cylinder with length  $l = 50 \text{ m}$ , internal radius  $R_1 = 3 \text{ m}$  and insulation thickness  $\Delta R = 0.07 \text{ m}$ , the heat transfer rate is given by:

$$\frac{dQ}{dt} = -kA \frac{dT}{dx} = -k2\pi Rl \frac{dT}{dR} = \tilde{C}_1$$

The solution for  $T(R)$  is:

$$T(R) = c_1 \ln(R) + c_2$$

Knowing that  $T(R = R_1) = T_1 = 25^\circ\text{C}$  and  $T(R = R_2) = T_2 = -40^\circ\text{C}$ , where  $R_2 = R_1 + \Delta R = 3 + 0.07 = 3.07 \text{ m}$ , we have:

$$c_1 \ln\left(\frac{R_2}{R_1}\right) = T_2 - T_1 \Rightarrow c_1 = \frac{T_2 - T_1}{\ln\left(\frac{R_2}{R_1}\right)}$$

Substituting:

$$c_1 = \frac{-40 - 25}{\ln\left(\frac{3.07}{3}\right)}$$

Calculating the heat transfer rate per unit length:

$$\left| \frac{dQ}{dt} \right| = k2\pi l \frac{T_2 - T_1}{\ln\left(\frac{R_2}{R_1}\right)}$$

Substituting  $k = 16.744 \times 10^{-3} \text{ W}/(\text{m}^\circ\text{C})$ :

$$\begin{aligned} \left| \frac{dQ}{dt} \right| &= 16.744 \times 10^{-3} \times 2\pi \times 50 \times \frac{25 - (-40)}{\ln\left(\frac{3.07}{3}\right)} \\ &\approx 14.82 \text{ kW} \end{aligned}$$

The heating rate required for keeping the temperature constant on the plane cabin is approximately  $14.8 \text{ kW}$ .

**Answer 1.4.3**

1. The power lost through radiative processes is:

$$P = (\dot{I}_{\text{out}} - \dot{I}_{\text{in}})A = \sigma \varepsilon A (T_1^4 - T_0^4)$$

Where:

- ▶  $\sigma = 5.67 \times 10^{-8} \text{ W/m}^2/\text{K}^4$ : de Stefan–Boltzmann constant.
- ▶  $\varepsilon = 1$ : Emissivity.
- ▶  $A = 1.5 \text{ m}^2$ : Surface area.
- ▶  $T_0 = 298.15 \text{ K}$ : Ambient temperature.
- ▶  $T_1 = 306.15 \text{ K}$ : Body temperature.

Substituting:

$$P = 5.67 \times 10^{-8} \times 1.5 \times (306.15^4 - 298.15^4) \approx 75.09 \text{ W}$$

2. Considering that the heat loss through conduction is negligible, and that heat loss through convection amounts for 50% of total losses:

$$P_{\text{tot}} = Q_{\text{rad}} + Q_{\text{conv}} \Rightarrow Q_{\text{conv}} = Q_{\text{rad}}$$

Therefore, the total power is:

$$P_{\text{tot}} = 2 \times 75.09 \approx 150.18 \text{ W}$$

Calculating the total daily energy:

$$E = P_{\text{tot}} \times \Delta t = 150.18 \times 60 \times 60 \times 24 = 1.3 \times 10^7 \text{ J}$$

Converting to calories:

$$E = \frac{1.30 \times 10^7}{4.1868} \approx 3100 \text{ kcal}$$

Therefore, the amount of daily calories a person should ingest to maintain metabolism is approximately 3100 kcal.

**Answer 1.4.4**

1. The power emitted by the filament lamp is:

$$I = \sigma T^4 = 5.67 \times 10^{-8} \times (3600 + 273.15)^4 \approx 1.28 \times 10^7 \text{ W/m}^2$$

Total power is:

$$P = I \times A = 1.28 \times 10^7 \times 10 \times 10^{-6} \approx 127.58 \text{ W}$$

2. The wavelength corresponding to the maximum intensity is:

$$\lambda_{\text{max}} = \frac{2.898 \times 10^{-3}}{T} = \frac{2.898 \times 10^{-3}}{3873.15} \approx 748.6 \text{ nm}$$

3. Time needed for heating water:

Water initial temperature:  $T_{\text{initial}} = 20^\circ\text{C} = 293.15 \text{ K}$ .

$$Q = c \cdot m \cdot \Delta T = 4186 \times 0.12 \times (373.15 - 293.15) = 40.186 \text{ kJ}$$

The incident radiative intensity on the glass is:

$$I(d) = \frac{P}{4\pi d^2} = \frac{127.58}{4\pi \times (0.5)^2} = 40.61 \text{ W/m}^2$$

Incident power on the glass area:

$$P_{\text{inc}} = I(d) \times S = 40.61 \times 0.005 = 0.2031 \text{ W}$$

Necessary time:

$$\Delta t = \frac{Q}{P_{\text{inc}}} = \frac{40.186 \times 10^3}{0.203} \approx 197\,959 \text{ s} = 54 \text{ h } 59 \text{ min } 18.6 \text{ s}$$

Accordingly the time needed for the water to start boiling is approximately 2 days, 6 hours, 59 minutes and 19 seconds.

#### Answer 1.4.5

1. Heat flux on the plate surface:

$$\dot{q} = \frac{\dot{Q}}{A_s} = \frac{100 \times 0.03 \text{ W}}{0.15 \times 0.10} = \frac{3}{0.015} = 200 \text{ W/m}^2$$

2. Surface temperature for the microchips:

$$\dot{Q} = hA_s(T_s - T_\infty) \Rightarrow T_s = T_\infty + \frac{\dot{Q}}{hA_s}$$

Substituting:

$$T_s = 40 + \frac{100 \times 0.03}{(10 \times 0.015)} = 40 + \frac{3}{0.15} = 40 + 20 = 60^\circ\text{C}$$

3. Thermal resistance between the plate surface and the cooling medium:

$$R_{\text{conv}} = \frac{1}{hA_s} = \frac{1}{(10 \text{ W/(m}^2\text{ }^\circ\text{C)})(0.015 \text{ m}^2)} = 6.67^\circ\text{C/W}$$

#### Answer 1.4.6

1. Amount of heat dissipated in 24h:

$$Q = \dot{Q}\Delta t = 0.30 \text{ W} \times 24 \text{ h} = 7.2 \text{ Wh}$$

2. Resistance surface area:

$$A_s = \pi D^2 + \pi DL = \pi \left( \frac{0.003}{2} \right)^2 + \pi \times 0.003 \times 0.017 = 0.000163 \text{ m}^2$$

Heat flux:

$$\dot{q} = \frac{\dot{Q}}{A_s} = \frac{0.30 \text{ W}}{0.000163 \text{ m}^2} = 1840.5 \text{ W/m}^2$$

3. Surface temperature:

$$\dot{Q} = hA_s(T_s - T_\infty) \Rightarrow T_s = T_\infty + \frac{\dot{Q}}{hA_s}$$

Substituting:

$$T_s = 50 + \frac{0.30 \text{ W}}{(9 \text{ W/(m}^2\text{ }^\circ\text{C)})(0.000163 \text{ m}^2)} = 50 + 202.8 = 252.8^\circ\text{C}$$

Therefore, the surface temperature for the resistance will be approximately 252.8°C.

**Answer 1.4.7**

The solar radiation impinging on the spacecraft surface is:

$$I_{\text{in}} = I_{\text{sun}} \times \alpha_{\text{sun}} = 1373 \times 0.2 = 274.6 \text{ W/m}^2$$

The radiation emitted by the spacecraft towards Outer Space is:

$$I_{\text{out}} = \sigma T^4 \times \varepsilon$$

In steady-state thermal equilibrium:

$$I_{\text{in}} + I_{\text{space, in}} = I_{\text{space, out}}$$

As the background radiation from Space is essentially zero ( $T_{\text{space}} = 0 \text{ K}$ ):

$$I_{\text{space, in}} = 0$$

The equilibrium temperature at the spacecraft surface is:

$$T_{\text{spacecraft}} = \left( \frac{I_{\text{sun}} \times \alpha_{\text{sun}}}{\varepsilon \times \sigma} \right)^{1/4}$$

Substituting the values and calculating:

$$T_{\text{spacecraft}} = \left( \frac{1373 \times 0.2}{0.6 \times 5.67 \times 10^{-8}} \right)^{1/4}$$

$$T_{\text{spacecraft}} \approx 299.7 \text{ K}$$

Therefore, the surface equilibrium temperature of the spacecraft will approximately be 299.7 K.

**Answer 1.4.8**

Given the R-134 liquid and saturated vapor volumes are in equilibrium at 30 °C:

- Liquid volume:  $V_{\text{liq}} = 0.2 \text{ m}^3$
- Vapor volume:  $V_{\text{vap}} = 1.0 \text{ m}^3$
- Total volume:  $V_{\text{total}} = 1.2 \text{ m}^3$

To calculate the liquid ( $m_{\text{liq}}$ ) and vapor ( $m_{\text{vap}}$ ) masses, we consider the specific properties of R-134a at 30 °C:

$$m_{\text{liq}} = \frac{V_{\text{liq}}}{v_f} = \frac{0.2}{0.000842} \approx 237.53 \text{ kg}$$

$$m_{\text{vap}} = \frac{V_{\text{vap}}}{v_g} = \frac{1.0}{0.026642} \approx 37.56 \text{ kg}$$

Total mass:

$$m = m_{\text{liq}} + m_{\text{vap}} = 237.53 + 37.56 = 275.09 \text{ kg}$$

The quality of the mixture is given by:

$$x = \frac{m_{\text{vap}}}{m} = \frac{37.56}{275.53} \approx 0.158$$

Therefore, the vapor fraction in terms of mass is approximately 15.8%.

**Answer 1.4.9**

To determine the water pressure at  $200^\circ\text{C}$  with a specific volume  $v = 0.37\text{ m}^3/\text{kg}$ , we need to consult the thermodynamic tables. We verify on table C.1:

- ▶ The specific volume for saturated vapor at  $200^\circ\text{C}$  is  $v_g = 0.12721\text{ m}^3/\text{kg}$ .
- ▶ As  $v = 0.37\text{ m}^3/\text{kg} > v_g$ , this means that we are in the superheated vapor region.

We consult table C.3 for temperature  $T = 200^\circ\text{C}$ . We start with:

- ▶ For  $p = 0.5\text{ MPa}$ , we have  $v = 0.42503\text{ m}^3/\text{kg}$ .
- ▶ For  $p = 0.6\text{ MPa}$ , we have  $v = 0.35212\text{ m}^3/\text{kg}$ .

The value  $v = 0.37\text{ m}^3/\text{kg}$  is comprised between  $0.5\text{ MPa}$  and  $0.6\text{ MPa}$ .

We do a linear interpolation between  $0.5\text{ MPa}$  and  $0.6\text{ MPa}$ :

$$p = 500 + (600 - 500) \times \frac{0.37 - 0.42503}{0.3521 - 0.42503}$$

$$p = 580.42\text{ kPa}$$

The pressure is approximately  $580.042\text{ kPa}$ .

**Answer 1.4.10**

Control volume: both tanks. Constant total volume and mass process.

- ▶ State  $A_{ini}$ :  $(p, v)$   $m_A = V_A/v_A = 1/0.5 = 2\text{ kg}$
- ▶ State  $B_{ini}$ :  $(p, T)$ , we need to consult the vapor tables in appendix C.2.  
 $T_{sat}(0.6\text{ MPa}) = 158.83^\circ\text{C} < T_{ini}^B = 500^\circ\text{C}$ . We have superheated vapor.

We obtain for the  $(p, T)$  properties pair:  $v = 0.592\text{ m}^3/\text{kg}$ .

We may write:  $V_B = m_B \times v_B = 4\text{ kg} \times 0.592\text{ m}^3/\text{kg} = 2.368\text{ m}^3$ .

The final state is:

- ▶  $m_{tot} = m_A + m_B = 6\text{ kg}$
- ▶  $V_{tot} = V_A + V_B = 3.368\text{ m}^3$
- ▶  $v_2 = V_{tot}/m_{tot} = 0.561\text{ m}^3/\text{kg}$

**Answer 1.4.11**

The volume  $V$  of the vessel remains constant, such as the mass  $m$  of the substance inside (water). Accordingly, the specific volume  $v = V/m$  will also remain constant.

- ▶ Initial State:  $V = V_l + V_g$  with  $V_l = 1/10 V_g$ .  
 We consult the saturated water tables in appendix C.2 at  $T = 100^\circ\text{C}$ :  
 $v_f = 0.001043\text{ m}^3/\text{kg}$ ,  $v_g = 1.6718\text{ m}^3/\text{kg}$ .

We have

$$V = V_l + V_g = V_g/10 + V_g = 11/10 V_g = 11/10 v_g \times m_g = 1.84 m_g$$

$$V = V_l + V_g = V_l + 10V_l = 11V_l = 11v_l \times m_l = 0.0115 m_l$$

since the quality of the flow  $x$  is given by:  $x = m_g/(m_g + m_f)$

$$x_{ini} = (V/1.84)/(V/1.84 + V/0.0115) = (1.84 + 0.0115)/1.84 - 1 = 0.00625$$

and

$$v_{ini} = v_f(1 - x_g) + v_g x_g = 0.0115 \text{ m}^3/\text{kg}$$

- Final State:  $p = 1.0 \text{ MPa}$  and  $v_{fin} = v_{ini} = 0.0115 \text{ m}^3/\text{kg}$ ,

We consult the saturated water tables in appendix C.2 at  $p = 1.0 \text{ MPa}$ :

$$T_2 = T_{sat}(2 \text{ MPa}) = 212.4 \text{ }^\circ\text{C}$$

$$v_f = 0.001127 \text{ m}^3/\text{kg}, v_g = 0.19436 \text{ m}^3/\text{kg}.$$

We have:

$$v_{fin} = v_f + x_{fin} v_{fg}$$

$$0.0115 = 0.001127 + x_2 \times 0.19436$$

$$\rightarrow x_{fin} = 0.0533$$

There is more vapor at the final state.

#### Answer 1.4.12

1. Forces Balance:

$$F = mg = \Delta p \times A$$

$$\rightarrow m = \Delta p \times A / g$$

Substituting:

$$\begin{aligned} m &= (200 - 101.3) \times 1000 \times 5 \times 10^{-6} / 9.81 [\text{N/m/s}^2] \\ &= 0.0503 \text{ kg} \approx 50 \text{ g} \end{aligned}$$

2. The temperature corresponds to the saturation temperature of water vapor (since we have water and vapor at the same time on the cooker)

$$T_{sat}(200 \text{ kPa}) = 120.21 \text{ }^\circ\text{C}$$

#### Answer 1.4.13

The energy equation for an equilibrium thermal mixture is given by:

$$U_2 - U_1 = \sum m_i(u_2 - u_1)_i = 0$$

For water:

$$v_f = 0.001023 \text{ m}^3/\text{kg}, \quad V = 3 \text{ L} = 0.003 \text{ m}^3$$

$$m = \frac{V}{v} = \frac{0.003}{0.001023} \approx 2.932 \text{ kg}$$

The different materials and their specific heats are:

- Water:  $m_{\text{H}_2\text{O}} = 2.932 \text{ kg}$ ,  $c_{\text{H}_2\text{O}} = 4.184 \text{ kJ}/(\text{kg K})$
- Aluminium:  $m_{\text{al}} = 1 \text{ kg}$ ,  $c_{\text{al}} = 0.90 \text{ kJ}/(\text{kg K})$
- Steel:  $m_{\text{steel}} = 1 \text{ kg}$ ,  $c_{\text{steel}} = 0.46 \text{ kJ}/(\text{kg K})$
- Glass:  $m_{\text{glass}} = 1 \text{ kg}$ ,  $c_{\text{glass}} = 0.80 \text{ kJ}/(\text{kg K})$

We apply the energy conservation equation to determine the final temperature  $T_2$ :

$$\sum m_i c_i (T_2 - T_1)_i = 0$$

Expanding the summation:

$$\begin{aligned} m_{\text{H}_2\text{O}} c_{\text{H}_2\text{O}} (T_{\text{fin}} - T_{\text{ini}}^{\text{H}_2\text{O}}) + (m_{\text{al}} c_{\text{al}} + m_{\text{steel}} c_{\text{steel}} + m_{\text{glass}} c_{\text{glass}}) (T_{\text{fin}} - T_{\text{ini}}^{\text{al, steel, glass}}) &= 0 \\ 2.932 \times 4.184 \times (T_{\text{fin}} - 75) + (1 \times 0.9 + 1 \times 0.46 + 1 \times 0.8) \times (T_{\text{fin}} - 25) &= 0 \\ 12.27 \times (T_{\text{fin}} - 75) + 2.16 \times (T_{\text{fin}} - 25) &= 0 \end{aligned}$$

Therefore

$$T_{\text{fin}} = \frac{12.27 \times 75 + 2.16 \times 25}{12.27 + 2.16} = 67.52^\circ\text{C}$$

and the system final temperature will be approximately  $67.5^\circ\text{C}$ .

Note: Since we are merely considering temperature differences, there is no need to convert temperatures to Kelvin units.

#### Answer 1.4.14

We have 5 kg of liquid water at  $25^\circ\text{C}$  and 100 kPa. When we add 20 kJ of energy:

##### 1. Final temperature of water:

The internal energy variation is:

$$\Delta E = Q \Rightarrow \Delta T = \frac{Q}{mc} = \frac{20 \times 10^3}{5 \times 4.184 \times 10^3} = \frac{20}{20.92} \approx 0.96^\circ\text{C}$$

Accordingly the final temperature of water is:

$$T_2 = T_1 + \Delta T = 25 + 0.96 \approx 25.96^\circ\text{C}$$

##### 2. Terminal velocity for water if the energy is used to generate horizontal movement:

The kinetic energy variation is given by:

$$\Delta E = \frac{1}{2} m v^2 \Rightarrow v = \sqrt{\frac{2\Delta E}{m}} = \sqrt{\frac{2 \times 20 \times 10^3}{5}} = \sqrt{8000} \approx 89.44 \text{ m/s}$$

##### 3. Final water height if the energy is used for raising it:

The potential energy variation is:

$$\Delta E = mgh \Rightarrow h = \frac{\Delta E}{mg} = \frac{20 \times 10^3}{5 \times 9.81} = \frac{20 \times 10^3}{49.05} \approx 407.8 \text{ m}$$

**Answer 1.4.15**

The total mass of water is  $m_{water} = \rho V = (1 \text{ kg/L}) \times (50 \times 20 \text{ L}) = 1000 \text{ kg}$

The energy balance for the house is

$$\begin{aligned} E_{in} - E_{out} &= \Delta E_{sys} \\ W_{in} - Q_{out} &= \Delta U = \Delta U_{water} + \Delta U_{air} \\ &= \Delta U_{water} \end{aligned}$$

or:

$$\dot{W}_{in} \Delta t - Q_{out} = \Delta U_{water} = mc(T_H - T_C)_{water}$$

Substituting,

$$(15 \text{ kJ/s})\Delta t - 50\,000 \text{ kJ/h} \times 10 \text{ h} = 1000 \text{ kg} \times 4.184 \text{ kJ/(kg } ^\circ\text{C)}(20 - 70)^\circ\text{C},$$

which gives

$$\Delta t = 19\,387 \text{ s} = 5.77 \text{ h}, 23 \text{ min}, 7 \text{ s}$$

If the house incorporated no solar heating, the energy balance relation above would simplify further to

$$\dot{W}_{in} \Delta t - Q_{out} = 0$$

Substituting,

$$(15 \text{ kJ/s})\Delta t - 50\,000 \text{ kJ/h} \times 10 \text{ h} = 0,$$

which gives

$$\Delta t = 33\,333 \text{ s} = 9 \text{ h}, 15 \text{ min}, 33 \text{ s}$$

## 1.5 Week 5

### 1.5.1 Exercises

#### Exercise 1.5.1

From the ideal gas law, we know that  $p = C \frac{T}{V}$  with  $C$  a given constant.

1. Plot  $p$  as a function of  $V$ , for  $T = T_1$  and  $T = T_2$ , with  $T_2 > T_1$
2. Calculate  $\left(\frac{\partial p}{\partial V}\right)_T$  and  $\left(\frac{\partial p}{\partial T}\right)_V$ . Then write  $dp$  as a function of  $dT$  and  $dV$ .
3. Calculate the work performed on a gas, as the result of a constant-temperature compression between  $V_1$  and  $V_2$
4. Calculate  $\int_{V_1}^{V_2} p dV$ , assuming  $\frac{dT}{T} = \frac{dV}{V}$
5. Calculate  $\int_{p_1}^{p_2} dp$  through the following pathways:
  - a)  $\int_{p_1}^{p_2} dp$
  - b)  $\int_{V_1}^{V_2} \left(\frac{\partial p}{\partial V}\right)_T dV + \int_{T_1}^{T_2} \left(\frac{\partial p}{\partial T}\right)_{V_2} dT$

#### Exercise 1.5.2

Consider a glass of water in a room at 25 °C with 60% relative humidity. If the water temperature is 20 °C, determine the vapor pressure:

1. In the exposed surface of the water.
2. In a place in the room, faraway from the glass.

**Note:** Consult the vapor tables in Appendix C.2.

#### Exercise 1.5.3

On a given day, the temperature and relative humidity of air over a large pool are, respectively 20 °C and 60%. Determine the temperature of the pool assuming a phase equilibrium between the pool water and the vapor in the air.

**Note:** Consult the vapor tables in Appendix C.2.

#### Exercise 1.5.4

A reservoir of 1 m<sup>3</sup> has 2.841 kg of vapor at 0.7 MPa. Determine the vapor temperature using:

1. The ideal gas law
2. The van der Waals gas law
3. the vapor tables (available in Appendix C.2.)

#### Exercise 1.5.5

A classroom which normally hosts 35 persons is cooled through window air-conditioning units with a cooling capacity of 7 kW. We assume that a resting person dissipates heat at a rate of about 530 kJ/h. There are 10 lamps in the room, each one with a power of 100 W. We estimate that the heat transfer rate to the room through the walls and windows is 15 000 kJ/h. If the air from the room is to be held at a constant temperature of 23 °C, determine the number of necessary air conditioning units.

**Exercise 1.5.6**

A man with 80 kg ate a double-hamburger, a normal french fries serving and a 200 mL Coca-Cola for lunch. Determine how long will he take to burn the lunch calories<sup>a</sup> if:

1. he is watching television;
2. he is quickly swimming.

How would your answers be for a woman weighting 60 kg?

**Note:** Check the tables F.3 e F.2.

<sup>a</sup> neglecting the energy spent to maintain the metabolic processes for the human body

**Exercise 1.5.7**

An overweight 80 kg person is on a strict diet in an attempt to reduce weight. His/her daily caloric intake is calibrated to be slightly below the mean daily allowance for a person of his/her weight, in an attempt to make the body consume the missing calories by burning stored fat in the person's body.

At some point during the day, the person succumbs to temptation and eats a croissant. Afterwards, ridden by guilt, the person decides to carry out extra exercise to make up for this additional calorie intake, and he/she decides to climb flights of stairs inside a very tall skyscraper.

Which height will the person reach once he/she will have consumed all the additional calories from the croissant? Assume an average efficiency of 7% in metabolizing the croissant energy and converting it to work<sup>a</sup>.

**Note:** Check the energy content of a croissant in table F.3.

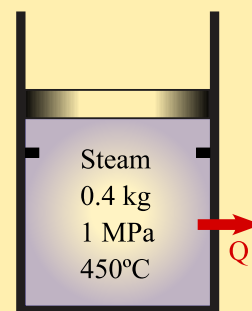
<sup>a</sup> According to Appendix F, we can assume the digestive metabolic efficiency is close to 100% since a croissant is a highly processed food; the efficiency of ATP reactions is 31.41%; and the efficiency of heat to mechanical work conversion is 22.5%. This yields a compound efficiency of 7%, assuming that the rest of the food intake of the person was used to sustain the daily basic metabolic needs of his/her body

**Exercise 1.5.8**

A piston-cylinder device with a set of end stops initially contains 0.4 kg of water at 1.0 MPa and 450 °C. The position of the end stops corresponds to 50% of the initial volume. Afterwards, the vapor is cooled. Determine the compression work if the final state is:

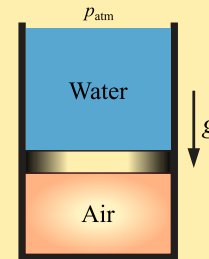
1. 1.0 MPa and 250 °C;
2. 400 kPa;
3. Determine also the final temperature for (b).

**Note:** Check the vapor tables in appendix C.2.



**Exercise 1.5.9**

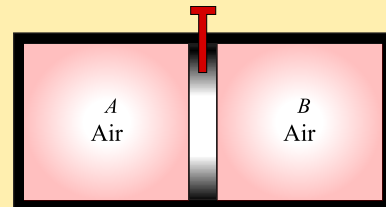
A cylinder with 5 m height and cross-section  $0.2 \text{ m}^2$  has, in its inferior part, a massless piston, covered with water at  $20^\circ\text{C}$ , as shown in the corresponding figure. Below the piston there is air, at a temperature of 300 K, and a volume  $0.3 \text{ m}^3$ . This air is heated, moving the piston upwards, and spilling all the water. Find out the amount of heat transferred to the air, once all the water is spilled.



**Note:** Consider air as a mixture of diatomic molecules with 5 excited degrees of freedom and molar mass  $m_m = 28.7 \text{ g/mol}$  and consider  $v_{\text{H}_2\text{O}} = 0.001002 \text{ m}^3/\text{kg}$ .

**Exercise 1.5.10**

An insulated cylinder is divided in two parts with  $1 \text{ m}^3$  each, by a piston initially held in position, as shown in the corresponding figure. Side A has air at 300 kPa and 350 K. Side B has air at 2.0 MPa and 1010 K. The piston is then unlocked, allowing its free movement, and allowing heat conduction so that thermal equilibrium may be reached. Calculate the air mass in A and B and the corresponding final temperature and pressure.



**Note:** Consider air as a mixture of diatomic molecules with 5 excited degrees of freedom and molar mass  $m_m = 28.7 \text{ g/mol}$ .

## Formulae

### Constants

$1 \text{ eV} = 1.602 \times 10^{-19} \text{ J}$   
 $k_B = 1.381 \times 10^{-23} \text{ m}^2/(\text{kg s}^2 \text{ K})$  or  $\text{J/K}$   
 $R = 8.314 \text{ J}/(\text{mol K})$   
 $N_A = 6.022 \times 10^{23} / \text{mol}$   
 $P_{\text{atm}} = 101.325 \text{ kPa}$   
 $T_{\text{amb}} = 20^\circ\text{C}$

### Formulas

**Specific Volume:**  $v = \frac{V}{m}$  **Average kinetic energy:**  $K = \frac{3}{2} k_B T$

**Ideal gas equation:**  $PV = nRT$

**van der Waals gas equation:**  $\left(P + a \frac{n^2}{V^2}\right)(V - nb) = nRT$

**Relation between  $a$  and  $b$  and the critical temperature and pressure:**  $a = \frac{27R^2 T_c^2}{64P_c}$ ,  $b = \frac{RT_c}{8P_c}$

$Q = mc\Delta T$

$Q = m\lambda$

$T(\text{K}) = 273.15 + T(^{\circ}\text{C})$

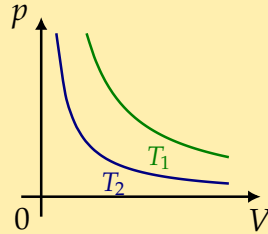
**1<sup>st</sup> Law of Thermodynamics:**  $\Delta U = Q - W$   $U = \frac{N}{2} nRT$ , with  $N$  the number of excited degrees of freedom.

**Work:**  $W = \int F dx = \int P dV$

## 1.5.2 Solutions to Exercises

## Answer 1.5.1

1.



2.

$$\left(\frac{\partial p}{\partial T}\right) = -C \frac{T}{V^2}$$

$$\left(\frac{\partial p}{\partial V}\right) = \frac{C}{V}$$

Accordingly, the differential will be:

$$dp = \left(\frac{\partial p}{\partial V}\right) dV + \left(\frac{\partial p}{\partial T}\right) dT = -C \frac{T}{V^2} dV + \frac{C}{V} dT$$

3.

$$W = \int p dV = \int_{V_1}^{V_2} \frac{CT}{V} dV = CT \ln\left(\frac{V_2}{V_1}\right)$$

4.

$$\int_{V_1}^{V_2} p dV = \int_{V_1}^{V_2} \frac{T}{V} dV = T \ln\left(\frac{V_2}{V_1}\right)$$

5. a)

$$\int_{p_1}^{p_2} dp = p_2 - p_1$$

b)

$$\begin{aligned} \int_{V_1}^{V_2} \frac{T_1}{V_1} dV + \int_{T_1}^{T_2} \left(\frac{\partial p}{\partial T}\right)_V dT &= CT_1 \left(\frac{1}{V_2} - \frac{1}{V_1}\right) + \Delta T \frac{C}{V_2} \\ &= CT_1 \left(\frac{1}{V_2} - \frac{1}{V_1}\right) + \frac{CT_2}{V_2} - \frac{CT_1}{V_2} = p_2 - p_1 \end{aligned}$$

## Answer 1.5.2

1. On the exposed surface with  $T = 20^\circ\text{C}$ , we have saturated water with:

$$p_{\text{sat}} = 2.3343 \text{ kPa}$$

2. On the atmosphere, we consider the pressure is given by the water on the saturated atmosphere, in other terms the pressure will be 60% of the pressure value for saturated water at  $T = 25^\circ\text{C}$ :

$$p_{\text{sat}} = 0.6 \times 3.1694 = 1.9016 \text{ kPa}$$

**Answer 1.5.3**

For equilibrium conditions, we have equivalence for water pressure on the atmosphere and water surface:

$$p_{sup} = p_{atm} = 0.6 \times 2.3393 \text{ kPa} = 1.40358 \text{ kPa}$$

Accordingly, we must perform an interpolation, since we verify that there is no tabulated value corresponding to this pressure:

$$p_{@10^\circ\text{C}} = 1.228 \text{ kPa} \quad \text{e} \quad p_{@15^\circ\text{C}} = 1.7058 \text{ kPa}$$

Interpolation for temperature:

$$T = 10 + \frac{1.7058 - 1.40358}{1.7058 - 1.2282} \times (15 - 10)$$

$$T \approx 13.16^\circ\text{C}$$

**Answer 1.5.4**

Data:

$$V = 1 \text{ m}^3$$

$$m = 2.8149 \text{ kg}$$

$$p = 0.7 \text{ MPa} = 7 \times 10^5 \text{ Pa}$$

$$v = \frac{V}{m} = \frac{1}{2.814} = 0.35520 \text{ m}^3/\text{kg}$$

$$R = 0.4615 \text{ kPa m}^3/(\text{kg K})$$

1.

$$T = \frac{pv}{R} = \frac{700 \times 0.352}{0.4615} = 533.91 \text{ K}$$

2.

$$T_{cr} = 647.1 \text{ K} \quad p_{cr} = 22.06 \text{ MPa}$$

$$a = \frac{27R^2T_{cr}^2}{64p_{cr}} = 1.705 \text{ m}^6/(\text{kPa kg})^3$$

$$b = \frac{RT_{cr}}{8p_{cr}} = 0.00169 \text{ m}^3/\text{kg}$$

$$T = \frac{1}{R} \left( p + \frac{a}{v^2} \right) (v - b) = \frac{1}{0.4615} \left( 700 + \frac{1.705}{[0.3552]^2} \right) (0.352 - 0.00169) = 541.79 \text{ K}$$

3. Looking at the thermodynamic tables, we only have  $p = 0.8 \text{ MPa}$  and  $p = 0.6 \text{ MPa}$ , and we need to perform an interpolation to achieve the required value. We verify that the specific volumes for both pressures are  $T = 200^\circ\text{C}$  and  $T = 350^\circ\text{C}$ , respectively. Averaging:

$$T \approx 275^\circ\text{C} = 548.15 \text{ K}$$

**Answer 1.5.5**

$$P = 7 \text{ kW} = 7000 \text{ W}$$

$$P_p = 580 \text{ kJ/h} = 147.22 \text{ W}$$

$$P_{pes} = 35 \times 147.22 \text{ W} = 5152.7 \text{ W}$$

$$P_l = 1000 \text{ W}$$

$$P_{par} = 15\,000 \text{ kJ/h} = 4166.67 \text{ W}$$

Total power:

$$P_t = 5152.7 + 1000 + 4166.67 = 10\,319.37 \text{ W}$$

Number of air conditioning units:

$$n^\circ = \frac{P_t}{P} \approx 2 \text{ units}$$

**Answer 1.5.6**

Data:

$$m = 80 \text{ kg}$$

$$E_{\text{food}} = 576 + 0.8 \times 110 + 236 = 900 \text{ kcal} = 3765.6 \times 10^3 \text{ kJ}$$

1. For a power of  $P = 1.16 \times 80 = 92.8 \text{ W}$ , the corresponding time is:

$$\Delta t = \frac{E_{\text{food}}}{P} = 11 \text{ h } 16 \text{ min}$$

For a mass of 60 kg:

$$P = 1.16 \times 60 = 69.6 \text{ W}$$

$$\Delta t = \frac{E_{\text{food}}}{P} = 15 \text{ h } 2 \text{ min}$$

2. For  $P = 9.8 \times 80 = 784 \text{ W}$ :

$$\Delta t = \frac{E_{\text{food}}}{P} = 1 \text{ h } 20 \text{ min}$$

For  $P = 9.8 \times 60 = 588 \text{ W}$ :

$$\Delta t = \frac{E_{\text{food}}}{P} = 1 \text{ h } 47 \text{ min}$$

**Answer 1.5.7**

The energy content of a croissant is according to Table F.3 is:

$$E_{\text{food}} = 231 \text{ Cal} = 4184 \times 231 \text{ J}$$

Climbing stairs brings a person to a higher height, with an increase of potential energy of:

$$\Delta E_{\text{pot}} = mgh$$

Since only 7% of the croissant energy was converted into useful work we have:

$$E_{\text{food}} \times 0.07 = \Delta E_{\text{pot}} = mgh$$

$$h = \frac{E_{\text{food}} \times 0.07}{mg}$$

Substituting:

$$h = \frac{4184 \times 231 \times 0.07}{80 \times 9.81} \\ = 86.2 \text{ m}$$

The person will have to climb 86.2 m in a building (this gets him/her typically to the 24<sup>th</sup> floor for a typical floor height of 3.5 m).

### Answer 1.5.8

1. Data:

$$m_{va} = 0.4 \text{ kg} \\ p = 1 \times 10^6 \text{ Pa} \\ T = 450^\circ \text{C}$$

Consulting the superheated water tables:

$$v_1 = 0.30661 \text{ m}^3/\text{kg} \\ v_2 = 0.23275 \text{ m}^3/\text{kg}$$

The work is:

$$W = mp(v_1 - v_2) = 0.4 \times 10^6 \times (0.30661 - 0.233275) = 29.5 \text{ kW}$$

2.

$$W = p(v_1 - 0.5v_1) = 153 \text{ kW}$$

3.

$$v_2 = 0.153305 \text{ m}^3/\text{kg} \\ p = 0.40 \text{ MPa}$$

Consulting the tables:

$$T = 143.6^\circ \text{C (Saturated)}$$

### Answer 1.5.9

The cylinder of volume  $V_{\text{tot}} = 5 \times 0.2 = 1.0 \text{ m}^3$  contains air and water. Air is heated until all the water is displaced away. The analysis is carried out for two states:

**State 1:**

- Initial air volume:  $V_{\text{air}} = 0.3 \text{ m}^3$
- water volume:  $V_{\text{H}_2\text{O}} = V_{\text{tot}} - V_{\text{air}} = 1 - 0.3 = 0.7 \text{ m}^3$
- water mass:  $\frac{V_{\text{H}_2\text{O}}}{v_{\text{H}_2\text{O}}} = \frac{0.7}{0.001002} = 698.6 \text{ kg}$
- Initial air pressure (calculated considering the piston force and the opposing atmospheric pressure):

$$p_0 = \frac{m_{\text{H}_2\text{O}} \times g}{A} + p_{\text{atm}} = \frac{698.6 \times 9.81}{0.2} + 101.325 = 135.59 \text{ kPa}$$

- Air mass:  $m_{\text{air}} = \frac{pV}{RT} = \frac{135.59 \times 0.3}{0.287 \times 300} \left[ \frac{\text{kPa} \times \text{m}^3}{\text{kJ}/(\text{kg K}) \times \text{K}} \right] = 0.472 \text{ kg}$

**State 2:**

- ▶ Final air volume:  $V_2 = 5 \times 0.2 = 1 \text{ m}^3$
- ▶ Final air pressure:  $p_2 = p_{\text{atm}} = 101.325 \text{ kPa}$
- ▶ Final air temperature (calculated according to the state equation, considering  $m_{\text{air}}$  is kept constant):

$$T_2 = T_1 \frac{p_2 V_2}{p_1 V_1} = 300 \times \frac{101.325 \times 10^3 \times 1}{135.59 \times 10^3 \times 0.3} = 747.29 \text{ K}$$

### Work performed by air

The work performed by air during the expansion process is:

$$W_{12} = \int p \, dV = \frac{p_1 + p_2}{2} (V_2 - V_1)$$

Substituting the values:

$$W = \frac{135.59 + 101.325}{2} (1 - 0.3) = 82.92 \text{ kJ}$$

### Heat transferred to air

The energy transferred to the system is decomposed into the heat which has been added to the gas and the work performed by the gas during its expansion:

$$\Delta U = Q + W = m_{\text{ar}} c_v (T_2 - T_1) + \int_1^2 p \, dV$$

Where  $c_v = 0.717 \text{ kJ}/(\text{kg K})$ . Carrying out the numerical application, the internal energy variation is given by:

$$\Delta U = 0.472 \text{ kg} \times 0.717 \text{ kJ}/(\text{kg K}) \times (747.59 - 300) \text{ K} + 82.92 \text{ kJ} = 234.40 \text{ kJ}$$

Therefore the total heat transfer to air is approximately 234.4 kJ.

### Answer 1.5.10

Two sections of an isolated cylinder are separated by a movable piston, containing air on sides A and B. The system reaches thermal and pressure equilibrium, and we need to calculate the air masses in both sides, as well as the final values for temperature and pressure.

#### Initial data

##### Side A:

- ▶ Volume:  $V_A = 1 \text{ m}^3$
- ▶ Initial pressure:  $p_A = 300 \text{ kPa}$
- ▶ Initial temperature:  $T_A = 350 \text{ K}$
- ▶ Air molar mass:  $m_m = 28.7 \text{ g/mol} = 0.0287 \text{ kg/mol}$
- ▶ Gas constant for air:  $R = 8.314/m_m = 0.287 \text{ kJ}/(\text{kg K})$

We calculate the air mass on A using the ideal gas equation:

$$m_A = \frac{p_A V_A}{R T_A} = \frac{300 \times 1}{0.287 \times 350} = 2.987 \text{ kg}$$

##### Side B:

- ▶ Volume:  $V_B = 1 \text{ m}^3$
- ▶ Initial pressure:  $p_B = 2.0 \text{ MPa} = 2000 \text{ kPa}$
- ▶ Initial temperature:  $T_B = 1010 \text{ K}$

Air mass in B:

$$m_B = \frac{p_B V_B}{RT_B} = \frac{2000 \times 1}{0.287 \times 1010} = 6.900 \text{ kg}$$

**Final equilibrium:**

For the final state, temperatures and pressures are evened out on both sides:

$$T_A = T_B = T_f \quad \text{and} \quad p_A = p_B = p_f$$

**Energy conservation:**

The system total internal energy is conserved:

$$U_i = U_f \quad \Rightarrow \quad m_A c_v T_A + m_B c_v T_B = (m_A + m_B) c_v T_f$$

Simplifying, we may write:

$$T_f = \frac{m_A T_A + m_B T_B}{m_A + m_B}$$

Substituting:

$$T_f = \frac{2.987 \times 350 + 6.900 \times 1010}{2.987 + 6.900} \approx 810.06 \text{ K}$$

**Final pressure:**

Using the ideal gas equation for the cylinder total volume:

$$p_f = \frac{(m_A + m_B) R T_f}{V_{\text{total}}}$$

Substituting:

$$p_f = \frac{(2.987 + 6.900) \times 0.287 \times 810.60}{2} \approx 1149.30 \text{ kPa}$$

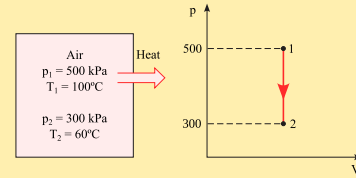
Accordingly the air mass in A and B is 3.0 kg and 6.9 kg, respectively. The final pressure is approximately 1149 kPa and the final temperature is 810 K.

## 1.6 Week 6

### 1.6.1 Exercises

#### Exercise 1.6.1

A rigid tank contains air at 500 kPa and 100 °C. As a result of heat transfer to the surroundings, the temperature and pressure inside the tank drop to 60 °C and 300 kPa, respectively. Determine the boundary work done during this process.

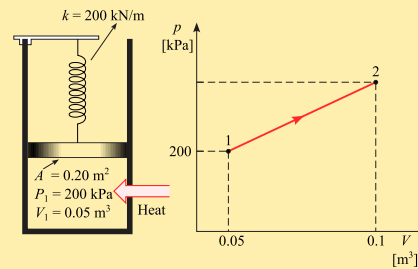


Schematic and pV diagram

#### Exercise 1.6.2

A piston-cylinder device contains 0.05 m<sup>3</sup> of a gas initially at 200 kPa. At this state, a linear spring that has a spring constant of 200 kN is touching the piston but exerting no force on it. Now heat is transferred to the gas, causing the piston to rise and to compress the spring until the volume inside the cylinder doubles. If the cross-sectional area of the piston is 0.2 m<sup>2</sup>, determine:

1. the final pressure inside the cylinder,
2. the total work done by the gas,
3. the fraction of this work done against the spring to compress it.

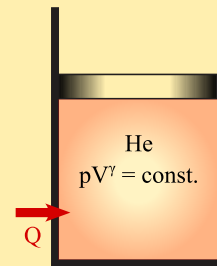


Schematic and pV diagram

#### Exercise 1.6.3

A piston-cylinder device contains Helium gas initially at 150 kPa, 25 °C, and 0.2 m<sup>3</sup>. Helium is now compressed in a polytropic process ( $pV^n = \text{const.}$ ) to 500 kPa and 180 °C. Determine the heat loss or gain during this process.

**Note:** To obtain the necessary constants for Helium, please check table B.1 in appendix.



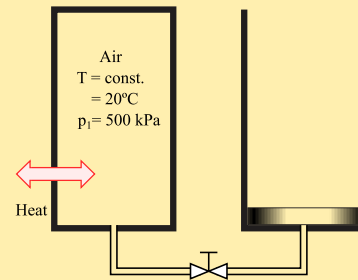
#### Exercise 1.6.4

Consider a piston-cylinder device that contains 0.5 kg air. Now, heat is transferred to the air at constant pressure and the air temperature increases by 15 °C. Determine the expansion work done during this process.

**Note:**  $M_{\text{air}} = 28.97 \text{ g/mol}$

**Exercise 1.6.5**

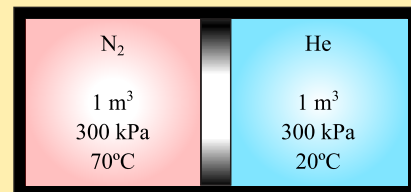
A rigid tank containing  $0.5 \text{ m}^3$  of air at  $500 \text{ kPa}$  and  $20^\circ\text{C}$  is connected by a valve to a piston-cylinder device with zero clearance. The temperature is kept at  $20^\circ\text{C}$ . The mass of the piston is such that a pressure of  $200 \text{ kPa}$  is required to raise the piston. The valve is now opened slightly, and air is allowed to flow into the cylinder until the pressure in the tank drops to  $200 \text{ kPa}$ . During this process, heat is exchanged with the surroundings such that the entire air remains at  $30^\circ\text{C}$  at all times. Determine the heat transfer for this process.



**Note:**  $M_{\text{air}} = 28.97 \text{ g/mol}$

**Exercise 1.6.6**

Consider a well-insulated horizontal rigid cylinder that is divided into two compartments by a piston that is free to move but does not allow either gas to leak into the other side. Initially, one side of the piston contains  $1 \text{ m}^3$  of  $\text{N}_2$  gas at  $300 \text{ kPa}$  and  $70^\circ\text{C}$  while the other side contains  $1 \text{ m}^3$  of  $\text{He}$  gas at  $300 \text{ kPa}$  and  $20^\circ\text{C}$ . Now thermal equilibrium is established in the cylinder as a result of heat transfer through the piston. Using constant specific heats at room temperature, determine the final equilibrium temperature in the cylinder. What would your answer be if the piston were not free to move?

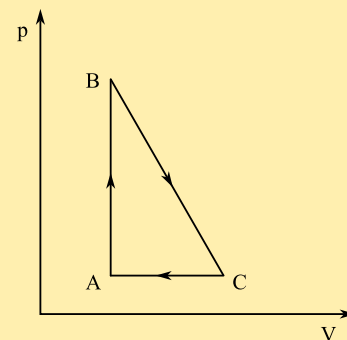


**Note:** To obtain the necessary constants for helium and nitrogen, please check tables B.1.

**Exercise 1.6.7**

$0.4 \text{ mol}$  of an atomic gas are subject to a cyclical process, described in the  $pV$  diagram according to the pathways  $A \rightarrow B \rightarrow C$ . Consider  $p_A = 1 \text{ bar}$ ,  $V_A = 10 \text{ L}$ ,  $p_B = 4 \text{ bar}$ ,  $V_C = 25 \text{ L}$ .

1. Determine the gas temperatures in  $A, B, C$
2. Determine the work performed by the cycle
3. Determine the heat rejected by the cycle
4. Determine the efficiency of the corresponding engine thermodynamic cycle

**Exercise 1.6.8**

Catastrophic explosions of steam boilers in the 1800s and early 1900s resulted in hundreds of deaths, which prompted the development of the “ASME Boiler and Pressure Vessel Code” in 1915. Considering that the pressurized fluid in a vessel eventually reaches equilibrium with its surroundings shortly after the explosion, the work that a pressurized fluid would do if allowed to expand adiabatically to the state of the surroundings can be viewed as the explosive energy of the pressurized fluid. Because of the very short time period of the explosion and the apparent stability afterward, the explosion process can be considered to be adiabatic with no changes in kinetic and potential energies. The closed-system conservation of energy relation in this case reduces to  $W_{\text{out}} = m(u_1 - u_2)$ . Then the explosive energy  $E_{\text{exp}}$  becomes

$$E_{exp} = m(u_1 - u_2) \quad (1.5)$$

where the subscripts 1 and 2 refer to the state of the fluid before and after the explosion, respectively. The specific explosion energy  $e_{exp}$  is usually expressed per unit volume, and it is obtained by dividing the quantity above by the total volume  $V$  of the vessel:

$$e_{exp} = \frac{u_1 - u_2}{v_1} \quad (1.6)$$

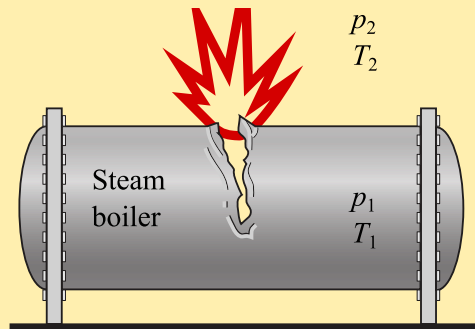
where  $v_1$  is the specific volume of the fluid before the explosion.

Show that the specific explosion energy of an ideal gas with constant specific heat is

$$e_{exp} = \frac{P_1}{k-1} \left( 1 - \frac{T_2}{T_1} \right) \quad (1.7)$$

Also, determine the total explosion energy of  $20 \text{ m}^3$  of air at  $5 \text{ MPa}$  and  $100^\circ\text{C}$  when the surroundings are at  $20^\circ\text{C}$ .

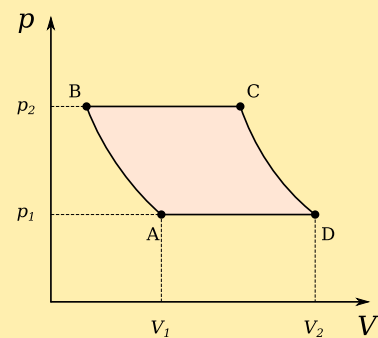
**Note:** To obtain the necessary constants for air, please check tables B.1.



Note: This exercise is taken from: Cengel, "Thermodynamics, and Engineering Approach", 5<sup>th</sup> Edition.

### Exercise 1.6.9

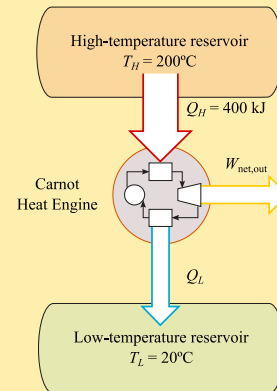
An ideal gas undergoes a process which consists in two reversible isobaric processes and two reversible isothermal processes, according to the figure below. Determine the work performed by the cycle.



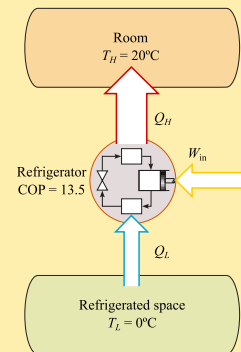
**Exercise 1.6.10**

A Carnot heat engine, shown in the next figure, receives 400 kJ of heat per cycle from a high-temperature source at  $700^\circ\text{C}$  and rejects heat to a low-temperature sink at  $20^\circ\text{C}$ . Determine:

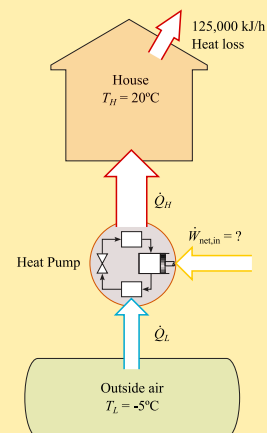
1. the thermal efficiency of this Carnot engine and
2. the amount of heat rejected to the sink per cycle.

**Exercise 1.6.11**

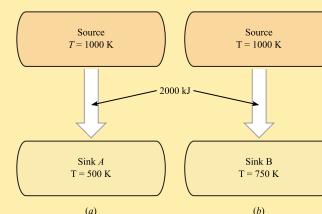
An inventor claims to have developed a refrigerator that maintains the refrigerated space at  $0^\circ\text{C}$  while operating in a room where the temperature is  $20^\circ\text{C}$  and that has a COP (Coefficient of Performance) of 13.5. Is this claim reasonable?

**Exercise 1.6.12**

A heat pump is to be used to heat a house during the winter, as shown in the Figure below. The house is to be maintained at  $20^\circ\text{C}$  at all times. The house is estimated to be losing heat at a rate of  $125\,000 \text{ kJ/h}$  when the outside temperature drops to  $-5^\circ\text{C}$ . Determine the minimum power required to drive this heat pump.

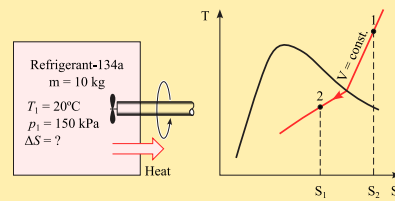
**Exercise 1.6.13**

A heat source at  $1000 \text{ K}$  loses  $2000 \text{ kJ}$  of heat to a sink at (a)  $500 \text{ K}$  and (b)  $750 \text{ K}$ . Determine which heat transfer process is more irreversible.



**Exercise 1.6.14**

A rigid tank contains 5 kg of refrigerant-134a initially at 20 °C and 150 kPa. The refrigerant is now cooled while being stirred until its pressure drops to 100 kPa. Determine the entropy change of the refrigerant during this process.



Schematic and TS diagram

**Note:** Please check the steam tables for R-134a in appendix C.3.

**Exercise 1.6.15**

Show the Carnot cycle on a  $T - S$  diagram and indicate the areas that represent the heat supplied  $Q_H$ , heat rejected  $Q_C$ , and the net work output  $W_{net,out}$  on this diagram.

## Formulae

### Constants

$$1\text{eV} = 1.602 \times 10^{-19} \text{ J}$$

$$k_B = 1.381 \times 10^{-23} \text{ m}^2/(\text{kg s}^2 \text{ K}) \text{ or J/K}$$

$$R = 8.31446 \text{ J/(mol K)}$$

$$N_A = 6.022 \times 10^{23} / \text{mol}$$

$$p_{atm} = 101.325 \text{ kPa}$$

$$T_{amb} = 20^\circ \text{C}$$

### Formulas

**Specific Volume:**  $v = \frac{V}{m}$

**Ideal Gas Equation:**  $pV = nRT$

**Specific Heats for an Ideal Gas:**  $C_p = C_v + R$

**1<sup>st</sup> Law of Thermodynamics:**  $\Delta U = Q - W$

**Work:**  $W = \int F dx = \int p dV$

**Internal Energy of a Gas:**  $U = \frac{N}{2} nRT$ ,  $N$ : degrees freedom.

$\Delta U = nC_v \Delta T$

$\Delta H = nC_p \Delta T$

**Value of a property  $y$  in a saturated liquid/vapor mixture:**  $y = y_f + x y_{fg}$  with  $x = \frac{m_g}{m_{tot}}$  the mass fraction for the gas in the mixture ( $y$  can represent the specific volume  $v$ , the energy density  $u$ , the entropy density  $s$ , among others)

**Engine Efficiency:**  $\eta = \frac{W}{Q_{in}}$

**COP of a Heat Pump:**  $COP = \frac{Q_{in}}{W}$

**COP of a Refrigerator:**  $COP = \frac{Q_{out}}{W}$

**Efficiency for a Carnot Cycle:**  $\eta = 1 - \frac{T_2}{T_1}$

For cycles operating between two heat sources, with temperatures  $T_H$  and  $T_C$ , with  $T_H > T_C$ .

$Q_H$  is the heat transferred between the gas and the heat source at temperature  $T_H$ .

$Q_C$  is the heat transferred between the gas and the heat sink at temperature  $T_C$ ;

**Entropy:**  $\Delta S = \frac{Q_{rev}}{T}$

**Density of entropy:**  $s = \frac{S}{m}$

$T(\text{K}) = 273.15 + T(^{\circ}\text{C})$

### 1.6.2 Solutions to Exercises

#### Answer 1.6.1

The work performed during a thermodynamic process is given by:

$$W = \int_{V_1}^{V_2} p \, dV$$

Since the volume remains constant ( $dV = 0$ ). Therefore the work performed is by definition:

$$W = 0$$

**Conclusion:** Since the volume remains constant, there is no work performed during this process.

#### Answer 1.6.2

The following initial conditions for a piston system are given:

- Initial volume:  $V_1 = 0.05 \, \text{m}^3$
- Initial pressure:  $p_1 = 200 \, \text{kPa}$
- Spring constant:  $k = 200 \, \text{kN/m}$
- Piston area:  $A = 0.2 \, \text{m}^2$
- Final volume:  $V_f = 0.10 \, \text{m}^3$

1. The force exerted by the spring is:

$$F = -k \cdot x$$

Where  $x$  is the displacement, calculated as:

$$x = \frac{\Delta V}{A} = \frac{0.10 - 0.05}{0.2} = 0.25 \, \text{m}$$

Accordingly, the force exerted by the spring is:

$$F = -200 \times 0.25 = -50 \, \text{kN}$$

The additional pressure exerted by the spring is:

$$p = \frac{F}{A} = \frac{50 \times 10^3}{0.2} = 2.5 \times 10^5 \, \text{Pa} = 250 \, \text{kPa}$$

The new final pressure will be the sum of both the initial and additional pressures:

$$p_f = p_1 + p = 200 \, \text{kPa} + 250 \, \text{kPa} = 450 \, \text{kPa}$$

2. To calculate the performed work, we examine the area under the curve on a  $p - V$  graph. The work performed is the sum of the rectangular and triangular areas:

$$W = 0.05 \times 200 + \frac{0.05 \times 250}{2} = 16.25 \, \text{kJ}$$

3. The area ratio corresponds to the piston movement (triangular area) divided by the total work:

$$\eta = \frac{A_{II}}{W} = \frac{6.25}{16.25} \approx 0.38$$

**Answer 1.6.3**

The following information is provided:

- ▶ Initial pressure:  $p_1 = 150 \text{ kPa}$
- ▶ Initial temperature:  $T_1 = 25^\circ = 298.15 \text{ K}$
- ▶ Initial volume:  $V_1 = 0.2 \text{ m}^3$
- ▶ Final pressure :  $p_2 = 500 \text{ kPa}$
- ▶ Final temperature:  $T_2 = 180^\circ = 453.15 \text{ K}$
- ▶ Gas constant (Helium):  $R_{He} = 2.0773 \text{ kJ}/(\text{kg K})$

consulting the gas specific heat properties (table B.1), we obtain:

$$R_{He} = 2.0773 \text{ kJ}/(\text{kg K})$$

Using the ideal gas law:

$$pV = mRT$$

We may obtain the mass  $m$ :

$$m = \frac{p_1 V_1}{RT_1} = \frac{150 \times 0.2}{2.0773 \times 298.15} = 0.048 \text{ kg}$$

Using the ideal gas law, the relationship between volume and temperature is:

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$$

Isolating  $V_2$ :

$$V_2 = V_1 \frac{p_1 T_2}{p_2 T_1} = 0.2 \times \frac{150 \times 453.15}{500 \times 298.15} = 0.09 \text{ m}^3$$

The transformation is polytropic, and we accordingly have:

$$p_2 V_2^n = p_1 V_1^n$$

Where  $n$  is the polytropic coefficient. Using the relationship:

$$\frac{V_2}{V_1} = \left( \frac{p_1}{p_2} \right)^{\frac{1}{n}} \Rightarrow 0.45596 = \left( \frac{150}{500} \right)^{\frac{1}{n}} \Rightarrow n = 1.533$$

The work performed during the polytropic process is:

$$W = \int_{V_i}^{V_f} p \, dV$$

We know that:

$$p_1 V_1^n = 12.72 \times 10^3 \Rightarrow p = \frac{12.72 \times 10^3}{V^n}$$

Substituting in the equation for work:

$$W = \int_{V_i}^{V_f} \frac{12.72 \times 10^3}{V^n} \, dV$$

The integral solution is:

$$W = \frac{12.72 \times 10^3}{n-1} \left( \frac{1}{V_f} - \frac{1}{V_i} \right)$$

Finally, substituting the numerical values:

$$W = 145.841 \text{ kJ}$$

Applying the 1<sup>st</sup> Law of Thermodynamics:

$$\Delta U = Q - W \quad \text{e} \quad \Delta U = mc_V \Delta T$$

Using  $c_V = 3.1159$  (obtained from the Helium properties – table B.1):

$$Q_{in} = 0.048 \times 3.1159 \times 155 - 145.841 = -122.659 \text{ kJ}$$

Therefore, heat has been lost during the process.

#### Answer 1.6.4

The following information is provided:

- ▶ Mass:  $m = 0.5 \text{ kg}$
- ▶ Pressure:  $p = \text{const.}$
- ▶ Temperature variation:  $\Delta T = 15^\circ\text{C} = 15 \text{ K}$
- ▶ Gas constant for air (table B.1):  $R_{ar} = 0.2870 \text{ kJ}/(\text{kg K})$

The work performed for an isobaric process is:

$$W = \int_{V_i}^{V_f} p \, dV$$

The ideal gas equation provides us with the relationship between pressure, volume and temperature:

$$pV_i = mRT_i$$

We further know that:

$$T_f = T_i + \Delta T = T_i + 15 \text{ K}$$

Therefore, the temperature variation may be expressed as:

$$\Delta T = \frac{p(V_f - V_i)}{mR}$$

We may then rewrite work as:

$$W = p(V_f - V_i) = mR\Delta T$$

Substituting:

$$W = 0.5 \times 0.2870 \times 15 = 2.15 \text{ kJ}$$

#### Answer 1.6.5

Initial conditions:

- ▶ Initial volume:  $V_i = 0.5 \text{ m}^3$
- ▶ Initial pressure:  $p_i = 500 \text{ kPa}$
- ▶ Initial temperature:  $T_i = 20^\circ\text{C} = 293.15 \text{ K}$

Using the ideal gas law and the 1<sup>st</sup> Law of thermodynamics (assuming temperature is constant, e.g.  $\Delta U = 0$ ):

$$Q = W$$

Using the ideal gas law:

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$$

Since the temperature is constant we may simplify:

$$V_2 = \frac{p_1}{p_2} V_1 = \frac{500}{200} \times 0.5 = 1.25 \text{ m}^3$$

Now, assuming the piston remains at a constant pressure  $p = 200 \text{ kPa}$ , the work performed is:

$$W = \int_{V_i}^{V_f} p \, dV = p(V_f - V_i)$$

Substituting:

$$W = 200 \times (1.25 - 0.5) = 200 \times 0.75 = 150 \text{ kJ}$$

Therefore, the performed work is 150 kJ, which also corresponds to the exchanged heat, since  $Q = W$ .

### Answer 1.6.6

The following information is provided:

- ▶ Volume of Nitrogen:  $V_{N_2} = 1 \text{ m}^3$
- ▶ Pressure of Nitrogen:  $p_{N_2} = 300 \text{ kPa}$
- ▶ Temperature of Nitrogen:  $T_{N_2} = 70^\circ \text{C} = 343.15 \text{ K}$
- ▶ Volume of Helium:  $V_{He} = 1 \text{ m}^3$
- ▶ Pressure of Helium:  $p_{He} = 300 \text{ kPa}$
- ▶ Temperature of Helium:  $T_{He} = 20^\circ \text{C} = 293.15 \text{ K}$

Constants obtained for specific heats of ideal gases (table B.1):

- ▶ Gas constant for Helium:  $R_{He} = 2.0773 \text{ kJ}/(\text{kg K})$
- ▶ Gas constant for Nitrogen:  $R_{N_2} = 0.2968 \text{ kJ}/(\text{kg K})$
- ▶ Constant volume specific heat for Helium:  $C_{vHe} = 3.1159 \text{ kJ}/(\text{kg K})$
- ▶ Constant volume specific heat for Nitrogen:  $C_{vN_2} = 0.7429 \text{ kJ}/(\text{kg K})$

We calculate the mass for each gas using the ideal gas equation:

$$m_{N_2} = \frac{p_{N_2} V_{N_2}}{R_{N_2} T_{N_2}} = \frac{300 \times 1}{0.2968 \times 343.15} = 2.9456 \text{ kg}$$

$$m_{He} = \frac{p_{He} V_{He}}{R_{He} T_{He}} = \frac{300 \times 1}{2.0773 \times 293.15} = 0.4899 \text{ kg}$$

Applying the 1<sup>st</sup> Law of thermodynamics and assuming there is no variation of internal energy ( $\Delta E = 0$ ), we may write the thermal balance equation:

$$m_{He} C_{vHe} (T_e - T_{He}) = -m_{N_2} C_{vN_2} (T_e - T_{N_2})$$

Solving for the equilibrium temperature  $T_e$ :

$$0.4899 \times 3.1159 \times (T_e - 293.15) = -2.9456 \times 0.7429 \times (T_e - 343.15)$$

Solving for  $T_e$ :

$$T_e = 322.465 \text{ K} = 49.30^\circ \text{C}$$

The temperature would be equal in case the piston would be held in place as this condition only affects the final pressure  $p$  and would not modify the specific heats for the gases.

**Answer 1.6.7**

Available data:

- ▶  $n = 0.4 \text{ mol}$
- ▶  $p_A = 1 \text{ bar} = 1 \times 10^5 \text{ Pa}$
- ▶  $V_A = 10 \text{ L} = 10 \times 10^{-3} \text{ m}^3$
- ▶  $p_B = 4 \text{ bar} = 4 \times 10^5 \text{ Pa}$
- ▶  $V_B = 25 \text{ L} = 25 \times 10^{-3} \text{ m}^3$

1.

$$p_A V_A = nRT_A \Rightarrow T_A = 300.70 \text{ K}$$

$$V_B = V_A \Rightarrow p_B V_B = nRT_B \Rightarrow T_B = 1202.79 \text{ K}$$

$$p_C = p_A \Rightarrow p_C V_C = nRT_C \Rightarrow T_C = 751.74 \text{ K}$$

2. Since we have a monoatomic gas,

$$C_V = \frac{3}{2}R = 12.471 \text{ J/(mol K)}$$

**From A to B:**No work is performed,  $W_{AB} = 0$ **From C to A:**

$$W_{CA} = \int p \, dV = 1.05 \times 10^5 \times 1.5 \times 10^{-2} = -1.5 \text{ kJ}$$

**From B to C:**

$$p = mV + b$$

$$m = \frac{p_C - p_A}{V_C - V_A} = \frac{1 \times 10^5 - 4 \times 10^5}{25 \times 10^{-3} - 10 \times 10^{-3}} = 2.0 \times 10^7 \text{ Pa/m}^3$$

$$p = 2 \times 10^7 V + b$$

$$4 \times 10^7 = -2 \times 10^7 \times 10^{-2} + b \Rightarrow p = -2 \times 10^7 V + 6 \times 10^5$$

$$\begin{aligned} W_{BC} &= \int p \, dV = -10^7 (V_C^2 - V_B^2) + 6 \times 10^5 (V_C - V_A) \\ &= 3750 \text{ J} \end{aligned}$$

Accordingly,

$$W = W_{AB} + W_{CA} + W_{BC} = 2.25 \text{ kJ}$$

3. For  $Q_{AB}$ :

$$\begin{aligned} Q_{AB} &= \Delta U_{AB} + W_{AB} = nC_V \Delta T = 0.4 \times 12.471 \times (1202.79 - 300.70) \\ &= 2250.02 \text{ J} \end{aligned}$$

For  $Q_{BC}$ :

$$\Delta Q_{BC} = \Delta U_{BC} + W_{BC} = 3750 + 0.4 \times 12.471 \times (751.74 - 1202.79) = 1500 \text{ J}$$

For  $Q_{CA}$ :

$$\Delta Q_{CA} = \Delta U_{CA} + W_{CA} = 0.4 \times 12.471 \times (300.70 - 751.74) - 1500 = -3749.49 \text{ J}$$

The total heat exchange:

$$\Delta Q = \Delta Q_{AB} + \Delta Q_{BC} + \Delta Q_{CA} = 0$$

4. Efficiency:

$$\eta = \frac{|W|}{W + |\Delta Q_{CA}|} = 0.50$$

### Answer 1.6.8

Initial conditions:

- ▶ Initial volume:  $V = 20 \text{ m}^3$
- ▶ Initial pressure:  $p_1 = 5 \text{ MPa} = 5 \times 10^6 \text{ Pa}$
- ▶ Initial temperature:  $T_1 = 100^\circ\text{C} = 373.15 \text{ K}$
- ▶ Final temperature:  $T_2 = 30^\circ\text{C} = 303.15 \text{ K}$
- ▶ specific heats ratio:  $\gamma = \frac{C_p}{C_v}$
- ▶ Gas constant for air:  $R_{ar} = 0.2870 \text{ kJ}/(\text{kg K})$

We know the expansion energy may be written as:

$$E_{exp} = m(u_2 - u_1)$$

where:

$$u_2 - u_1 = C_v(T_2 - T_1), \text{ for an ideal gas}$$

Additionally, for an ideal gas we have the relationship between  $C_p$  and  $C_v$ :

$$C_p - C_v = R$$

Therefore,  $\gamma$  (ratio of specific heats) may be written as:

$$\gamma = \frac{C_p}{C_v} \Rightarrow \frac{C_v}{R} = \frac{1}{\gamma - 1}$$

Accordingly we may calculate the expansion energy per unit mass as:

$$e_{exp} = \frac{p_1}{\gamma - 1} \left( 1 - \frac{T_2}{T_1} \right)$$

Substituting:

$$E_{exp} = e_{exp} \cdot V = \frac{5 \times 10^6 \times 20}{1.4 - 1} \times \left( 1 - \frac{373.15}{303.15} \right) = 4.7 \times 10^7 \text{ J} = 47 \text{ MJ}$$

### Answer 1.6.9

We consider a thermodynamic process composed of four individual processes:

- ▶ AB: isothermal process
- ▶ BC: isobaric process
- ▶ CD: isothermal process
- ▶ DA: isobaric process

#### Process AB: Isothermal

The work performed during the isothermal process is given by:

$$w_{AB} = \int_{V_A}^{V_B} p \, dV = nRT_A \ln \left( \frac{V_B}{V_A} \right)$$

#### Process BC: Isobaric

For the isobaric process, the work is:

$$w_{BC} = \int_{V_B}^{V_C} p \, dV = p_B(V_C - V_B)$$

#### Process CD: Isothermal

Similarly to process AB, the isothermal work on CD is:

$$w_{CD} = \int_{V_C}^{V_D} p \, dV = nRT_C \ln \left( \frac{V_D}{V_C} \right)$$

#### Process DA: Isobaric

For the isobaric process DA, the work is:

$$w_{DA} = \int_{V_D}^{V_A} p \, dV = p_D(V_A - V_D)$$

#### Total work during the cycle

The total work is the sum of the work for each process:

$$W = w_{AB} + w_{BC} + w_{CD} + w_{DA}$$

Substituting:

$$W = nRT_A \ln \left( \frac{V_B}{V_A} \right) + p_B(V_C - V_B) + nRT_C \ln \left( \frac{V_D}{V_C} \right) + p_D(V_A - V_D)$$

Using the equation for ideal gas and knowing that  $p_B = p_C = p_2, p_A = p_D = p_1, V_A = V_1, V_D = V_2$ , we have:

$$p_1 V_1 = nRT_A \Rightarrow T_A = \frac{p_1 V_1}{nR}$$

$$p_1 V_2 = nRT_D \Rightarrow T_D = \frac{p_1 V_2}{nR}$$

$$p_2 V_C = nRT_C \Rightarrow V_C = \frac{p_1}{p_2} V_2$$

$$p_2 V_B = nRT_A \Rightarrow V_B = \frac{p_1}{p_2} V_1$$

Substituting:

$$W = p_1 V_1 \ln \left( \frac{\frac{p_1}{p_2} V_1}{V_1} \right) + p_2 \left( \frac{p_1}{p_2} V_2 - \frac{p_1}{p_2} V_1 \right) + p_2 \frac{p_1}{p_2} V_2 \ln \left( \frac{V_2}{\frac{p_1}{p_2} V_2} \right) + p_1 (V_1 - V_2)$$

$$W = p_1 V_1 \ln \left( \frac{p_1}{p_2} \right) + p_1 (V_2 - V_1) + p_1 V_2 \ln \left( \frac{p_1}{p_2} \right) + p_1 (V_1 - V_2)$$

The final expression for the work is therefore:

$$W = p_1 \ln \left( \frac{p_1}{p_2} \right) (V_1 - V_2)$$

**Answer 1.6.10**

Available data:

- Heat source temperature:  $T_H = 700^\circ\text{C} = 923.15\text{ K}$
- Heat sink temperature:  $T_C = 20^\circ\text{C} = 293.15\text{ K}$

1. The efficiency of a Carnot cycle is given by:

$$\eta = 1 - \frac{T_C}{T_H}$$

Substituting:

$$\eta = 1 - \frac{293.15}{923.15} = 0.68$$

2. We know the efficiency may also be expressed as:

$$\eta = \frac{W_{net}}{Q_H}$$

Accordingly  $W_{net} = Q_H \times \eta = 272.98\text{ kJ}$ , and we have:

$$\begin{aligned} Q_C &= Q_H - W_{net} \\ &= 127.02\text{ kJ} \end{aligned}$$

**Answer 1.6.11**

Given a refrigerator operating between the following temperatures:

- Heat source temperature:  $T_H = 20^\circ\text{C} = 293.15\text{ K}$
- Heat sink temperature:  $T_F = 0^\circ\text{C} = 273.15\text{ K}$

The Coefficient of Performance (COP) of a Carnot refrigerator, which would be the refrigerator with the largest COP, is given by the formula:

$$COP = \frac{1}{\left(\frac{T_H}{T_F} - 1\right)}$$

Substituting for  $T_H$  e  $T_F$ :

$$COP = \frac{1}{\left(\frac{293.15}{273.15} - 1\right)} = \frac{1}{0.0732} = 13.65$$

Since  $13.65 > 13.5$ , the statement that the COP is higher than 13.5 is reasonable.

**Answer 1.6.12**

Available data:

- Heat source temperature:  $T_H = 20^\circ\text{C} = 293.15\text{ K}$
- Heat sink temperature:  $T_C = -5^\circ\text{C} = 268.15\text{ K}$
- Heat rejected by the heat pump:  $\dot{Q}_H = 34\,722.22\text{ W} = 34.7\text{ kW}$

The Coefficient of Performance (COP) for the Carnot heat pump is given by:

$$COP = \frac{1}{1 - \frac{T_C}{T_H}}$$

Substituting:

$$\text{COP} = \frac{1}{1 - \frac{268.15}{293.15}} = 11.726$$

**Determining Work:**

The work performed by the heat pump may be determined through the relationship between the rejected heat and the COP:

$$\dot{W} = \frac{\dot{Q}_H}{\text{COP}}$$

Substituting:

$$\dot{W} = \frac{34.7}{11.726} = 2.96 \text{ kW}$$

### Answer 1.6.13

For case (a):

$$\Delta S_{\text{source}} = \frac{-2000 \text{ J}}{1000 \text{ K}} = -2.0 \text{ kJ/K}$$

$$\Delta S_{\text{sink}} = \frac{2000 \text{ J}}{500 \text{ K}} = 4.0 \text{ kJ/K}$$

The total entropy variation for the system is:

$$S_a = 4.0 - 2.0 = 2.0 \text{ kJ/K}$$

For case (b):

$$\Delta S_{\text{source}} = -2.0 \text{ kJ/K}$$

$$\Delta S_{\text{sink}} = \frac{2000 \text{ J}}{750 \text{ K}} = 2.66 \text{ kJ/K}$$

The total entropy variation for the system is:

$$S_b = 2.66 - 2.0 = 0.66 \text{ kJ/K}$$

Since  $S_a > S_b$ , (a) is more irreversible.

### Answer 1.6.14

Available data:

- ▶  $m = 10 \text{ kg}$
- ▶  $T_1 = 20^\circ\text{C} = 293.15 \text{ K}$
- ▶  $p_1 = 150 \text{ kPa}$

For the given conditions, we need to interpolate the tabulated values:

**For  $p_1 = 140 \text{ kPa}$  and  $T = 20^\circ\text{C}$ :**

$$s_1 = 1.858 \text{ kJ/(kg K)}, \quad v_1 = 0.16544 \text{ m}^3/\text{kg}$$

**For  $p_2 = 180 \text{ kPa}$  and  $T = 20^\circ\text{C}$ :**

$$s_2 = 1.8355 \text{ kJ/(kg K)}, \quad v_2 = 0.12748 \text{ m}^3/\text{kg}$$

$$S = \frac{s_1 - s_2}{p_1 - p_2} P + b \Rightarrow b = 1.93675 \Rightarrow s = 1.852 \text{ kJ}/(\text{kg K})$$

$$V = \frac{v_1 - v_2}{p_1 - p_2} P + b \Rightarrow b = 0.2983 \Rightarrow v = 0.15595 \text{ m}^3/\text{kg}$$

For  $p_2 = 100 \text{ kPa}$ :

$$v_f = 0.000726 \text{ m}^3/\text{kg}, \quad v_g = 0.19256 \text{ m}^3/\text{kg}$$

$$s_f = 0.8884 \text{ kJ}/(\text{kg K}), \quad s_{fg} = 0.8551 \text{ kJ}/(\text{kg K})$$

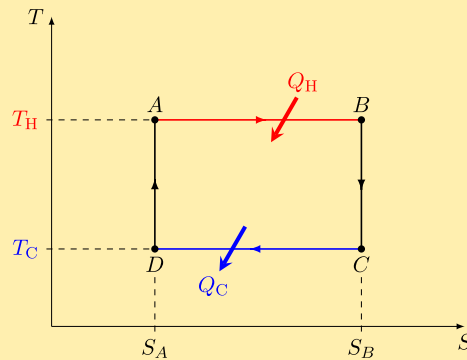
**Determining the Quality:**

$$x_2 = \frac{v - v_f}{v_{fg}} = \frac{0.15595 - 0.000726}{0.19256 - 0.000726} = 0.809$$

$$s' = s_f + x_2 s_g = 0.8678 + 0.8799 \times 0.809 = 1.579 \text{ kJ}/(\text{kg K})$$

$$\Delta S = m(s' - s) = 10(1.579 - 1.852) = -2.73 \text{ kJ/K}$$

**Answer 1.6.15**



## 1.7 Week 7

### 1.7.1 Exercises

#### Exercise 1.7.1

An ideal Otto cycle has a compression ratio of 10. At the beginning of the compression process, air is at 100 kPa and 20 °C. 750 kJ/kg of heat is transferred to air through a constant volume heat addition process. Accounting for the variation of the specific heat of air with temperature, determine:

1. the maximum temperature and pressure which occur during the cycle;
2. the total work;
3. the thermal efficiency;
4. the mean effective pressure (MEP) for the cycle.

**Note:** Use the *ideal gas air* table D.1 to determine the variation of the air properties for each process.

#### Exercise 1.7.2

A standard air cycle with variable specific heats is performed on a closed system. It is composed of the following four processes:

- ▶ A → B: Adiabatic Compression from 100 kPa and 20 °C to 750 kPa
- ▶ B → C: Heat addition at  $V = \text{const.}$  up to 1500 K
- ▶ C → D: Adiabatic Expansion to 100 kPa
- ▶ D → A: Heat rejection at  $p = \text{const.}$  to the initial state

1. Draw the p–V and T–S diagrams;
2. Calculate the work per unit mass;
3. Determine the thermal efficiency.

**Note:** Use the *ideal gas air* table D.1 to determine the variation of the air properties for each process.

#### Exercise 1.7.3

A standard air cycle with variable specific heats is performed on a closed system. It is composed of the following four processes:

- ▶ A → B: Adiabatic Compression from 100 kPa e 20 °C to 700 kPa;
- ▶ B → C: Heat addition of 1600 kJ/kg at  $p = \text{const.}$ ;
- ▶ C → D: Heat rejection at  $V = \text{const.}$  until 100 kPa;
- ▶ D → A: Heat rejection at  $p = \text{const.}$  until the initial state.

1. Draw the p–V and T–S diagrams;
2. Calculate the maximum temperature for the cycle;
3. Determine the thermal efficiency. What can you conclude about the viability for this cycle?

**Note:** Use the *ideal gas air* table D.1 to determine the variation of the air properties for each process.

#### Exercise 1.7.4

Refrigerant R-134a enters the capillary tube of a refrigerator as a saturated liquid at 850 kPa, and the pressure is then reduced to 150 kPa. Assume a steady-state process and determine the quality of the refrigerant in its final state and the temperature decrease for this process. Suggestion: Consider the Energy Conservation Law for a control volume.

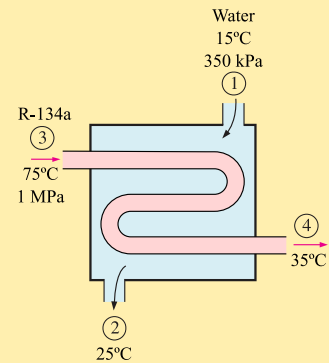
**Note:** Use the required tables (see Appendix C.3) to obtain the properties and values of Refrigerant-134a).

### Exercise 1.7.5

Refrigerant R-134a is cooled by water in a condenser. The refrigerant enters the condenser with a mass flow of 5 kg/min at 1 MPa and 75 °C and exits at 35 °C. The cooling water enters at 350 kPa and 15 °C and exits at 25 °C. Neglecting any pressure losses, determine:

1. the required mass flow of cooling water;
2. the heat transfer rate from the refrigerant to the water.

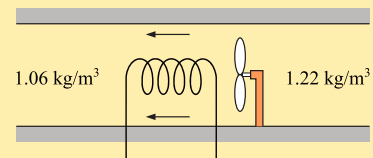
**Note:** Use the required tables to obtain the properties and values of Refrigerant-134a and water (see tables C.3 and C.2).



### Exercise 1.7.6

A hair dryer is essentially a constant diameter duct where a few layers of electrical resistances are placed into. A small fan draws the air and forces it through the resistances which in turn will heat it.

If the air density is 1.22 kg/m<sup>3</sup> at the entrance and 1.06 kg/m<sup>3</sup> at the exit, determine the percent increase of the air velocity as it flows through the dryer.



### Exercise 1.7.7

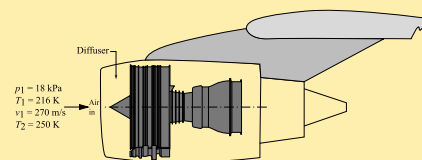
A house is kept at 1 atm and 22 °C. Hot air is forced outside the house at a rate of 150 m<sup>3</sup>/h as the result of exterior air at 4 °C entering inside the house through cracks in the structure. Determine the rate of energy loss for the house as the result of mass transfer.

**Note:**  $R_{air} = 0.287 \text{ kPa m}^3/(\text{kg K})$  and  $C_p = 1.005 \text{ kJ}/(\text{kg } ^\circ\text{C})$ .



### Exercise 1.7.8

As shown in the side picture, air enters the diffuser of a jet engine operating at steady-state at 18 kPa, 216 K and a velocity of 270 m/s (typical values for a high-altitude flight). The air flows adiabatically through the diffuser and reaches a temperature of 250 K at the exit. Using the ideal gas law, determine the airflow velocity at the diffuser exit, in m/s.



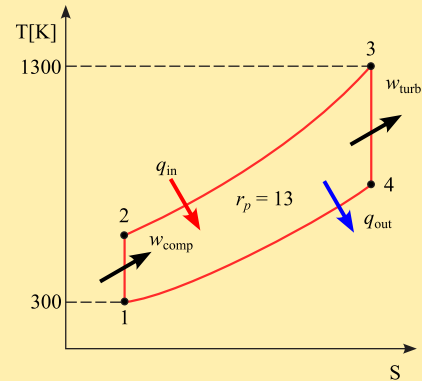
**Note:** Check the thermodynamic tables for ideal air on appendix D.1.

**Exercise 1.7.9**

A gas turbine power plant operates on the ideal Brayton cycle, with a pressure ratio of 13. The gas temperature is 300 K at compressor inlet and 1400 K at turbine inlet. Using the standard air assumptions, determine:

1. the gas temperature at compressor and turbine outlet;
2. the Back Work Ratio (BWR);
3. the thermal efficiency.

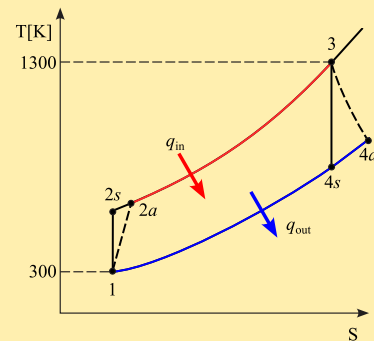
**Note:** Use the table for *air as an ideal gas*, see table D.1, to determine the air properties after the adiabatic processes.

**Exercise 1.7.10**

Assuming a compressor efficiency of 85% and a turbine efficiency of 90%, determine:

1. the Back Work Ratio (BWR);
2. the efficiency;
3. the turbine outlet temperature for the gas turbine cycle discussed in exercise 1.7.9.

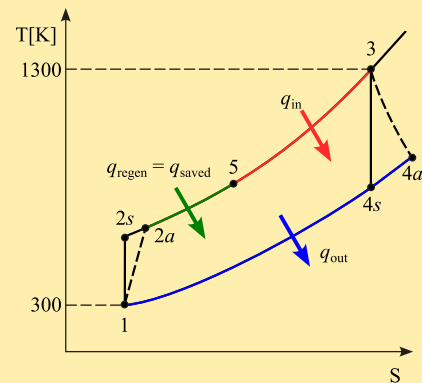
**Note:** Use the table for *air as an ideal gas*, see table D.1, to determine the air properties after the adiabatic processes.

**Exercise 1.7.11**

In the previous exercise (Ex. 1.7.10) we verified that the fluid exits the turbine at a high temperature, compared to the ambient one. We may harness this energy, which otherwise would go spoiled, to pre-heat the gas at compressor exit and save fuel during the heating process.

Determine the efficiency of the gas turbine described in Ex. 1.7.9 in case a regenerator with 85% efficiency is added to the system.

**Note:** Use the table for *air as an ideal gas*, see table D.1, to determine the air properties after the adiabatic processes.

**Exercise 1.7.12**

A mole of a diatomic gas experiences the following transformations:

- ▶ A → B: isometric from  $V_A = 0.04 \text{ m}^3$  e  $p_A = 1 \text{ atm}$  to  $p_B = 0.5 \text{ atm}$ ;
- ▶ B → C: isobaric to  $V_C = 0.06 \text{ m}^3$ ;
- ▶ C → D: isometric to the initial pressure;
- ▶ D → A: isobaric to the original point.

1. Calculate the temperatures  $T_A$ ,  $T_B$ ,  $T_C$  e  $T_D$ ;
2. Determine the heat and work exchanges with the surroundings, indicating the processes for which these occur;
3. What kind of machine do we have? Calculate its efficiency.

**Note:** Assume that the gas calorific properties are independent from  $T$ .

### Exercise 1.7.13

Gas turbines operate on the basis of the ideal Brayton cycle. a mole of a monoatomic gas, initially at 300 K and 1 atm is subjected to the following cycle:

- ▶ A  $\rightarrow$  B: adiabatic compression to  $3/5^{\text{th}}$  of its initial volume;
- ▶ B  $\rightarrow$  C: constant pressure process, leading to a temperature increase to 1300 K;
- ▶ C  $\rightarrow$  D: adiabatic expansion until the initial pressure is reached;
- ▶ D  $\rightarrow$  A: constant pressure process, resulting on a temperature decrease to its initial value.

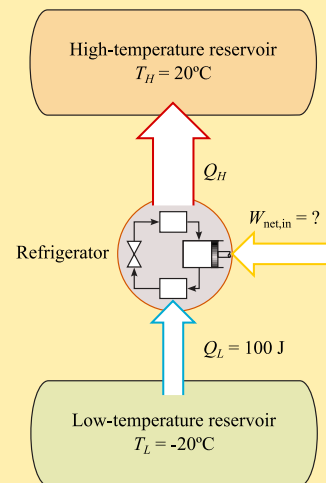
1. Sketch the cycle on the (p,V) plane and determine the pressure, volume and temperature at the edges of the cycle (A, B, C e D);
2. Determine the heat and work exchanges carried out during each phase fo the cycle;
3. Determine the efficiency of the cycle.

### Exercise 1.7.14

Consider a Carnot refrigerator which operates between the temperatures  $-20^\circ\text{C}$  and  $20^\circ\text{C}$ . During half an hour, the fluid receives 100 J from the freezer. Admit that the machine is reversible.

1. Calculate the efficiency of the machine;
2. Calculate the value of the mechanical energy given to the machine, and the thermal energy released to the heat source, during this half an hour;
3. Calculate the power of the machine;
4. Calculate (in g/s) the machine flow rate, assuming it is Refrigerant R-134a.

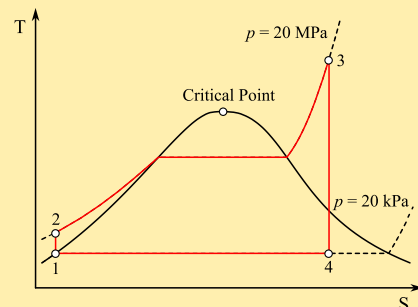
**Note:**  $\lambda_{\text{vap.}}^{R-134a} = 200 \text{ kJ/kg}$ .



### Exercise 1.7.15

Assume a simple Rankine cycle using water as a fluid operating between minimum and maximum pressures of 0.2 bar and 200 bar, respectively, the maximum superheated vapor temperature being  $500^\circ\text{C}$ . Use the vapor tables (see appendix C.2) to calculate the cycle efficiency. Compare it with the efficiency of an equivalent Carnot cycle operating between the same limit temperatures.

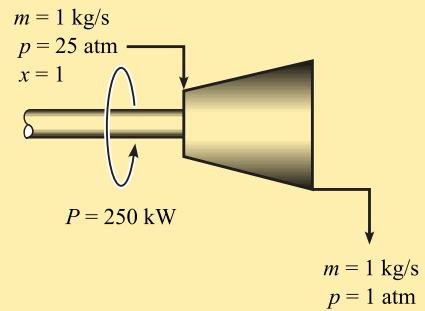
**Note:**  $1 \text{ bar} \approx 1 \text{ atm} = 101325 \text{ Pa}$ .



**Exercise 1.7.16**

A vapor turbine operates at steady-state and receives a water vapor flux of 1 kg/s at 25 atm, delivering 250 kW of power. The turbine outlet pressure is 1 atm. Assume that the turbine is embedded on a system which may approximately be described as a simple, lossless Rankine cycle.

1. Draw the cycle on a T-S diagram
2. Determine the specific enthalpy of saturated liquid water at 1 atm, using the saturated vapor tables.
3. Determine the vapor and liquid fractions at turbine outlet.



**Note:** Consult the vapor tables in Appendix C.2.

## Formulae

### Constants

$$\begin{aligned}
 1\text{eV} &= 1.602 \times 10^{-19} \text{ J} \\
 k_B &= 1.381 \times 10^{-23} \text{ m}^2/(\text{kg s}^2 \text{ K}) \text{ or J/K} \\
 R &= 8.31446 \text{ J/(mol K)} \\
 N_A &= 6.022 \times 10^{23} / \text{mol} \\
 p_{\text{atm}} &= 101.325 \text{ kPa} \\
 T_{\text{amb}} &= 20^\circ \text{C} \\
 \sigma &= 5.67 \times 10^{-8} \text{ W/m}^2/\text{K}^4
 \end{aligned}$$

### Formulas

**Specific Volume:**  $v = \frac{V}{m}$

**Ideal Gas Equation:**  $pV = nRT$

**Specific Heats for an Ideal Gas:**  $C_p = C_v + R$

**1<sup>st</sup> Law of Thermodynamics:**  $\Delta U = Q - W$

**Work:**  $W = \int F dx = \int p dV$

**Internal Energy of a Gas:**  $U = \frac{N}{2} nRT$ ,  $N$ : degrees freedom.

$\Delta U = nC_v \Delta T$

$\Delta H = nC_p \Delta T$

**Energy balance for a control volume:**  $\dot{E}_{cv} = \dot{Q}_{cv} - \dot{W}_{cv} + \sum_i \dot{m}_i \left( h_i + \frac{v_i^2}{2} + gz_i \right) - \sum_e \dot{m}_e \left( h_e + \frac{v_e^2}{2} + gz_e \right)$

**Value of a property  $y$  in a saturated liquid/vapor mixture:**  $y = y_f + x y_{fg}$  with  $x = \frac{m_g}{m_{\text{tot}}}$  the mass fraction for the gas in the mixture ( $y$  can represent the specific volume  $v$ , the energy density  $u$ , the entropy density  $s$ , among others)

**Engine Efficiency:**  $\eta = \frac{W}{Q_{in}}$

**COP of a Heat Pump:**  $COP = \frac{Q_{in}}{W}$

**COP of a Refrigerator:**  $COP = \frac{Q_{out}}{W}$

**Efficiency for a Carnot Cycle:**  $\eta = 1 - \frac{T_2}{T_1}$

For cycles operating between two heat sources, with temperatures  $T_H$  and  $T_C$ , with  $T_H > T_C$ .

$Q_H$  is the heat transferred between the gas and the heat source at temperature  $T_H$ .

$Q_C$  is the heat transferred between the gas and the heat sink at temperature  $T_C$ ;

**Entropy:**  $\Delta S = \frac{Q_{rev}}{T}$

**Density of entropy:**  $s = \frac{S}{m}$

At constant pressure:  $q = \Delta h$

**Thermal Efficiency:**  $\eta = \frac{W_{out}}{q_{in}}$

**Compression Ratio:**  $R = \frac{V_{max}}{V_{min}}$

**Mean Effective Pressure of a Cycle:**  $MEP = \frac{W}{V_{max} - V_{min}}$

**Back Work Ratio on a Gas Turbine:**  $r_{bw} = \frac{W_{compressor}}{W_{turbine}}$

**Efficiency of the compressor on a real gas turbine:**  $\eta_C = \frac{W_{ideal}}{W_{real}}$

**Efficiency of the turbine on a real gas turbine:**  $\eta_T = \frac{W_{real}}{W_{ideal}}$

**Efficiency of a regenerator:**  $\epsilon = \frac{q_{regenerado}}{q_{max}}$

## 1.7.2 Solutions

### Answer 1.7.1

We consider the value of  $R$  for air:  $R = 0.287 \text{ kJ}/(\text{kg K})$ .

#### ► Process A → B

- Isentropic Compression, lets determine  $T_B$ ,  $p_B$  e  $u_B$ .

We use the variables  $s^\circ$  and  $s^+$  for an isentropic process (see discussion in appendix D.1):

$$s_B^\circ - s_A^\circ = +R \ln \frac{p_B}{p_A}$$

$$s_B^+ - s_A^+ = -R \ln \frac{v_B}{v_A}$$

The compression ratio is 10, therefore  $\frac{v_B}{v_A} = 1/10$  e  $s_B^+ - s_A^+ = -R \ln \frac{v_B}{v_A} = -R \ln(1/10)$ .

Doing a table lookup, we obtain  $s_A^+ = 1.7548 \text{ kJ}/(\text{kg K})$ . Using the former expression we obtain  $s_B^+ = 2.4156 \text{ kJ}/(\text{kg K})$ .

Doing a reverse table lookup ( $s^+ \rightarrow T$ ) we obtain  $T_B = 713.6 \text{ K}$ . Through direct table lookup ( $T \rightarrow s^\circ$ ) we then obtain  $s_B^\circ = 4.3013 \text{ kJ}/(\text{kg K})$ .

We determine the pressure ratios according to:  $\frac{p_B}{p_A} = \exp\left(\frac{s_B^\circ - s_A^\circ}{R}\right) = \exp\left(\frac{4.3013 - 3.3850}{0.287}\right)$

Then  $p_B = 100 \times 24.3537 = 2435.37 \text{ kPa}$ .

Doing a table lookup (for  $T_A$  and  $T_B$ ) we additionally obtain the specific internal energies for states A and B:  $u_A = 209.549 \text{ kJ/kg}$ ;  $u_B = 523.41 \text{ kJ/kg}$ .

#### ► Process B → C

- Isochoric Heat Addition Process, we are going to determine  $T_C$ ,  $p_C$  and  $u_C$ .

Internal Energy in C:

$$u_C = q_{in} + u_B = 750 + 523.41 = 1273.41 \text{ kJ/kg}$$

We obtain the temperature  $T_C$  through a reverse table lookup ( $u \rightarrow T$ );  $T_C = 1573.5 \text{ K}$ .

The heat addition process is carried at constant volume for an ideal Otto cycle, therefore  $V_C = V_B$ . Knowing the temperatures  $T_B$  and  $T_C$  we may determine  $p_C$  through:

$$\frac{p_B V_B}{T_B} = \frac{p_C V_C}{T_C} \Rightarrow \frac{p_B}{T_B} = \frac{p_C}{T_C} \Rightarrow p_C = \frac{2435.37}{713.6} \times 1573.5 \Rightarrow p_C = 5370 \text{ kPa}$$

We are additionally determining  $s_C^\circ$  through a direct table lookup ( $T \rightarrow s^\circ$ ). We obtain  $s_C^\circ = 5.2099 \text{ kJ}/(\text{kg K})$ , a value which we will later use to determine process C → D.

#### ► Process C → D

- Isentropic Expansion, we are determining  $T_D$ ,  $p_D$  e  $u_D$ , following the same steps than for A → B.

$$\frac{v_D}{v_C} = 10 \Rightarrow s_D^+ - s_C^+ = -R \ln \frac{v_D}{v_C} = -R \ln(10).$$

Performing a table lookup with  $T_C = 1573.5 \text{ K}$ , we obtain  $s_C^+ = 3.0973 \text{ kJ}/(\text{kg K})$ . Using the expression  $R \ln \frac{v_D}{v_C} = s_D^+ - s_C^+$  we obtain  $s_D^+ = 2.4365 \text{ kJ}/(\text{kg K})$ .

Doing a reverse table lookup ( $s^+ \rightarrow T$ ) we obtain  $T_D = 732.5 \text{ K}$ . Through a direct table lookup ( $T \rightarrow s^\circ$ ) we then obtain  $s_D^\circ = 4.3295 \text{ kJ}/(\text{kg K})$ .

We determine the pressure ratios according to:  $\frac{p_D}{p_C} = \exp\left(\frac{s_D^\circ - s_C^\circ}{R}\right) = \exp\left(\frac{4.3295 - 5.2099}{0.287}\right) = 0.0465$

Therefore  $p_D = p_C \times \frac{p_D}{p_C} = 5370 \times 0.0465 = 249.88 \text{ kPa}$ .

Doing a table lookup we further obtain the specific internal energy in  $D$ :  $u_D = 538.471 \text{ kJ/kg}$ .

2)

- Heat that exits the system:

$$q_{out} = u_D - u_A = 538.471 - 209.549 = 328.922 \text{ kJ/kg}$$

- Total Work:

$$W = q_{in} - q_{out} = 750 - 328.922 = 421.078 \text{ kJ/kg}$$

3)

- Efficiency:

$$\eta = \frac{w_{tot}}{q_{in}} = \frac{421.078}{750} = 0.56$$

- Otto cycle efficiency, assuming ideal gas properties:

$$\eta = 1 - r^{1-\gamma} = 1 - 10^{1-1.4} = 0.6$$

The values are different, one should be careful in deciding when to apply the ideal gas approximation!!!

4)

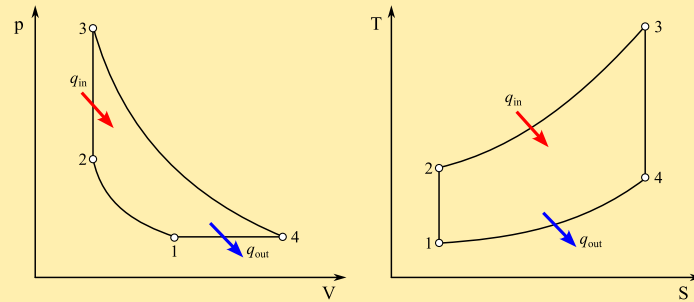
- Averaged effective pressure for the cycle (MEP):

$$V_1 = \frac{RT_1}{p_1} = \frac{0.287 \text{ kPa m}^3/(\text{kg K}) \times 293 \text{ K}}{100 \text{ kPa}} = 0.841 \text{ m}^3/\text{kg}$$

$$\text{MEP} = \frac{w_{tot}}{V_1 - V_2} = \frac{w_{tot}}{V_1 - V_1/r} = \frac{w_{tot}}{V_1(1 - 1/r)} = \frac{421.078 \text{ kJ/kg}}{0.841 \text{ m}^3/\text{kg}(1 - 1/10)} = 556.31 \text{ kPa}$$

**Answer 1.7.2**

1)



We consider the value of  $R$  for air:  $R = 0.287 \text{ kJ}/(\text{kg K})$ .

► **Process 1  $\rightarrow$  2**

- Reversible Adiabatic Compression. This corresponds to an Isentropic Compression. Lets calculate  $T_2$  and  $u_2$ .

We use the variables  $s^\circ$ :

$$s_2^\circ - s_1^\circ = +R \ln \frac{p_2}{p_1}$$

We determine  $s_1^\circ$  through a table lookup for  $T_1 = 293 \text{ K}$ :  $s_1^\circ = 3.385 \text{ kJ}/(\text{kg K})$ . Since we know  $p_1$  and  $p_2$  we have:  $s_2^\circ = R \ln \frac{p_2}{p_1} + s_1^\circ = 3.9633 \text{ kJ}/(\text{kg K})$ .

Performing a reverse table lookup ( $s^\circ \rightarrow T$ ) we obtain  $T_2 = 518 \text{ K}$ .

We further obtain the specific internal energies in 1 and 2:  $u_1 = 209.549 \text{ kJ/kg}$ ;  $u_2 = 373.397 \text{ kJ/kg}$ .

► **Process 2  $\rightarrow$  3**

- Isochoric Heat Addition Process.

Conhecemos  $T_3 = 1500 \text{ K}$ . By definition  $V_3 = V_2$  and we obtain  $u_3 = 1205.087 \text{ kJ/kg}$  from the tables. We determine  $p_3$  through the relationship:

$$\frac{p_2 V_2}{T_2} = \frac{p_3 V_3}{T_3} \Rightarrow \frac{p_2}{T_2} = \frac{p_3}{T_3} \Rightarrow p_3 = \frac{750}{518} \times 1500 \Rightarrow p_3 = 2171.8 \text{ kPa}$$

Performing a table lookup we additionally obtain the specific internal energy in 3:  $u_3 = 1205.087 \text{ kJ/kg}$ .

► **Process 3  $\rightarrow$  4**

- Reversible Adiabatic Expansion, which corresponds to an Isentropic expansion, we determine  $T_4$  and  $u_4$  following the same reasoning than for process 1  $\rightarrow$  2.

Performing a table lookup with  $T_3 = 1500 \text{ K}$ , we obtain  $s_3^\circ = 5.1518 \text{ kJ}/(\text{kg K})$ . Since we know  $p_3$  and  $p_4$  we have:  $s_4^\circ = R \ln \frac{p_4}{p_3} + s_3^\circ = 4.2684 \text{ kJ}/(\text{kg K})$ .

Performing a reverse table lookup ( $s^\circ \rightarrow T$ ) we obtain  $T_4 = 692 \text{ K}$ .

Doing a table lookup we additionally obtain the specific internal energy in 4:  $u_4 = 506.471 \text{ kJ/kg}$ .

2) Calculation of the Heat added to the cycle:

$$q_{in} = u_3 - u_2 = 1205.087 - 373.397 = 831.69 \text{ kJ/kg}$$

Note: Since heat rejection  $q_{out}$  is carried at constant pressure, we need to consider enthalpy instead of internal energy (constant volume processes):

$$q_{out} = h_4 - h_1$$

We obtain  $h_4$  and  $h_1$  through a table lookup using  $T_4 = 692 \text{ K}$  and  $T_1 = 293 \text{ K}$

$$q_{out} = h_4 - h_1 = 705.075 - 293.64 = 411.435 \text{ kJ/kg}$$

Total Work:

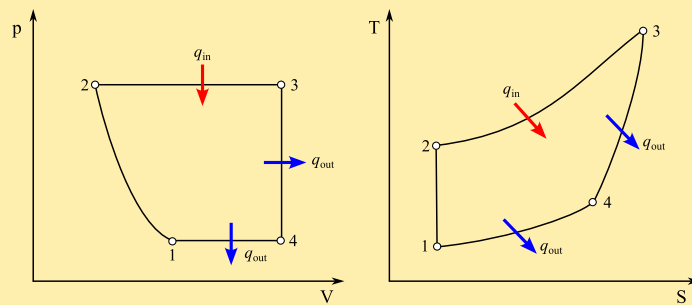
$$w_{net} = q_{in} - q_{out} = 420.255 \text{ kJ/kg}$$

Finally, the efficiency  $\eta$  is:

$$\eta = \frac{w_{net}}{q_{in}} = 0.505$$

### Answer 1.7.3

1)



We consider the value of  $R$  for air:  $R = 0.287 \text{ kJ}/(\text{kg K})$ .

#### ► Process 1 → 2

- Reversible Adiabatic Compression corresponds to an Isentropic Compression, let us obtain  $T_2$  and  $u_2$ .

We use the variables  $s^\circ$ :

$$s_2^\circ - s_1^\circ = +R \ln \frac{p_2}{p_1}$$

We determine  $s_1^\circ$  through a table lookup at  $T_1 = 293 \text{ K}$ :  $s_1^\circ = 3.385 \text{ kJ}/(\text{kg K})$ . Since we know  $p_1$  and  $p_2$  we have:  $s_2^\circ = R \ln \frac{p_2}{p_1} + s_1^\circ = 3.9435 \text{ kJ}/(\text{kg K})$ .

Performing an inverse table lookup ( $s^\circ \rightarrow T$ ) we obtain  $T_2 = 509 \text{ K}$ .

We also obtain the specific internal energies in 1 and 2:  $u_1 = 209.549 \text{ kJ/kg}$ ;  $u_2 = 366.695 \text{ kJ/kg}$ .

#### ► Process 2 → 3

- Isobaric Heat Addition process.

$u_3 = u_2 + \dot{q}_{in}$  with  $\dot{q}_{in} = 1600 \text{ kJ/kg}$ . Then  $u_3 = 366.695 + 1600 = 1966.695 \text{ kJ/kg}$ .  
( $u \rightarrow T$ ) we obtain  $T_3 = 2298.5 \text{ K}$ .

► **Process 3  $\rightarrow$  4**

- Isochoric Heat Rejection process.

We have  $V_4 = V_3$ ,  $p_3 = p_2 = 700 \text{ kPa}$  and  $p_4 = 100 \text{ kPa}$ . Then:

$$\frac{p_3 V_3}{T_3} = \frac{p_4 V_4}{T_4} \Rightarrow \frac{p_3}{T_3} = \frac{p_4}{T_4} \Rightarrow T_4 = \frac{700}{100} \times 2299 \Rightarrow T_4 = 328.36 \text{ K}$$

Performing a table lookup we obtain the specific internal energy in 4:  $u_4 = 234.938 \text{ kJ/kg}$ .

► **Process 4  $\rightarrow$  1**

- Isobaric Heat Rejection process.

We know both temperatures in the limiting points of the process ( $T_4 = 328.36 \text{ K}$  and  $T_1 = 293 \text{ K}$ ) and therefore all the gas properties may be determined through a table lookup.

2) Maximum temperature:  $T_3 = 2299 \text{ K}$ . The gas CANNOT be considered an ideal gas in this temperature range.

3)

- Transferred heats calculation:

The total heat  $q_{out}$  is given by:

$$\begin{aligned} q_{in} &= 1600 \text{ kJ/kg} \\ q_{out} &= q_{3 \rightarrow 4} + q_{4 \rightarrow 1} \\ &= (234.938 - 1966.695) + (293.64 - 329.177) \\ &= -1767.3 \text{ kJ/kg}, \end{aligned}$$

where we consider the constant volume energy difference for process  $q_{3 \rightarrow 4}$  and the constant pressure energy difference for process  $q_{4 \rightarrow 1}$ .

- Thermodynamic efficiency:

$$\eta = 1 - \frac{|q_{out}|}{q_{in}} = 1 - \frac{1767.3}{1600} = -0.105$$

The efficiency is negative since  $q_{out} > q_{in}$ . This is an impossible thermodynamic cycle.

**Answer 1.7.4**

- Considering the energy balance for the control volume in steady-state:

$$q_e = \dot{m} h_e \Rightarrow \dot{m} h_f = \dot{m} h_e \Rightarrow h_1 = h_e$$

- We use the R-134a vapor tables for  $p_1 = 850 \text{ kPa}$

$$\begin{aligned} h_1 &= 246.77 \text{ kJ/kg} \\ T_1 &= 33.47^\circ \text{C} \end{aligned}$$

- We use the R-134a vapor tables for  $p_2 = 150 \text{ kPa}$

$$h_f = 177.31 \text{ kJ/kg}$$

$$h_g = 388.30 \text{ kJ/kg}$$

$$T_2 = -17.18^\circ\text{C}$$

- Vapor fraction  $x$ :

$$x = \frac{246.77 - 177.31}{388.30 - 177.31} = 0.33$$

- Temperature variation  $\Delta T$ :

$$\Delta T = 33.47 - (-17.18) = -50.65^\circ\text{C}$$

### Answer 1.7.5

1)

- $\dot{m}_R = 5 \text{ kg/min}$   
 ► Entry and exit conditions:

water:  $p_{1,2} = 300 \text{ kPa}$ ,  $T_1 = 15^\circ\text{C}$ ,  $T_2 = 25^\circ\text{C}$ , comp. liq.

R-134a:  $p_{3,4} = 1 \text{ MPa}$ ,  $T_3 = 75^\circ\text{C}$ ,  $T_4 = 35^\circ\text{C}$ , superheated vap.  $\rightarrow$  comp. liq.

- For a stationary system we assume:

$$\dot{m}_{\text{in}} = \dot{m}_{\text{out}} \quad (\text{constant mass flux})$$

$$Q = 0, \quad W = 0$$

- Energy balance:

$$\dot{m}_R(h_4 - h_3) = \dot{m}_w(h_2 - h_1)$$

- Enthalpy values (we used the compressed liquid tables of water for  $h_{1,2}$ , the superheated vapor tables of R-134a for  $h_3$ , and the saturated vapor tables of R-134a for  $h_4$ , approximating  $h_{4,\text{liq.}}(T) \approx h_{4,\text{sat. liq.}}(T)$  as there are no compressed liquid tables for R-134a):

$$h_3 = 303.85 \text{ kJ/kg}$$

$$h_4 = 100.87 \text{ kJ/kg}$$

$$h_1 = 104.83 \text{ kJ/kg}$$

$$h_2 = 62.982 \text{ kJ/kg}$$

- Calculating the exit mass flux  $\dot{m}_w$ :

$$(5) \left( \frac{100.87 - 303.85}{62.982 - 104.83} \right) = \dot{m}_w \approx 29.1 \text{ kg/min}$$

2)

$$\begin{aligned} \dot{Q} &= \dot{m}_w h_2 - \dot{m}_w h_w \\ &= 29.1 \times (145.83 - 62.982) \approx 1218 \text{ kJ/min} \end{aligned}$$

**Answer 1.7.6**

$$\dot{m}_1 = \dot{m}_2 \Rightarrow \rho_1 \dot{V} = \rho_2 \dot{V} \Rightarrow \rho_1 v_1 A = \rho_2 v_2 A \Rightarrow \frac{v_2}{v_1} = \frac{1.22}{1.06} = 1.15$$

It increased 15%

**Answer 1.7.7**

$$\rho_{ar} = \frac{p}{RT} = \frac{101.325}{0.287 \times (22 + 273)} = 1.192 \text{ kg/m}^3$$

**Mass calculation:**

$$\dot{m} = \text{constant}$$

$$\dot{m} = \rho \cdot \dot{V} = 1.192 \times 150 = 179.5 \text{ kg/h} = 0.0499 \text{ kg/s}$$

**Energy calculation:**

$$\begin{aligned} \Delta E &= \dot{m}(h_f - h_i) = \dot{m} \cdot C_p(T_f - T_i) \\ &= 0.0499 \times 1.005 \times (22 - 4) \\ &= 0.903 \text{ kJ} \end{aligned}$$

**Answer 1.7.8**

Air enters the diffuser at a velocity of 270 m/s, we want to determine the exit speed when the air reaches 250 K. The process is adiabatic and mass conservation is valid.

**Inputs:**

- ▶ Inlet pressure:  $p_1 = 18 \text{ kPa}$
- ▶ Inlet temperature:  $T_1 = 216 \text{ K}$
- ▶ Inlet velocity:  $V_1 = 270 \text{ m/s}$
- ▶ Outlet pressure:  $p_2 = 18 \text{ kPa}$  (assuming isobaric process)
- ▶ Outlet temperature:  $T_2 = 250 \text{ K}$

**1<sup>st</sup> Law of Thermodynamics:**

The 1<sup>st</sup> Law for a control volume is:

$$\dot{m} \left( h_1 + \frac{v_1^2}{2} \right) = \dot{m} \left( h_2 + \frac{v_2^2}{2} \right)$$

Since  $\dot{m}_{in} = \dot{m}_{out}$ , we may simplify:

$$\frac{v_1^2}{2} + h_1 = \frac{v_2^2}{2} + h_2$$

Rearranging for the outlet velocity  $v_2$ :

$$v_2 = \sqrt{v_1^2 + 2(h_1 - h_2)}$$

**Specific enthalpies calculation**

We use table D.1 of air as an ideal gas to determine the enthalpies.

For  $T_1 = 216 \text{ K}$ , we interpolate to determine  $h_1$ :

$$h_1 = 215.97 \text{ kJ/kg}$$

For  $T_2 = 250 \text{ K}$ , the specific enthalpy is:

$$h_2 = 250.05 \text{ kJ/kg}$$

### Outlet velocity calculation

Substituting the values in the equation for  $v_2$ :

$$v_2 = \sqrt{(270 \text{ m/s})^2 + 2 \times (215.97 - 250.05) \times 1000} \approx 68.9 \text{ m/s}$$

The outlet velocity is approximately  $68.9 \text{ m/s}$ .

### Answer 1.7.9

We consider the  $R$  value for air:  $R = 0.287 \text{ kJ/(kg K)}$ .

1)

#### ► Process 1 → 2

- Isentropic Compression, we have  $T_1 = 300 \text{ K}$ . We now determine  $T_2$ .

Using the variables  $s^\circ$ :

$$s_2^\circ - s_1^\circ = +R \ln \frac{p_2}{p_1}$$

We determine  $s_1^\circ$  through a table lookup at  $T_1 = 300 \text{ K}$ :  $s_1^\circ = 3.4088 \text{ kJ/(kg K)}$ . Since  $r = \frac{p_2}{p_1} = 13$

we have:  $s_2^\circ = R \ln \frac{p_2}{p_1} + s_1^\circ = 4.1449 \text{ kJ/(kg K)}$ .

Performing a reverse table lookup ( $s^\circ \rightarrow T$ ) we obtain  $T_2 = 616 \text{ K}$ .

We further obtain the specific enthalpies in 1 and 2:  $h_1 = 300.669 \text{ kJ/kg}$ ;  $h_2 = 624.279 \text{ kJ/kg}$ .

#### ► Process 2 → 3

- Heat Addition Process.

$T_3 = 1400 \text{ K}$  is an exercise input, we have  $h_3 = 1515.116 \text{ kJ/kg}$  from a table lookup.

#### ► Process 3 → 4

- Isentropic Expansion.

As for the Isentropic Compression case we determine  $s_3^\circ$  from a table lookup at  $T_3 = 1400 \text{ K}$ :

$s_3^\circ = 5.0685 \text{ kJ/(kg K)}$ . Since  $r = \frac{p_4}{p_3} = 1/13$  we have:  $s_4^\circ = R \ln \frac{p_4}{p_3} + s_3^\circ = 4.3324 \text{ kJ/(kg K)}$ .

Performing a reverse table lookup ( $s^\circ \rightarrow T$ ) we obtain  $T_4 = 734.5 \text{ K}$ .

We additionally obtain the specific enthalpies in 4 and 3:  $h_4 = 750.863 \text{ kJ/kg}$ ;  $h_3 = 1395.777 \text{ kJ/kg}$ .

The compressor and turbine outlet temperatures are, respectively,  $T_2 = 616 \text{ K}$  e  $T_4 = 751 \text{ K}$ .

2)

$$\text{► } w_{\text{comp}} : h_2 - h_1 = 624.279 - 300.669 = -323.61 \text{ kJ/kg}$$

- ▶  $w_{\text{turb}} : h_3 - h_4 = 1515.116 - 750.863 = 764.26 \text{ kJ/kg}$
- ▶  $\text{BWR} = w_{\text{comp}}/w_{\text{turb}} = 323.61/764.26 = 0.42$

3)

- ▶  $q_{\text{in}} : h_3 - h_2 = 1515.116 - 624.279 = 890.84 \text{ kJ/kg}$
- ▶  $w_{\text{net}} = w_{\text{turb}} + w_{\text{comp}} : 764.26 - 323.61 = 440.65 \text{ kJ/kg}$
- ▶  $\eta_t = w_{\text{net}}/q_{\text{in}} = 440.65/890.84 = 0.495$

**Answer 1.7.10**

1)

Considering that the compressor and the turbine have efficiencies of 0.85, we need to correct the input work received by the compressor (which increases) and the work delivered by the turbine (which decreases)

- ▶  $w'_{\text{comp}} = \frac{w_{\text{comp}}}{\eta_c} = \frac{323.61}{0.85} = 380.72 \text{ kJ/kg}$
- ▶  $w'_{\text{turb}} = \frac{w_{\text{turb}}}{\eta_t} = 764.26 \times 0.9 = 687.83 \text{ kJ/kg}$
- ▶  $\text{BWR} = \frac{380.72}{687.83} = 0.55$

2)

- ▶ Work received by the compressor:  $w_{\text{comp}} = h_{2a} - h_1 \rightarrow h_{2a} = h_1 + w_{\text{comp}}$

$$h_{2a} = 300.669 + 380.72 = 681.39 \text{ kJ/kg}$$

- ▶ Added heat:  $q_{\text{in}} = h_3 - h_{2a} = 1515.116 - 681.39 = 833.73 \text{ kJ/kg}$
- ▶ Net work:  $w_{\text{net}} = w_{\text{out}} - w_{\text{in}} = 687.83 - 380.72 = 307.11 \text{ kJ/kg}$
- ▶ Thermal efficiency:  $\eta = \frac{w_{\text{net}}}{q_{\text{in}}} = \frac{307.11}{833.73} = 0.37$

3)

- ▶ Turbine work:  $w_{\text{turb}} = h_3 - h_{4a} \rightarrow h_{4a} = h_3 - w_{\text{turb}}$

$$h_{4a} = 1515.116 - 687.83 = 827.29 \text{ kJ/kg}$$

- ▶ Performing a table reverse lookup ( $h \rightarrow T$ ) we obtain the outlet temperature:  $T_4 = 804.5 \text{ K}$ .

**Answer 1.7.11**

We determine the air enthalpy at the regenerator exit, using the corresponding efficiency formula:

$$\varepsilon = \frac{h_5 - h_{2a}}{h_{4a} - h_{2a}} = \frac{h_s - 681.39}{827.29 - 681.39} \Rightarrow h_5 = 798.11 \text{ kJ/kg}$$

$$q_{\text{in}} = h_3 - h_5 = 1515.116 - 798.11 = 717.01 \text{ kJ/kg}$$

$$\eta_{\text{new}} = \frac{307.11}{717.01} = 0.43$$

The addition of the regenerator increases the cycle efficiency from  $\eta = 0.37$  to  $\eta = 0.43$ .

**Answer 1.7.12**

$$11) \quad C_p = \frac{5}{2}R \quad C_v = \frac{3}{2}R \quad m = 1 \quad R = 8.314 \text{ J/(mol K)}$$

1)

$$pV = nRT \Rightarrow p_A V_A = nRT_A$$

$$T_A = \frac{P_A V_A}{nR} = \frac{101325 \times 0.4}{8.314} = 487.9 \text{ K}$$

$$T_B = 243.75 \text{ K}$$

$$T_C = 365.62 \text{ K}$$

$$T_D = 931.24 \text{ K}$$

2)

$$w = \int p dV$$

$$w_{AB} = w_{CD} = 0$$

$$w_{BC} = 1013.25$$

$$w_{DA} = -2026.5$$

$$W_{\text{Tot}} = -1013.25$$

$$q_{AB} = mC_v \Delta T$$

$$q_{CD} = mC_v \Delta T$$

$$q_{BC} = mC_p \Delta T$$

$$q_{DA} = mC_p \Delta T$$

$$q_{AB} = 20.785 \times 243.65 = -5064.27 \text{ J}$$

$$q_{BC} = 29.092 \times 121.87 = 3546.3 \text{ J}$$

$$q_{CD} = 7546.73 \text{ J}$$

$$q_{DA} = -7095.5 \text{ J}$$

3) This is a refrigerator.

**Answer 1.7.13**

1. Inputs:

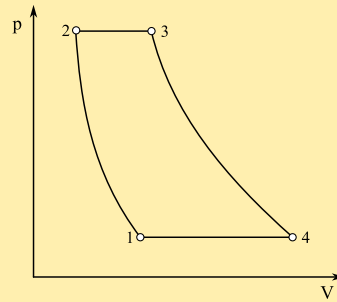
$$m = 1 \text{ mol}$$

$$C_p = \frac{5}{2}R$$

$$C_v = \frac{3}{2}R$$

$$\gamma = \frac{5}{3} = \frac{C_p}{C_v}$$

$p - V$  diagram:



- $p_1 V_1 = mRT_1 \Rightarrow V_1 = 0.025 \text{ m}^3 \Rightarrow V_2 = 0.015 \text{ m}^3$
- $p_1 V_1^\gamma = \text{const} \Rightarrow p_1 V_1^\gamma = 216.58 \Rightarrow p_2 = \frac{216.58}{V_2^\gamma} = 237\,341 \text{ Pa}$
- $T_2 = \frac{p_2 V_2}{mR} = 428.3 \text{ K}$
- $T_3 = 1300 \text{ K}$
- $p_3 = p_2 = 237\,341 \text{ Pa}$
- $V_3 = 0.046 \text{ m}^3$
- $p_4 = 1 \text{ atm}$
- $p_4 V_4^\gamma = p_3 V_3^\gamma = 1401.93 \Rightarrow V_4 = 0.017 \text{ m}^3$
- $T_4 = \frac{p_4 V_4}{mR} = 938.42 \text{ K}$

2. Heat quantities calculation:

$$Q_{2-3} = mC_p \Delta T = 18118.3$$

$$Q_{4-1} = mC_p \Delta T = -13269.6$$

3. Efficiency:

$$\eta = 1 - \frac{Q_{\text{out}}}{Q_{\text{in}}} = 0.26$$

#### Answer 1.7.14

1)

$$\text{COP} = \frac{1}{\frac{T_q}{T_f} - 1} = 6.33$$

2)

$$6.33 = \frac{100}{w} \Rightarrow w = 15.80 \text{ J}$$

$$W = Q_h - Q_f = 115.80$$

3)

$$P = \frac{W}{\Delta T} = 7.4 \text{ W}$$

4)

$$P = \frac{100}{0.5h} = 0.56 \text{ W} \Rightarrow P = \dot{m} c_v \Delta T \Rightarrow \dot{m} = 2.8 \times 10^{-5} \text{ g/s}$$

**Answer 1.7.15**

1. ► Pressures:

$$p_H = 20 \text{ MPa}$$

$$p_L = 20 \text{ kPa}$$

- State 1 → 2:

$$h_1 = 251.42 \text{ kJ/kg}$$

$$w_{\text{pump}} = h_2 - h_1 \simeq \left( \frac{v_{f,1} + v_{f,2}}{2} \right) \Delta p = \frac{0.001017 + 0.002040}{2} \times (20 \text{ MPa} - 20 \text{ kPa}) = 30.539 \text{ kJ/kg}$$

$$h_2 = h_1 + w_{\text{pump}} = 30.791 \text{ kJ/kg}$$

- State 2 → 3: Using the superheated vapor tables for  $p_H = 20 \text{ MPa}$  e  $T_3 = 500^\circ\text{C}$ :

$$h_3 = 3241.2 \text{ kJ/kg}$$

$$q_{\text{in}} = h_3 - h_2 = 3241.2 - 30.791 = 3210.41 \text{ kJ/kg}$$

- State 4:

$$s_4 = s_3 = 6.1446 \text{ kJ/(kg K)} \Rightarrow x_4 = \frac{s_4 - s_f}{s_g - s_f} = \frac{6.1446 - 0.832}{7.9072 - 0.832} = 0.75$$

$$h_4 = h_f + h_{fg} \times x_4 = 251.42 + 2357.5 \times 0.75 = 2019.545 \text{ kJ/kg}$$

- Work performed by the turbine:

$$w_{\text{turb}} = h_3 - h_4 = 3241.2 - 2019.545 = 1221.655 \text{ kJ/kg}$$

2. Total work and thermal efficiency:

$$w_{\text{net}} = w_{\text{turb}} - w_{\text{pump}} = 1221.655 - 30.791 = 1190.864 \text{ kJ/kg}$$

$$\eta = w_{\text{net}}/q_{\text{in}} = 1190.864/3210.41 = 0.37$$

3. Thermal efficiency of an equivalent Carnot cycle:

- $T_H = 500^\circ\text{C}$

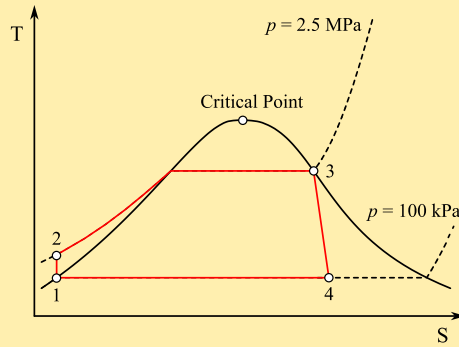
- $T_L = T_{\text{sat}, 20 \text{ kPa}} = 60.06^\circ\text{C}$

$$\eta_{\text{Carnot}} = 1 - \frac{T_L}{T_H} = \frac{60.06 + 273.15}{500 + 273.15} = 0.43$$

**Answer 1.7.16**

1.  $T - S$  graph with pressures:

- $p_r = 25 \text{ atm} \approx 2.5 \text{ MPa}$
- $p_e = 1 \text{ atm} \approx 100 \text{ kPa}$



2. Turbine work:

$$w_t = h_3 - h_4 = \frac{250 \text{ kJ/s}}{1 \text{ kg/s}}$$

$$h_4 = 2801.9 - 250 = 2551.9 \text{ kJ/kg}$$

3. Knowing the enthalpy value at point 4 we may determine the fluid quality through a table lookup for  $p = 100 \text{ kPa}$ :

$$x = \frac{h_4 - h_f}{h_g - h_f} = \frac{2551.9 - 417.50}{2674.9 - 417.50} = 0.81 \quad (\text{vapor})$$

$$\Rightarrow 0.19 \quad (\text{liquid})$$

# APPENDIX

# Thermodynamic Processes Relations

# A

Table A.1: Useful Relations for Different Thermodynamic Processes

Process	Properties	Polytropic Idx.	Properties	dU	W	Q
Adiabatic	Q = 0	$pV^\gamma = \text{const.}$	$TV^{\gamma-1} = \text{const.}$	-W	$-(p_f V_f - p_i V_i) / (\gamma - 1)$	0
Isothermal	T = const.	$pV^1 = \text{const.}$	$TV^0 = \text{const.}$	0	$nRT \ln(V_f / V_i)$	-W
Isobaric	p = const.	$pV^0 = \text{const.}$	$TV^{-1} = \text{const.}$	Q - W	$p(V_f - V_i)$	$nC_p(T_f - T_i)$
Isochoric	V = const.	$pV^{+\infty} = \text{const.}$	$TV^{-1} = \text{const.}$	Q	0	$nC_v(T_f - T_i)$
Isentropic*	S = const.	$pV^\gamma = \text{const.}$	$TV^{\gamma-1} = \text{const.}$	-W	$-(p_f V_f - p_i V_i) / (\gamma - 1)$	0
Isenthalpic	H = const.	—	—	—	—	—
Politropic	$pV^n = \text{const.}$	$pV^n = \text{const.}$	$TV^{n-1} = \text{const.}$	Q - W	$-(p_f V_f - p_i V_i) / (n - 1)$	—
Reversible	dS - dQ/T = 0	—	—	—	—	—

\*: An Isentropic process corresponds to an Adiabatic and Reversible process

Table A.2: Isentropic Relations for an Ideal Gas

$\frac{T_2}{T_1}$	$=$	$\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$	$=$	$\left(\frac{V_1}{V_2}\right)^{\gamma-1}$
$\left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$	$=$	$\frac{p_2}{p_1}$	$=$	$\left(\frac{V_1}{V_2}\right)^\gamma$
$\left(\frac{T_2}{T_1}\right)^{\frac{1}{\gamma-1}}$	$=$	$\left(\frac{p_2}{p_1}\right)^{\frac{1}{\gamma}}$	$=$	$\frac{V_1}{V_2}$

# Substance Properties

# B

Table B.1: Ideal-gas specific heats of various gaseous substances at 300 K

Gas	Formula	M (g/mole)	R (kJ/kg.K)	C <sub>p</sub> (kJ/kg.K)	C <sub>v</sub> (kJ/kg.K)	γ
Air	-	28.9647	0.2870	1.0044	0.7174	1.400
Argon	Ar	39.948	0.2081	0.5203	0.3122	1.667
Carbon dioxide	CO <sub>2</sub>	44.0095	0.1889	0.8457	0.6567	1.288
Carbon monoxide	CO	28.0101	0.2968	1.0405	0.7437	1.399
Ethene/Ethylene	C <sub>2</sub> H <sub>4</sub>	28.0532	0.2964	1.5342	1.2379	1.239
Hydrogen	H <sub>2</sub>	2.015 88	4.1245	14.3111	10.1866	1.405
Deuterium	D <sub>2</sub>	4.028 204	2.0641	7.2474	5.1834	1.398
Helium	He	4.002 602	2.0773	5.1931	3.1159	1.667
Krypton	Kr	83.798	0.0992	0.2480	0.1488	1.667
Methane	CH <sub>4</sub>	16.0425	0.5183	2.2260	1.7078	1.303
Neon	Ne	20.1797	0.4120	1.0300	0.6180	1.667
Nitrogen	N <sub>2</sub>	28.0134	0.2968	1.0397	0.7429	1.400
Oxygen	O <sub>2</sub>	31.9988	0.2598	0.9185	0.6587	1.394
Steam	H <sub>2</sub> O	18.0153	0.4615	1.8649	1.4033	1.329
Xenon	Xe	131.293	0.0633	0.1583	0.0950	1.667

Table B.2: Triple-point (in blue) and critical-point (in red) properties of common substances (liquid phase)

Gas	Formula	$M$ g/mol	$R$ kJ/(kg K)	$T$ K	$p$	$\rho_f$ mol/L	$u_f$ kJ/mol	$h_f$ kJ/mol	$s_f$ J/(mol K)	$C_{v,f}$ J/(mol K)	$C_{p,f}$ J/(mol K)
Air (Dry)	–	28.9664	0.2870	59.75 132.63	5.27 kPa 3.79 MPa	33.059 10.447	– –	– –	– –	34.007 0	55.065 0
Ammonia	NH <sub>3</sub>	17.0305	0.4882	195.49 405.56	6.05 kPa 11.36 MPa	43.091 13.696	0 20.421	0 21.251	0 68.04	50.538 –	73.322 –
Argon	Ar	39.948	0.2081	83.806 150.69	68.89 kPa 4.86 MPa	35.465 13.407	–4.853 –0.536	–4.851 –0.173	53.11 89.789	21.956 –	44.57 –
Benzene	C <sub>6</sub> H <sub>6</sub>	78.1118	0.1064	278.67 562.02	4.78 kPa 4.91 MPa	11.445 3.901	–10.419 41.819	–10.419 43.077	–33.021 91.197	91.629 –	131.62 –
N-Butane	C <sub>4</sub> H <sub>10</sub>	58.1222	0.1431	134.9 425.12	0.67 Pa 3.80 MPa	12.645 3.923	–5.221 39.364	–5.221 40.331	–27.033 137.35	83.783 –	114.67 –
Carbon Dioxide	CO <sub>2</sub>	44.0095	0.1889	216.59 304.13	517.96 kPa 7.38 MPa	26.777 10.625	3.503 13.928	3.522 14.622	22.943 63.094	42.895 –	85.96 –
Carbon Monoxide	CO	28.0101	0.2968	68.16 132.86	15.54 kPa 3.50 MPa	30.33 10.85	–0.812 4.291	–0.811 4.614	–10.82 40.039	35.351 –	60.43 –
Dichlorodifluoromethane (R-12)	CCl <sub>2</sub> F <sub>2</sub>	120.914	0.0688	116.1 385.12	0.24 Pa 4.14 MPa	15.125 4.673	8.020 41.164	8.020 42.049	33.619 172.7	69.219 –	103.52 –
Dichlorofluoromethane (R21)	CHCl <sub>2</sub> F	102.923	0.0808	200 451.48	0.87 kPa 5.18 MPa	15.36 5.111	13.221 45.082	13.221 46.097	71.486 170.48	99.638 –	127.22 –
Ethane	C <sub>2</sub> H <sub>6</sub>	30.0690	0.2765	90.368 305.32	1.14 Pa 4.87 MPa	21.668 6.857	–6.591 12.49	–6.591 13.2	–49.754 50.82	48.264 –	69.935 –
Ethene/Ethylene	C <sub>2</sub> H <sub>4</sub>	28.0532	0.2964	103.99 282.35	0.12 kPa 5.04 MPa	23.334 7.637	–4.435 10.545	–4.435 11.206	–33.074 46.609	45.503 –	68.155 –
n-Hexane	C <sub>6</sub> H <sub>14</sub>	86.1754	0.0965	177.83 507.82	1.19 Pa 3.04 MPa	8.802 2.706	–30.535 46.012	–30.535 47.137	–120.09 108.16	129.4 –	169.5 –
Hydrogen	H <sub>2</sub>	2.0159	4.1245	13.957 33.145	7.36 kPa 1.30 MPa	38.199 15.508	–0.109 0.517	–0.109 0.601	–6.193 19.442	10.405 –	14.154 –
Deuterium	D <sub>2</sub>	4.0283	2.0641	18.724 38.34	17.19 kPa 1.68 MPa	43.351 17.23	–0.123 0.742	–0.123 0.840	–5.701 23.892	13.513 –	22.665 –
Helium	He	4.0026	2.0773	2.1768 5.1953	5.04 kPa 0.23 MPa	36.48 17.384	–0.028 0.033	–0.0275 0.0462	–7.707 8.468	12.603 –	12.63 –
Hydrogen Sulfide	H <sub>2</sub> S	34.0810	0.2440	187.7 373.1	23.26 kPa 9.00 MPa	29.116 10.19	–1.721 14.47	–1.720 15.353	–8.588 49.374	44.39 –	68.835 –
Krypton	Kr	83.7980	0.0992	115.78 209.48	73.53 kPa 5.53 MPa	29.197 10.85	–0.175 5.883	–0.173 6.392	–1.458 35.568	21.225 –	43.334 –
Methane	CH <sub>4</sub>	16.0425	0.5183	90.694 190.56	11.70 kPa 4.60 MPa	28.142 10.139	–1.153 6.214	–1.152 6.667	–11.389 41.109	34.776 –	54.029 –
Methyl Alcohol	CH <sub>3</sub> OH	32.0419	0.2595	175.61 512.6	0.19 Pa 8.10 MPa	28.23 10.249	–12.44 24.628	–12.44 25.419	–49.524 57.139	56.727 –	70.39 –
Neon	Ne	20.1797	0.4120	24.556 44.4	43.36 kPa 2.66 MPa	61.931 24.1	–0.097 1.078	–0.096 1.189	–3.695 30.066	18.692 –	37.165 –
Nitrogen	N <sub>2</sub>	28.0134	0.2968	63.151 126.19	12.52 kPa 3.40 MPa	30.957 11.184	–4.223 0.515	–4.223 0.819	67.951 118.07	32.951 –	56.033 –
Nitrous Oxide	N <sub>2</sub> O	44.0128	0.1889	182.33 309.52	87.84 kPa 7.24 MPa	28.113 10.27	–0.181 12.745	–0.178 13.45	–0.969 51.414	42.295 –	75.603 –
Oxygen	O <sub>2</sub>	31.9988	0.2598	54.361 154.58	0.15 kPa 5.04 MPa	40.816 13.63	–6.195 0.668	–6.195 1.038	66.946 134.42	38.252 –	53.541 –
Propane	C <sub>3</sub> H <sub>8</sub>	44.0956	0.1886	85.525 369.89	0.17 mPa 4.25 MPa	16.626 5.00	–8.671 23.633	–8.671 24.483	–61.554 90.468	59.761 –	84.481 –
Propylene/Propene	C <sub>3</sub> H <sub>6</sub>	42.0797	0.1976	87.953 364.21	0.75 mPa 4.56 MPa	18.255 5.457	–8.39 21.513	–8.39 22.348	–59.752 83.559	66.011 –	91.83 –
Sulfur Dioxide	SO <sub>2</sub>	64.0640	0.1298	197.7 430.64	1.67 kPa 7.89 MPa	25.411 8.078	–5.727 19.562	–5.727 20.539	25.019 56.206	56.208 –	88.811 –
Tetrafluoroethane (R-134a)	CF <sub>3</sub> CH <sub>2</sub> F	102.0309	0.0815	169.85 374.21	0.39 kPa 4.06 MPa	15.594 5.017	7.291 38.947	7.291 39.756	42.1 159.38	80.83 –	120.79 –
Trichloromonofluoromethane (R-11)	CCl <sub>3</sub> F	137.368	0.0605	162.68 471.11	6.51 Pa 4.41 MPa	12.874 4.033	15.036 57.788	15.036 58.881	79.3 218.86	74.722 –	105.06 –
Water	H <sub>2</sub> O	18.0153	0.4615	273.16 647.1	0.61 kPa 22.06 MPa	55.497 17.874	0 36.314	0 37.548	0 79.393	75.978 –	76.023 –
Xenon	Xe	131.293	0.0633	161.41 289.73	81.77 kPa 5.84 MPa	22.592 8.40	–0.166 8.242	–0.163 8.938	–0.992 36.074	22.063 –	44.446 –

Table B.3: Triple-point (in blue) and critical-point (in red) properties of common substances (gas phase)

Gas	Formula	$M$ g/mol	$R$ kJ/(kg K)	$T$ K	$p$	$\rho_g$ mol/L	$u_g$ kJ/mol	$h_g$ kJ/mol	$s_g$ J/(mol K)	$C_{v,g}$ J/(mol K)	$C_{p,g}$ J/(mol K)
Air (Dry)	–	28.9664	0.2870	59.75 132.63	5.27 kPa 3.79 MPa	$4.9 \times 10^{-3}$ 10.447	– –	– –	– –	20.807 –	29.227 –
Ammonia	NH <sub>3</sub>	17.0305	0.4882	195.49 405.56	6.05 kPa 11.36 MPa	$3.7 \times 10^{-3}$ 13.696	23.739 20.421	25.357 21.251	129.71 68.04	25.822 –	34.452 –
Argon	Ar	39.948	0.2081	83.806 150.69	68.89 kPa 4.86 MPa	0.102 13.407	1.010 –0.536	1.689 –0.173	131.15 89.789	12.972 –	22.172 –
Benzene	C <sub>6</sub> H <sub>6</sub>	78.1118	0.1064	278.67 562.02	4.78 kPa 4.91 MPa	$2.1 \times 10^{-3}$ 3.901	22.242 41.819	24.551 43.077	92.466 91.197	67.07 –	75.507 –
N-Butane	C <sub>4</sub> H <sub>10</sub>	58.1222	0.1431	134.9 425.12	0.67 Pa 3.80 MPa	$5.935 \times 10^{-7}$ 3.923	22.482 39.364	23.604 40.331	186.65 137.35	55.993 –	64.307 –
Carbon Dioxide	CO <sub>2</sub>	44.0095	0.1889	216.59 304.13	517.96 kPa 7.38 MPa	0.313 10.625	17.286 13.928	18.943 14.622	94.138 63.094	27.691 –	39.992 –
Carbon Monoxide	CO	28.0101	0.2968	68.16 132.86	15.54 kPa 3.50 MPa	0.028 10.85	5.125 4.291	5.686 4.614	84.499 40.039	21.089 –	29.785 –
Dichlorodifluoromethane (R-12)	CCl <sub>2</sub> F <sub>2</sub>	120.914	0.0688	116.1 385.12	0.24 Pa 4.14 MPa	$2.513 \times 10^{-7}$ 4.673	33.279 41.164	34.244 42.049	259.49 172.7	34.581 –	42.896 –
Dichlorofluoromethane (R21)	CHCl <sub>2</sub> F	102.923	0.0808	200 451.48	0.87 kPa 5.18 MPa	$5.255 \times 10^{-4}$ 5.111	40.303 45.082	41.964 46.097	215.2 170.48	41.955 –	50.329 –
Ethane	C <sub>2</sub> H <sub>6</sub>	30.0690	0.2765	90.368 305.32	1.14 Pa 4.87 MPa	$1.520 \times 10^{-6}$ 6.857	10.542 12.49	11.293 13.2	148.15 50.82	26.809 –	35.124 –
Ethene/Ethylene	C <sub>2</sub> H <sub>4</sub>	28.0532	0.2964	103.99 282.35	0.12 kPa 5.04 MPa	$1.411 \times 10^{-4}$ 7.637	10.621 10.545	11.486 11.206	120.03 46.609	24.972 –	33.295 –
n-Hexane	C <sub>6</sub> H <sub>14</sub>	86.1754	0.0965	177.83 507.82	1.19 Pa 3.04 MPa	$8.042 \times 10^{-7}$ 2.706	6.561 46.012	8.039 47.137	96.825 108.16	97.884 –	106.2 –
Hydrogen	H <sub>2</sub>	2.0159	4.1245	13.957 33.145	7.36 kPa 1.30 MPa	0.064 15.508	0.692 0.517	0.806 0.601	59.344 19.442	12.586 –	21.296 –
Deuterium	D <sub>2</sub>	4.0283	2.0641	18.724 38.34	17.19 kPa 1.68 MPa	0.113 17.23	1.001 0.742	1.153 0.840	62.413 23.892	12.662 –	21.607 –
Helium	He	4.0026	2.0773	2.1768 5.1953	5.04 kPa 0.23 MPa	0.293 17.384	0.046 0.033	0.0635 0.0462	34.073 8.468	12.432 –	21.802 –
Hydrogen Sulfide	H <sub>2</sub> S	34.0810	0.2440	187.7 373.1	23.26 kPa 9.00 MPa	0.015 10.19	16.328 14.47	17.876 15.353	95.815 49.374	25.347 –	34 –
Krypton	Kr	83.7980	0.0992	115.78 209.48	73.53 kPa 5.53 MPa	0.078 10.85	7.974 5.883	8.911 6.392	77 35.568	13.27 –	22.597 –
Methane	CH <sub>4</sub>	16.0425	0.5183	90.694 190.56	11.70 kPa 4.60 MPa	0.016 10.139	6.831 6.214	7.579 6.667	84.885 41.109	25.243 –	33.851 –
Methyl Alcohol	CH <sub>3</sub> OH	32.0419	0.2595	175.61 512.6	0.19 Pa 8.10 MPa	$1.276 \times 10^{-7}$ 6.810	28.219 27.813	29.679 29.003	190.32 64.132	31.874 –	40.287 –
Neon	Ne	20.1797	0.4120	24.556 44.4	43.36 kPa 2.66 MPa	0.219 24.1	1.497 1.078	1.695 1.189	69.252 30.066	12.838 –	22.087 –
Nitrogen	N <sub>2</sub>	28.0134	0.2968	63.151 126.19	12.52 kPa 3.40 MPa	0.024 11.184	1.295 0.515	1.815 0.819	163.55 118.07	21.007 –	29.647 –
Nitrous Oxide	N <sub>2</sub> O	44.0128	0.1889	182.33 309.52	87.84 kPa 7.24 MPa	0.059 10.27	14.931 12.745	16.411 13.45	90.017 51.414	25.545 –	34.993 –
Oxygen	O <sub>2</sub>	31.9988	0.2598	54.361 154.58	0.15 kPa 5.04 MPa	$3.237 \times 10^{-4}$ 13.63	1.120 0.668	1.571 1.038	209.82 134.42	21.241 –	29.631 –
Propane	C <sub>3</sub> H <sub>8</sub>	44.0956	0.1886	85.525 369.89	0.17 mPa 4.25 MPa	$2.419 \times 10^{-10}$ 5	15.439 23.633	16.15 24.483	228.67 90.468	30.456 –	38.77 –
Propylene/Propene	C <sub>3</sub> H <sub>6</sub>	42.0797	0.1976	87.953 364.21	0.75 mPa 4.56 MPa	$1.022 \times 10^{-9}$ 5.457	15.517 21.513	16.248 22.348	220.37 83.559	29.613 –	37.928 –
Sulfur Dioxide	SO <sub>2</sub>	64.0640	0.1298	197.7 430.64	1.67 kPa 7.89 MPa	$1.015 \times 10^{-3}$ 8.078	21.093 19.562	22.735 20.539	118.95 56.206	28.043 –	36.425 –
Tetrafluoroethane (R-134a)	CF <sub>3</sub> CH <sub>2</sub> F	102.0309	0.0815	169.85 374.21	0.39 kPa 4.06 MPa	$2.761 \times 10^{-4}$ 5.0171	32.764 38.947	34.175 39.756	200.38 159.38	51.318 –	59.719 –
Trichloromonofluoromethane (R-11)	CCl <sub>3</sub> F	137.368	0.0605	162.68 471.11	6.51 Pa 4.41 MPa	$4.813 \times 10^{-6}$ 4.033	44.923 57.788	46.276 58.881	271.34 218.86	49.535 –	57.857 –
Water	H <sub>2</sub> O	18.0153	0.4615	273.16 647.1	0.61 kPa 22.06 MPa	$2.695 \times 10^{-4}$ 17.874	42.785 36.314	45.055 37.548	164.94 79.393	25.553 –	33.947 –
Xenon	Xe	131.293	0.0633	161.41 289.73	81.77 kPa 5.84 MPa	0.063 8.4	11.189 8.242	12.495 8.9377	77.428 36.074	13.322 –	22.699 –

# Thermodynamic Properties of Selected Substances

# C

## C.1 Using Thermodynamic Tables

As discussed in Chapter 6 of Volume 1, pure substances may exist in a single phase (solid, liquid, vapour/gas<sup>1</sup>), or as a mixture of two phases (solid/liquid; liquid/vapour; solid/vapour).

The thermodynamic state for such a pure substance is fixed by a limited amount of intensive properties, from which the other remaining properties may be derived:

- For a single-phase substance, if two intensive properties among  $p$ ,  $v$ ,  $T$ ,  $u$ ,  $h$ ,  $s$  are known, then the state is fixed.
- For a two-phase substance, the quality  $x$  further needs to be known in addition to the two intensive properties among the previous list, for the state to also be fixed.

Thermodynamic tables allow determining all the other intensive properties for the thermodynamic state of a substance. One typically starts by examining the relevant combinations of measurable intensive properties  $p$ ,  $v$ ,  $T$ , and  $x$  for the substance, to determine whether it is a single-phase or a two-phase substance, and to assign the phase or phase pair to which the thermodynamic state belongs.

The standard definition for  $x$  in a phase change corresponds to  $x = 0$  if the substance lies in the saturation curve adjacent to the lower energy/entropy state, and  $x = 1$  if the substance lies in the saturation curve adjacent to the higher energy/entropy state. For a saturated solid-liquid phase change,  $x = 0$  corresponds to the solid saturation curve, and  $x = 1$  to the liquid saturation curve, for a liquid-vapor phase change,  $x = 0$  corresponds to the liquid saturation curve, and  $x = 1$  to the vapor saturation curve, and finally for the solid-vapor phase change,  $x = 0$  corresponds to the solid saturation curve, and  $x = 1$  to the vapor saturation curve.

The choice of intensive variables comes out naturally<sup>2</sup> as thermodynamic tables are mostly useful for the analysis of thermodynamic processes in control volumes. Typically Pressure  $p$  and Temperature  $T$  are given, as these are the measurable intensive properties that are easier to probe during a thermodynamic process (the third measurable quantity, Specific Volume  $v$ , being harder to determine in control volumes).

The non-commensurable properties  $u$ ,  $h$ , and  $s$  are provided with respect to a given reference state. Since different authors may consider different reference states, the values among tables for these properties may differ, however the differentials  $\Delta u$ ,  $\Delta h$ , and  $\Delta s$  remain equivalent, and this is what most of thermodynamic analysis are concerned with.

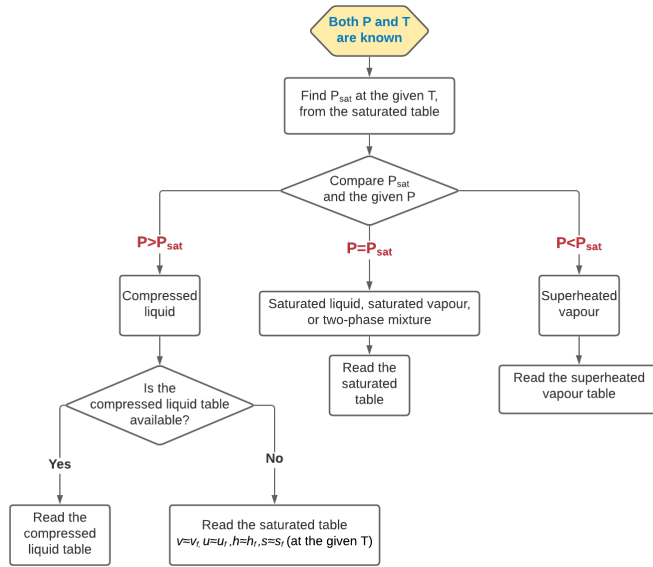
The procedure for determining the full set of intensive properties for a substance depend on the set of intensive properties that are initially known. The flowcharts for the four possible cases, adapted from Ref. [1], are presented in this section.

C.1 Using Thermodynamic Tables . . . . .	99
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C.3 R-134a Property Tables .	110

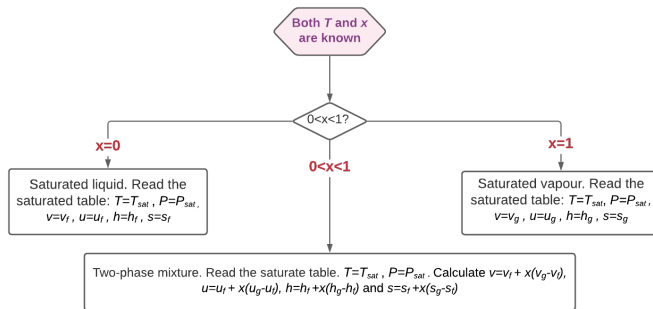
1: below/above the substance critical  
temperature

2: Intensive variables do not rely on well-  
defined frontiers, recall Chapter 1 of Vol-  
ume 1

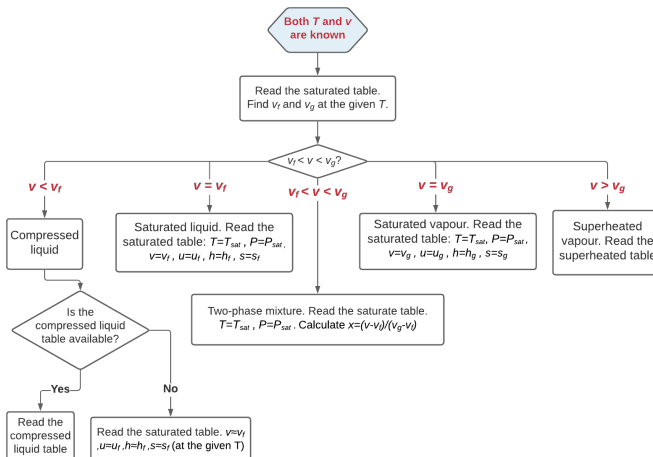
- Case 1: both  $T$  and  $p$  are given.



- Case 2: both  $T$  and  $x$  are given.



- Case 3: both  $T$  and  $v$  are given.

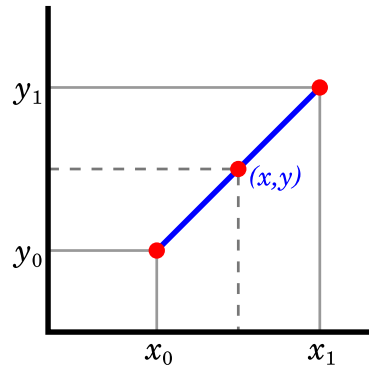


- Case 4: Temperature  $T$  and one of  $u$ ,  $h$  and  $s$  are given. The procedure is exactly the same as for case 3. Replace  $v$  with the given  $u$ ,  $h$ , or  $s$  in the flow chart.

## Interpolating properties

The tabulated properties are used in conjunction to a simple linear interpolation method to find the properties of a  $(x, y)$  state located between the  $(x_0, y_0)$  and  $(x_1, y_1)$  states, with  $x$  given and  $y$  unknown. Then the property  $y$  can be determined as

$$y = y_0 + (x - x_0) \frac{y_1 - y_0}{x_1 - x_0}.$$



## References

- [1] Claire Yu Yan. *Introduction to Engineering Thermodynamics*. 1st Edition. Open textbook library. BCcampus, 2022 (cited on page 99).

## C.2 Water Property Tables

Water properties are based on the formulation by the *International Association for the Properties of Water and Steam, IAPWS R6-95, Revised Release on the IAPWS Formulation 1995 for the Thermodynamic Properties of Ordinary Water Substance for General and Scientific Use*, 2016. <http://www.iapws.org/relguide/IAPWS-95.html>.

Numerical data properties for the water saturated, superheated, and compressed tables were retrieved from the *NIST Standard Reference Database 69: NIST Chemistry WebBook*, link: <https://webbook.nist.gov/cgi/fluid.cgi?ID=C7732185&Action=Page>.

Data for the water sublimation curve were calculated from the theoretical models proposed in Ref. [1] (sublimation curve properties); Ref. [2] (saturated ice properties); and Refs. [3, 4] (saturated vapor properties). Data was calculated down to  $-100^{\circ}\text{C}$ , the limit to which the virial coefficients used in the calculation were fitted [4].

The reference state is the “IAPWS-95” standard, with entropy  $s$  and internal energy  $u$  set to zero for the water triple point ( $T_t = 273.16\text{ K}$  and  $p_t = 611.657\text{ Pa}$ ).

## References

- [1] Jianhua Huang. ‘A Simple Accurate Formula for Calculating Saturation Vapor Pressure of Water and Ice’. In: *Journal of Applied Meteorology and Climatology* 57.6 (2018), pp. 1265–1272 (cited on page 102).
- [2] Rainer Feistel and Wolfgang Wagner. ‘A New Equation of State for  $\text{H}_2\text{O}$  Ice Ih’. In: *Journal of Physical and Chemical Reference Data* 35.2 (June 2006), pp. 1021–1047 (cited on page 102).
- [3] W. Wagner et al. ‘The IAPWS Industrial Formulation 1997 for the Thermodynamic Properties of Water and Steam’. In: *Journal of Engineering for Gas Turbines and Power* 122.1 (Jan. 2000), pp. 150–184 (cited on page 102).
- [4] H. F. Nelson and H. J. Sauer. ‘Formulation of High-Temperature Properties for Moist Air’. In: *HVAC&R Research* 8.3 (2002), pp. 311–334 (cited on page 102).

Table C.1: Saturated water – Temperature table

Temp., $T$ °C	Sat. press., $P_s$ kPa	Specific volume, $\text{m}^3/\text{kg}$		Internal energy, $\text{kJ/kg}$			Enthalpy, $\text{kJ/kg}$			Entropy, $\text{kJ/kg}\cdot\text{K}$	
		Sat. liquid, $v_f$	Sat. vapor, $v_g$	Sat. liquid, $u_f$	Evap., $u_{fg}$	Sat. vapor, $u_g$	Sat. liquid, $h_f$	Evap., $h_{fg}$	Sat. vapor, $h_g$	Sat. liquid, $s_f$	Sat. vapor, $s_g$
0	0.6117	0.001 000	205.99	0.000	2374.9	2374.9	0.001	2500.9	2500.9	0.0000	9.1555
5	0.8726	0.001 000	147.01	21.019	2360.8	2381.8	21.020	2489.1	2510.1	0.0763	8.9485
10	1.2282	0.001 000	106.30	42.020	2346.6	2388.6	42.021	2477.2	2519.2	0.1511	8.7487
15	1.7058	0.001 001	77.875	62.980	2332.5	2395.5	62.981	2465.3	2528.3	0.2245	8.5558
20	2.3393	0.001 002	57.757	83.912	2318.4	2402.3	83.914	2453.5	2537.4	0.2965	8.3695
25	3.1699	0.001 003	43.337	104.83	2304.3	2409.1	104.83	2441.7	2546.5	0.3672	8.1894
30	4.247	0.001 004	32.878	125.73	2290.2	2415.9	125.73	2429.8	2555.5	0.4368	8.0152
35	5.629	0.001 006	25.205	146.63	2276.1	2422.7	146.63	2417.9	2564.5	0.5051	7.8466
40	7.3849	0.001 008	19.515	167.53	2261.9	2429.4	167.53	2406.0	2573.5	0.5724	7.6831
45	9.595	0.001 010	15.252	188.43	2247.7	2436.1	188.43	2394.0	2582.4	0.6386	7.5247
50	12.352	0.001 012	12.027	209.33	2233.4	2442.7	209.34	2382.0	2591.3	0.7038	7.3710
55	15.762	0.001 015	9.5643	230.24	2219.1	2449.3	230.26	2369.8	2600.1	0.7680	7.2218
60	19.946	0.001 017	7.6672	251.16	2204.7	2455.9	251.18	2357.6	2608.8	0.8313	7.0768
65	25.042	0.001 020	6.1935	272.09	2190.3	2462.4	272.12	2345.4	2617.5	0.8937	6.9359
70	31.201	0.001 023	5.0395	293.03	2175.9	2468.9	293.07	2333.0	2626.1	0.9551	6.7989
75	38.595	0.001 026	4.1289	313.99	2161.2	2475.2	314.03	2320.6	2634.6	1.0158	6.6654
80	47.414	0.001 029	3.4052	334.96	2146.6	2481.6	335.01	2308.0	2643.0	1.0756	6.5355
85	57.867	0.001 032	2.8258	355.95	2131.9	2487.8	356.01	2295.3	2651.3	1.1346	6.4088
90	70.182	0.001 036	2.3591	376.97	2117.0	2494.0	377.04	2282.5	2659.5	1.1929	6.2852
95	84.608	0.001 040	1.9806	398.00	2102.0	2500.0	398.09	2269.5	2667.6	1.2504	6.1647
100	101.42	0.001 043	1.6718	419.06	2086.9	2506.0	419.17	2256.4	2675.6	1.3072	6.0469
105	120.9	0.001 047	1.4184	440.15	2071.8	2511.9	440.27	2243.1	2683.4	1.3633	5.9319
110	143.38	0.001 052	1.2093	461.26	2056.4	2517.7	461.42	2229.7	2691.1	1.4188	5.8193
115	169.18	0.001 056	1.0358	482.41	2040.9	2523.3	482.59	2216.0	2698.6	1.4737	5.7091
120	198.67	0.001 060	0.891 21	503.60	2025.3	2528.9	503.81	2202.1	2705.9	1.5279	5.6012
125	232.24	0.001 065	0.770 03	524.83	2009.5	2534.3	525.07	2188.0	2713.1	1.5816	5.4954
130	270.28	0.001 070	0.668 00	546.09	1993.4	2539.5	546.38	2173.7	2720.1	1.6346	5.3918
135	313.23	0.001 075	0.581 73	567.41	1977.3	2544.7	567.74	2159.2	2726.9	1.6872	5.2900
140	361.54	0.001 080	0.508 45	588.77	1960.8	2549.6	589.16	2144.2	2733.4	1.7392	5.1901
145	415.68	0.001 085	0.445 96	610.19	1944.2	2554.4	610.64	2129.2	2739.8	1.7907	5.0919
150	476.16	0.001 091	0.392 45	631.66	1927.4	2559.1	632.18	2113.7	2745.9	1.8418	4.9953
155	543.5	0.001 096	0.346 46	653.19	1910.3	2563.5	653.79	2098.0	2751.8	1.8924	4.9002
160	618.23	0.001 102	0.306 78	674.79	1893.0	2567.8	675.47	2081.9	2757.4	1.9426	4.8065
165	700.93	0.001 108	0.272 43	696.46	1875.4	2571.9	697.24	2065.6	2762.8	1.9923	4.7143
170	792.19	0.001 114	0.242 59	718.20	1857.5	2575.7	719.08	2048.8	2767.9	2.0417	4.6233
175	892.6	0.001 121	0.216 58	740.02	1839.4	2579.4	741.02	2031.7	2772.7	2.0906	4.5335
180	1002.8	0.001 127	0.193 84	761.92	1820.9	2582.8	763.05	2014.1	2777.2	2.1392	4.4448
185	1123.5	0.001 134	0.173 90	783.91	1802.1	2586.0	785.19	1996.2	2781.4	2.1875	4.3572
190	1255.2	0.001 141	0.156 36	806.00	1783.0	2589.0	807.43	1977.9	2785.3	2.2355	4.2704
195	1398.8	0.001 149	0.140 89	828.18	1763.5	2591.7	829.79	1959.0	2788.8	2.2832	4.1846
200	1554.9	0.001 156	0.127 21	850.47	1743.7	2594.2	852.27	1939.7	2792.0	2.3305	4.0997
205	1724.3	0.001 164	0.115 08	872.87	1723.5	2596.4	874.88	1919.9	2794.8	2.3777	4.0153
210	1907.7	0.001 173	0.104 29	895.39	1702.9	2598.3	897.63	1899.7	2797.3	2.4245	3.9318
215	2105.8	0.001 181	0.094 679	918.04	1681.9	2599.9	920.53	1878.8	2799.3	2.4712	3.8488
220	2319.6	0.001 190	0.086 092	940.82	1660.4	2601.2	943.58	1857.3	2800.9	2.5177	3.7663
225	2549.7	0.001 199	0.078 403	963.74	1638.5	2602.2	966.80	1835.3	2802.1	2.5640	3.6843
230	2797.1	0.001 209	0.071 503	986.81	1616.1	2602.9	990.19	1812.7	2802.9	2.6101	3.6027
235	3062.5	0.001 219	0.065 298	1010.0	1593.2	2603.2	1013.8	1789.4	2803.2	2.6561	3.5214
240	3346.9	0.001 229	0.059 705	1033.4	1569.7	2603.1	1037.6	1765.4	2803.0	2.7020	3.4403
245	3651.2	0.001 240	0.054 654	1057.0	1545.7	2602.7	1061.5	1740.7	2802.2	2.7478	3.3594
250	3976.2	0.001 252	0.050 083	1080.8	1521.0	2601.8	1085.8	1715.1	2800.9	2.7935	3.2786
255	4322.9	0.001 264	0.045 938	1104.8	1495.7	2600.5	1110.2	1688.9	2799.1	2.8392	3.1977
260	4692.3	0.001 276	0.042 173	1129.0	1469.7	2598.7	1135.0	1661.6	2796.6	2.8849	3.1167
265	5085.3	0.001 289	0.038 746	1153.4	1443.1	2596.5	1160.0	1633.5	2793.5	2.9307	3.0354
270	5503	0.001 303	0.035 621	1178.1	1415.6	2593.7	1185.3	1604.4	2789.7	2.9765	2.9539
275	5946.4	0.001 318	0.032 766	1203.1	1387.2	2590.3	1210.9	1574.3	2785.2	3.0224	2.8720
280	6416.6	0.001 333	0.030 153	1228.3	1358.1	2586.4	1236.9	1543.0	2779.9	3.0685	2.7894
285	6914.7	0.001 349	0.027 756	1253.9	1327.9	2581.8	1263.2	1510.5	2773.7	3.1147	2.7062
290	7441.8	0.001 366	0.025 555	1279.9	1296.6	2576.5	1290.0	1476.7	2766.7	3.1612	2.6222
295	7999.1	0.001 385	0.023 529	1306.2	1264.3	2570.5	1317.3	1441.4	2758.7	3.2080	2.5371
300	8587.9	0.001 404	0.021 660	1332.9	1230.7	2563.6	1345.0	1404.6	2749.6	3.2552	2.4507
305	9209.4	0.001 425	0.019 933	1360.2	1195.7	2555.9	1373.3	1366.1	2739.4	3.3028	2.3629
310	9865.1	0.001 448	0.018 335	1387.9	1159.2	2547.1	1402.2	1325.7	2727.9	3.3510	2.2734
315	10556	0.001 472	0.016 851	1416.3	1120.9	2537.2	1431.8	1283.3	2715.1	3.3998	2.1818
320	11284	0.001 499	0.015 471	1445.3	1080.7	2526.0	1462.2	1238.4	2700.6	3.4494	2.0878
325	12051	0.001 528	0.014 183	1475.1	1038.3	2513.4	1493.5	1190.8	2684.3	3.5000	1.9908
330	12858	0.001 561	0.012 979	1505.8	993.4	2499.2	1525.9	1140.1	2666.0	3.5518	1.8904
335	13707	0.001 597	0.011 847	1537.6	945.4	2483.0	1559.5	1085.9	2645.4	3.6050	1.7856
340	14601	0.001 638	0.010 781	1570.6	893.8	2464.4	1594.5	1027.3	2621.8	3.6601	1.6755
345	15541	0.001 685	0.009 769	1605.3	837.8	2443.1	1631.5	963.4	2594.9	3.7176	1.5586
350	16529	0.001 740	0.008 802	1642.1	776.0	2418.1	1670.9	892.7	2563.6	3.7784	1.4326
355	17570	0.001 808	0.007 868	1682.0	706.4	2388.4	1713.7	812.9	2526.6	3.8439	1.2941
360	18666	0.001 895	0.006 949	1726.3	625.5	2351.8	1761.7	719.8	2481.5	3.9167	1.1369
365	19821	0.002 017	0.006 012	1777.8	526.0	2303.8	1817.8	605.1	2422.9	4.0014	0.9483
370	21044	0.002 215	0.004 954	1844.1	386.2	2230.3	1890.7	443.8	2334.5	4.1112	0.6900
374	22064	0.003 106	0.003 106	2015.7	0.0	2015.7	2084.3	0.0	2084.3	4.4070	0.0000

Table C.2: Saturated water – Pressure table

Press., <i>P</i> kPa	Sat. temp., <i>T<sub>s</sub></i> °C	Specific volume, m <sup>3</sup> /kg		Internal energy, kJ/kg			Enthalpy, kJ/kg			Entropy, kJ/kg·K		
		Sat. liquid, <i>v<sub>f</sub></i>	Sat. vapor, <i>v<sub>g</sub></i>	Sat. liquid, <i>u<sub>f</sub></i>	Evap., <i>u<sub>fg</sub></i>	Sat. vapor, <i>u<sub>g</sub></i>	Sat. liquid, <i>h<sub>f</sub></i>	Evap., <i>h<sub>fg</sub></i>	Sat. vapor, <i>h<sub>g</sub></i>	Sat. liquid, <i>s<sub>f</sub></i>	Evap., <i>s<sub>fg</sub></i>	Sat. vapor, <i>s<sub>g</sub></i>
1.0	6.97	0.001 000	129.18	29.298	2355.2	2384.5	29.299	2484.4	2513.7	0.1059	8.8690	8.9749
1.5	13.02	0.001 001	87.959	54.681	2338.1	2392.8	54.683	2470.0	2524.7	0.1956	8.6314	8.8270
2.0	17.50	0.001 001	66.987	73.426	2325.5	2398.9	73.428	2459.5	2532.9	0.2606	8.4620	8.7226
2.5	21.08	0.001 002	54.240	88.417	2315.4	2403.8	88.420	2451.0	2539.4	0.3118	8.3302	8.6420
3.0	24.08	0.001 003	45.653	100.97	2306.9	2407.9	100.98	2443.8	2544.8	0.3543	8.2221	8.5764
4.0	28.96	0.001 004	34.791	121.38	2293.1	2414.5	121.39	2432.3	2553.7	0.4224	8.0510	8.4734
5.0	32.87	0.001 005	28.185	137.74	2282.1	2419.8	137.75	2422.9	2560.7	0.4762	7.9176	8.3938
7.5	40.29	0.001 008	19.233	168.74	2261.1	2429.8	168.75	2405.2	2574.0	0.5763	7.6738	8.2501
10	45.81	0.001 010	14.670	191.80	2245.4	2437.2	191.81	2392.1	2583.9	0.6492	7.4996	8.1488
15	53.97	0.001 014	10.020	225.93	2222.1	2448.0	225.94	2372.4	2598.3	0.7549	7.2522	8.0071
20	60.06	0.001 017	7.6480	251.40	2204.6	2456.0	251.42	2357.5	2608.9	0.8320	7.0752	7.9072
25	64.96	0.001 020	6.2032	271.93	2190.5	2462.4	271.96	2345.4	2617.4	0.8932	6.9370	7.8302
30	69.09	0.001 022	5.2284	289.24	2178.5	2467.7	289.27	2335.2	2624.5	0.9441	6.8234	7.7675
40	75.86	0.001 026	3.9930	317.58	2158.7	2476.3	317.62	2318.5	2636.1	1.0261	6.6429	7.6690
50	81.32	0.001 030	3.2400	340.49	2142.7	2483.2	340.54	2304.7	2645.2	1.0912	6.5018	7.5930
75	91.76	0.001 037	2.2170	384.36	2111.7	2496.1	384.44	2278.0	2662.4	1.2132	6.2425	7.4557
100	99.61	0.001 043	1.6939	417.40	2088.2	2505.6	417.50	2257.4	2674.9	1.3028	6.0560	7.3588
101.325	99.97	0.001 043	1.6732	418.95	2087.1	2506.0	419.06	2256.4	2675.5	1.3069	6.0475	7.3544
125	105.97	0.001 048	1.3749	444.22	2068.8	2513.0	444.35	2240.6	2684.9	1.3741	5.9099	7.2840
150	111.35	0.001 053	1.1593	466.97	2052.2	2519.2	467.13	2226.0	2693.1	1.4337	5.7893	7.2230
175	116.04	0.001 057	1.0036	486.82	2037.7	2524.5	487.00	2213.1	2700.1	1.4850	5.6865	7.1715
200	120.21	0.001 061	0.885 68	504.49	2024.6	2529.1	504.70	2201.5	2706.2	1.5302	5.5967	7.1269
225	123.97	0.001 064	0.793 20	520.47	2012.7	2533.2	520.71	2190.9	2711.6	1.5706	5.5170	7.0876
250	127.41	0.001 067	0.718 66	535.08	2001.7	2536.8	535.34	2181.2	2716.5	1.6072	5.4452	7.0524
275	130.58	0.001 070	0.657 25	548.56	1991.5	2540.1	548.86	2172.0	2720.9	1.6408	5.3799	7.0207
300	133.52	0.001 073	0.605 76	561.10	1982.1	2543.2	561.43	2163.5	2724.9	1.6717	5.3199	6.9916
325	136.27	0.001 076	0.561 94	572.84	1973.1	2545.9	573.19	2155.4	2728.6	1.7005	5.2644	6.9649
350	138.86	0.001 079	0.524 18	583.88	1964.6	2548.5	584.26	2147.7	2732.0	1.7274	5.2127	6.9401
375	141.30	0.001 081	0.491 29	594.32	1956.6	2550.9	594.73	2140.4	2735.1	1.7526	5.1645	6.9171
400	143.61	0.001 084	0.462 38	604.22	1948.9	2553.1	604.65	2133.4	2738.1	1.7765	5.1190	6.8955
450	147.90	0.001 088	0.413 90	622.65	1934.4	2557.1	623.14	2120.3	2743.4	1.8205	5.0355	6.8560
500	151.83	0.001 092	0.374 81	639.54	1921.2	2560.7	640.09	2108.0	2748.1	1.8604	4.9603	6.8207
550	155.46	0.001 097	0.342 60	655.16	1908.7	2563.9	655.76	2096.5	2752.3	1.8970	4.8916	6.7886
600	158.83	0.001 101	0.315 58	669.72	1897.1	2566.8	670.38	2085.7	2756.1	1.9308	4.8284	6.7592
650	161.98	0.001 104	0.292 59	683.36	1886.0	2569.4	684.08	2075.5	2759.6	1.9623	4.7699	6.7322
700	164.95	0.001 108	0.272 77	696.23	1875.6	2571.8	697.00	2065.8	2762.8	1.9918	4.7153	6.7071
750	167.75	0.001 111	0.255 51	708.40	1865.6	2574.0	709.24	2056.4	2765.6	2.0195	4.6641	6.6836
800	170.41	0.001 115	0.240 34	719.97	1856.0	2576.0	720.86	2047.4	2768.3	2.0457	4.6159	6.6616
850	172.94	0.001 118	0.226 89	731.00	1846.9	2577.9	731.95	2038.9	2770.8	2.0705	4.5704	6.6409
900	175.35	0.001 121	0.214 89	741.55	1838.0	2579.6	742.56	2030.4	2773.0	2.0940	4.5273	6.6213
950	177.66	0.001 124	0.204 10	751.67	1829.5	2581.2	752.74	2022.4	2775.1	2.1165	4.4862	6.6027
1000	179.88	0.001 127	0.194 36	761.39	1821.3	2582.7	762.52	2014.6	2777.1	2.1381	4.4469	6.5850
1100	184.06	0.001 133	0.177 45	779.78	1805.7	2585.5	781.03	1999.6	2780.6	2.1785	4.3735	6.5520
1200	187.96	0.001 138	0.163 26	796.96	1790.8	2587.8	798.33	1985.4	2783.7	2.2159	4.3058	6.5217
1300	191.60	0.001 144	0.151 19	813.11	1776.8	2589.9	814.60	1971.9	2786.5	2.2508	4.2428	6.4936
1400	195.04	0.001 149	0.140 78	828.36	1763.4	2591.8	829.97	1958.8	2788.8	2.2835	4.1840	6.4675
1500	198.29	0.001 154	0.131 71	842.83	1750.6	2593.4	844.56	1946.4	2791.0	2.3143	4.1287	6.4430
1750	205.73	0.001 166	0.113 43	876.13	1720.6	2596.7	878.17	1917.0	2795.2	2.3845	4.0032	6.3877
2000	212.38	0.001 177	0.099 585	906.14	1693.0	2599.1	908.50	1889.8	2798.3	2.4468	3.8922	6.3390
2250	218.41	0.001 187	0.088 715	933.57	1667.3	2600.9	936.24	1864.3	2800.5	2.5029	3.7925	6.2954
2500	223.95	0.001 197	0.079 949	958.91	1643.2	2602.1	961.91	1840.0	2801.9	2.5543	3.7015	6.2558
3000	233.85	0.001 217	0.066 664	1004.7	1598.5	2603.2	1008.3	1794.9	2803.2	2.6455	3.5401	6.1856
3500	242.56	0.001 235	0.057 058	1045.5	1557.4	2602.9	1049.8	1752.8	2802.6	2.7254	3.3989	6.1243
4000	250.35	0.001 253	0.049 776	1082.5	1519.2	2601.7	1087.5	1713.3	2800.8	2.7968	3.2728	6.0696
5000	263.94	0.001 286	0.039 446	1148.2	1448.8	2597.0	1154.6	1639.6	2794.2	2.9210	3.0527	5.9737
6000	275.58	0.001 319	0.032 448	1206.0	1383.9	2589.9	1213.9	1570.7	2784.6	3.0278	2.8623	5.8901
7000	285.83	0.001 352	0.027 378	1258.2	1322.8	2581.0	1267.7	1504.9	2772.6	3.1224	2.6924	5.8148
8000	295.01	0.001 385	0.023 526	1306.2	1264.3	2570.5	1317.3	1441.4	2758.7	3.2081	2.5369	5.7450
9000	303.34	0.001 418	0.020 490	1351.1	1207.4	2558.5	1363.9	1379.0	2742.9	3.2870	2.3921	5.6791
10 000	311.00	0.001 453	0.018 030	1393.5	1151.7	2545.2	1408.1	1317.4	2725.5	3.3606	2.2554	5.6160
11 000	318.08	0.001 489	0.015 990	1434.1	1096.4	2530.5	1450.4	1255.9	2706.3	3.4303	2.1242	5.5545
12 000	324.68	0.001 526	0.014 264	1473.1	1041.2	2514.3	1491.5	1193.9	2685.4	3.4967	1.9972	5.4939
13 000	330.85	0.001 566	0.012 780	1511.1	985.4	2496.5	1531.5	1131.2	2662.7	3.5608	1.8728	5.4336
14 000	336.67	0.001 610	0.011 485	1548.4	928.7	2477.1	1571.0	1066.9	2637.9	3.6232	1.7495	5.3727
15 000	342.16	0.001 657	0.010 338	1585.3	870.3	2455.6	1610.2	1000.5	2610.7	3.6846	1.6260	5.3106
16 000	347.35	0.001 709	0.009 308 8	1622.3	809.5	2431.8	1649.7	931.1	2580.8	3.7457	1.5006	5.2463
17 000	352.29	0.001 769	0.008 370 9	1659.9	745.3	2405.2	1690.0	857.5	2547.5	3.8077	1.3710	5.1787
18 000	356.99	0.001 840	0.007 501 7	1699.0	675.8	2374.8	1732.1	777.7	2509.8	3.8718	1.2343	5.1061
19 000	361.47	0.001 927	0.006 677 3	1740.5	598.6	2339.1	1777.2	688.8	2466.0	3.9401	1.0855	5.0256
20 000	365.75	0.002 040	0.005 865 2	1786.4	508.6	2295.0	1827.2	585.1	2412.3	4.0156	0.9158	4.9314
21 000	369.83	0.002 205	0.004 996 1	1841.2	392.5	2233.7	1887.6	451.0	2338.6	4.1064	0.7015	4.8079
22 000	373.71	0.002 704	0.003 647 5	1951.8	141.0	2092.8	2011.3	161.8	2173.1	4.2945	0.2501	4.5446
22 064	373.95	0.003 106	0.003 105 6	2015.7	0.0	2015.7	2084.3	0.0	2084.3	4.4070	0.0000	4.4070

Table C.3: Superheated water vapor – Pressure table (0.01–1.4MPa)

T °C	v m <sup>3</sup> /kg	u kJ/kg	h kJ/kg	s kJ/kg·K	T °C	v m <sup>3</sup> /kg	u kJ/kg	h kJ/kg	s kJ/kg·K	T °C	v m <sup>3</sup> /kg	u kJ/kg	h kJ/kg	s kJ/kg·K
p=0.01 MPa (45.81 °C)					p=0.05 MPa (81.32 °C)					p=0.10 MPa (99.61 °C)				
Sat.	14.67	2437.2	2583.9	8.1488	Sat.	3.2401	2483.2	2645.2	7.593	Sat.	1.6939	2505.6	2675	7.3589
50	14.867	2443.3	2592	8.1741	100	3.4187	2511.5	2682.4	7.6953	100	1.6959	2506.2	2675.8	7.361
100	17.196	2515.5	2687.5	8.4489	150	3.8897	2585.7	2780.2	7.9413	150	1.9367	2582.9	2776.6	7.6148
150	19.513	2587.9	2783	8.6892	200	4.3562	2660	2877.8	8.1592	200	2.1724	2658.2	2875.5	7.8356
200	21.826	2661.3	2879.6	8.9049	250	4.8206	2735.1	2976.1	8.3568	250	2.4062	2733.9	2974.5	8.0346
250	24.136	2736.1	2977.4	9.1015	300	5.284	2811.6	3075.8	8.5386	300	2.6388	2810.6	3074.5	8.2172
300	26.446	2812.3	3076.7	9.2827	400	6.2094	2968.9	3279.3	8.8659	400	3.1027	2968.3	3278.6	8.5452
400	31.063	2969.3	3279.9	9.6094	500	7.1338	3132.6	3489.3	9.1566	500	3.5655	3132.2	3488.7	8.8361
500	35.68	3132.9	3489.7	9.8998	600	8.0576	3303.1	3706	9.4201	600	4.0279	3302.8	3705.6	9.0998
600	40.296	3303.3	3706.3	10.163	700	8.9812	3480.6	3929.7	9.6625	700	4.49	3480.4	3929.4	9.3424
700	44.911	3480.8	3929.9	10.406	800	9.9047	3665.2	4160.4	9.8882	800	4.9519	3665	4160.2	9.5681
800	49.527	3665.3	4160.6	10.631	900	10.828	3856.8	4398.2	10.1	900	5.4137	3856.6	4398	9.78
900	54.142	3856.9	4398.3	10.843	1000	11.751	4055.1	4642.7	10.3	1000	5.8754	4055	4642.6	9.98
1000	58.758	4055.2	4642.8	11.043	1100	12.674	4259.9	4893.7	10.49	1100	6.3371	4259.8	4893.5	10.17
1100	63.373	4260	4893.7	11.233	1200	13.598	4470.8	5150.7	10.67	1200	6.7988	4470.7	5150.6	10.35
1200	67.988	4470.9	5150.7	11.413	1300	14.521	4687.3	5413.3	10.843	1300	7.2604	4687.2	5413.2	10.523
1300	72.604	4687.4	5413.4	11.586										
p=0.20 MPa (120.21 °C)					p=0.30 MPa (133.52 °C)					p=0.40 MPa (143.61 °C)				
Sat.	0.885 69	2529.1	2706.2	7.1269	Sat.	0.605 76	2543.2	2724.9	6.9916	Sat.	0.462 39	2553.1	2738.1	6.8955
150	0.959 86	2577.1	2769.1	7.281	150	0.634 01	2571	2761.2	7.0791	150	0.470 88	2564.4	2752.8	6.9306
200	1.0805	2654.6	2870.7	7.5081	200	0.716 42	2651	2865.9	7.3131	200	0.534 33	2647.2	2860.9	7.1723
250	1.1989	2731.4	2971.2	7.71	250	0.796 44	2728.9	2967.9	7.518	250	0.5952	2726.4	2964.5	7.3804
300	1.3162	2808.8	3072.1	7.8941	300	0.875 34	2807	3069.6	7.7037	300	0.654 89	2805.1	3067.1	7.5677
400	1.5493	2967.1	3277	8.2236	400	1.0315	2966	3275.5	8.0347	400	0.772 64	2964.9	3273.9	7.9002
500	1.7814	3131.4	3487.7	8.5152	500	1.1867	3130.6	3486.6	8.3271	500	0.889 36	3129.8	3485.5	8.1933
600	2.013	3302.2	3704.8	8.7792	600	1.3414	3301.6	3704	8.5914	600	1.0056	3301	3703.2	8.458
700	2.2443	3479.9	3928.8	9.022	700	1.4958	3479.5	3928.2	8.8344	700	1.1215	3479	3927.6	8.7012
800	2.4755	3664.7	4159.8	9.2479	800	1.65	3664.3	4159.3	9.0604	800	1.2373	3663.9	4158.8	8.9273
900	2.7066	3856.3	4397.6	9.4598	900	1.8042	3856	4397.3	9.2724	900	1.353	3855.7	4396.9	9.1394
1000	2.9375	4054.8	4642.3	9.6599	1000	1.9582	4054.5	4642	9.4726	1000	1.4686	4054.3	4641.7	9.3396
1100	3.1685	4259.6	4893.3	9.8497	1100	2.1122	4259.4	4893.1	9.6624	1100	1.5841	4259.2	4892.8	9.5295
1200	3.3994	4470.5	5150.4	10.03	1200	2.2662	4470.3	5150.2	9.8431	1200	1.6997	4470.1	5150	9.7102
1300	3.6302	4687	5413.1	10.203	1300	2.4202	4686.9	5412.9	10.016	1300	1.8152	4686.7	5412.8	9.8828
p=0.50 MPa (151.83 °C)					p=0.60 MPa (158.83 °C)					p=0.80 MPa (170.41 °C)				
Sat.	0.374 81	2560.7	2748.1	6.8207	Sat.	0.315 59	2566.8	2756.2	6.7593	Sat.	0.240 34	2576	2768.3	6.6616
200	0.425 03	2643.3	2855.8	7.061	200	0.352 12	2639.3	2850.6	6.9683	200	0.260 88	2631	2839.7	6.8176
250	0.474 43	2723.8	2961	7.2724	250	0.3939	2721.2	2957.6	7.1832	250	0.2932	2715.9	2950.4	7.0401
300	0.522 61	2803.2	3064.6	7.4614	300	0.434 42	2801.4	3062	7.374	300	0.324 16	2797.5	3056.9	7.2345
350	0.570 15	2883	3168.1	7.6346	350	0.474 27	2881.6	3166.1	7.5481	350	0.354 42	2878.6	3162.2	7.4106
400	0.6173	2963.7	3272.3	7.7955	400	0.513 74	2962.5	3270.8	7.7097	400	0.384 28	2960.2	3267.6	7.5734
500	0.710 94	3129	3484.5	8.0892	500	0.592	3128.2	3483.4	8.0041	500	0.443 32	3126.6	3481.3	7.8692
600	0.804 09	3300.4	3702.5	8.3543	600	0.669 76	3299.8	3701.7	8.2695	600	0.501 85	3298.7	3700.1	8.1354
700	0.896 96	3478.5	3927	8.5977	700	0.747 25	3478.1	3926.4	8.5131	700	0.560 11	3477.2	3925.3	8.3794
800	0.989 66	3663.6	4158.4	8.824	800	0.824 57	3663.2	4157.9	8.7395	800	0.6182	3662.4	4157	8.6061
900	1.0823	3855.4	4396.6	9.0362	900	0.901 78	3855.1	4396.2	8.9518	900	0.676 19	3854.5	4395.5	8.8185
1000	1.1748	4054	4641.4	9.2364	1000	0.978 93	4053.7	4641.1	9.1521	1000	0.734 11	4053.2	4640.5	9.0189
1100	1.2673	4259	4892.6	9.4263	1100	1.056	4258.7	4892.4	9.342	1100	0.791 97	4258.3	4891.9	9.2089
1200	1.3597	4470	5149.8	9.6071	1200	1.1331	4469.8	5149.6	9.5228	1200	0.8498	4469.4	5149.2	9.3898
1300	1.4521	4686.6	5412.6	9.7797	1300	1.2101	4686.4	5412.5	9.6954	1300	0.9076	4686.1	5412.2	9.5625
p=1.00 MPa (179.88 °C)					p=1.20 MPa (187.96 °C)					p=1.40 MPa (195.04 °C)				
Sat.	0.194 36	2582.8	2777.1	6.585	Sat.	0.163 26	2587.8	2783.8	6.5217	Sat.	0.140 78	2591.8	2788.8	6.4675
200	0.206 02	2622.2	2828.3	6.6955	200	0.169 34	2612.9	2816.1	6.5909	200	0.143 03	2602.7	2803	6.4975
250	0.232 75	2710.4	2943.1	6.9265	250	0.192 41	2704.7	2935.6	6.8313	250	0.163 56	2698.9	2927.9	6.7488
300	0.257 99	2793.6	3051.6	7.1246	300	0.213 86	2789.7	3046.3	7.0335	300	0.182 32	2785.7	3040.9	6.9552
350	0.2825	2875.7	3158.2	7.3029	350	0.234 55	2872.7	3154.2	7.2139	350	0.200 29	2869.7	3150.1	7.1379
400	0.306 61	2957.9	3264.5	7.4669	400	0.254 82	2955.5	3261.3	7.3793	400	0.217 82	2953.1	3258.1	7.3046
500	0.354 11	3125	3479.1	7.7641	500	0.294 64	3123.4	3476.9	7.6779	500	0.252 16	3121.8	3474.8	7.6047
600	0.401 11	3297.5	3698.6	8.031	600	0.333 94	3296.3	3697	7.9455	600	0.285 97	3295.1	3695.4	7.873
700	0.447 83	3476.2	3924.1	8.2755	700	0.372 97	3475.3	3922.9	8.1904	700	0.319 51	3474.4	3921.7	8.1183
800	0.494 38	3661.7	4156.1	8.5024	800	0.411 84	3661	4155.2	8.4176	800	0.352 87	3660.2	4154.3	8.3457
900	0.540 83	3853.9	4394.8	8.715	900	0.450 59	3853.3	4394	8.6303	900	0.386 14	3852.7	4393.3	8.5587
1000	0.587 21	4052.7	4639.9	8.9155	1000	0.489 28	4052.2	4639.4	8.831	1000	0.419 33	4051.7	4638.8	8.7594
1100	0.633 54	4257.9	4891.4	9.1056	1100	0.527 92	4257.5	4891	9.0212	1100	0.452 47	4257	4890.5	8.9497
1200	0.679 83	4469	5148.9	9.2866	1200	0.566 52	4468.7	5148.5	9.2022	1200	0.485 58	4468.3	5148.1	9.1308
1300	0.7261	4685.8	5411.9	9.4593	1300	0.605 09	4685.4	5411.5	9.3749	1300	0.518 66	4685.1	5411.2	9.3036

Table C.4: Superheated water vapor – Pressure table (1.6–8MPa)

T °C	v m <sup>3</sup> /kg	u kJ/kg	h kJ/kg	s kJ/kg·K	T °C	v m <sup>3</sup> /kg	u kJ/kg	h kJ/kg	s kJ/kg·K	T °C	v m <sup>3</sup> /kg	u kJ/kg	h kJ/kg	s kJ/kg·K
p=1.60 MPa (201.37 °C)					p=1.80 MPa (207.11 °C)					p=2.00 MPa (212.38 °C)				
Sat.	0.123 74	2594.8	2792.8	6.42	Sat.	0.110 37	2597.3	2795.9	6.3775	Sat.	0.099 588	2599.1	2798.3	6.3391
225	0.132 93	2645.1	2857.8	6.5537	225	0.116 78	2637	2847.2	6.4824	225	0.103 81	2628.5	2836.1	6.416
250	0.1419	2692.9	2919.9	6.6753	250	0.125 02	2686.7	2911.7	6.6087	250	0.1115	2680.2	2903.2	6.5475
300	0.158 66	2781.6	3035.4	6.8863	300	0.140 25	2777.4	3029.9	6.8246	300	0.125 51	2773.2	3024.2	6.7684
350	0.174 59	2866.6	3146	7.0713	350	0.1546	2863.6	3141.8	7.012	350	0.1386	2860.5	3137.7	6.9583
400	0.190 07	2950.7	3254.9	7.2394	400	0.168 49	2948.3	3251.6	7.1814	400	0.151 21	2945.9	3248.3	7.1292
500	0.220 29	3120.1	3472.6	7.5409	500	0.195 51	3118.5	3470.4	7.4845	500	0.175 68	3116.9	3468.2	7.4337
600	0.249 99	3293.9	3693.9	7.81	600	0.222	3292.7	3692.3	7.7543	600	0.199 61	3291.5	3690.7	7.7043
700	0.2794	3473.5	3920.5	8.0557	700	0.248 21	3472.6	3919.4	8.0004	700	0.223 26	3471.6	3918.2	7.9509
800	0.308 65	3659.5	4153.3	8.2834	800	0.274 26	3658.8	4152.4	8.2284	800	0.246 74	3658	4151.5	8.179
900	0.3378	3852.1	4392.6	8.4965	900	0.3002	3851.5	4391.9	8.4416	900	0.270 12	3850.9	4391.1	8.3925
1000	0.366 87	4051.2	4638.2	8.6974	1000	0.326 06	4050.7	4637.6	8.6426	1000	0.293 42	4050.2	4637	8.5936
1100	0.395 89	4256.6	4890	8.8878	1100	0.351 88	4256.2	4889.5	8.8331	1100	0.316 67	4255.7	4889.1	8.7842
1200	0.424 87	4467.9	5147.7	9.0689	1200	0.377 66	4467.5	5147.3	9.0143	1200	0.339 89	4467.2	5147	8.9654
1300	0.453 83	4684.8	5410.9	9.2417	1300	0.403 41	4684.5	5410.6	9.1872	1300	0.363 08	4684.1	5410.3	9.1384
p=2.50 MPa (223.95 °C)					p=3.00 MPa (233.85 °C)					p=3.50 MPa (242.56 °C)				
Sat.	0.079 951	2602.1	2801.9	6.2558	Sat.	0.066 665	2603.2	2803.2	6.1856	Sat.	0.057 059	2602.9	2802.7	6.1244
225	0.080 26	2604.8	2805.5	6.2629	225	0.070 627	2644.7	2856.5	6.2893	225	0.058 757	2624	2829.7	6.1764
250	0.087 053	2663.3	2880.9	6.4107	250	0.081 179	2750.8	2994.3	6.5412	250	0.068 453	2738.8	2978.4	6.4484
300	0.098 937	2762.2	3009.6	6.6459	300	0.090 556	2844.4	3116.1	6.7449	300	0.076 804	2836	3104.8	6.6601
350	0.109 79	2852.5	3127	6.8424	350	0.099 379	2933.5	3231.7	6.9234	350	0.084 556	2927.2	3223.2	6.8427
400	0.120 12	2939.8	3240.1	7.017	400	0.107 89	3021.2	3344.8	7.0856	400	0.091 978	3016.1	3338	7.0074
450	0.130 15	3026.2	3351.6	7.1767	450	0.1162	3108.6	3457.2	7.2359	450	0.099 195	3104.5	3451.6	7.1593
500	0.139 99	3112.8	3462.7	7.3254	500	0.123 45	3285.5	3682.8	7.5103	500	0.113 25	3282.5	3678.9	7.4356
600	0.159 31	3288.5	3686.8	7.5979	600	0.132 45	3467	3912.2	7.759	600	0.127 02	3464.7	3909.3	7.6854
700	0.178 35	3469.3	3915.2	7.8455	700	0.148 41	3654.3	4146.9	7.9885	700	0.140 61	3652.5	4144.6	7.9156
800	0.197 21	3656.2	4149.2	8.0743	800	0.1642	3847.9	4387.5	8.2028	800	0.1541	3846.4	4385.7	8.1303
900	0.215 97	3849.4	4389.3	8.2882	900	0.179 88	4047.7	4634.1	8.4045	900	0.167 51	4046.4	4632.7	8.3324
1000	0.234 66	4048.9	4635.6	8.4896	1000	0.195 49	4253.6	4886.7	8.5955	1000	0.180 87	4252.5	4885.6	8.5235
1100	0.2533	4254.7	4887.9	8.6804	1100	0.211 05	4462.5	5142.2	8.777	1100	0.1942	4464.4	5144.1	8.7053
1200	0.2719	4466.2	5146	8.8618	1200	0.226 57	4682.5	5408.8	8.9502	1200	0.2075	4681.7	5408	8.8785
1300	0.290 47	4683.3	5409.5	9.0349	1300	0.242 07	4882.5	5708.8	9.1143	1300	0.224 07	4882.5	5708.8	9.1143
p=4.00 MPa (250.35 °C)					p=4.50 MPa (257.44 °C)					p=5.00 MPa (263.94 °C)				
Sat.	0.049 776	2601.7	2800.8	6.0696	Sat.	0.044 06	2599.7	2798	6.0197	Sat.	0.039 447	2597	2794.2	5.9737
275	0.054 607	2668.9	2887.3	6.2312	275	0.047 333	2651.3	2864.3	6.1429	275	0.041 439	2632.3	2839.5	6.0571
300	0.058 87	2726.2	2961.7	6.3639	300	0.051 378	2713	2944.2	6.2854	300	0.045 346	2699	2925.7	6.211
350	0.066 473	2827.4	3093.3	6.5843	350	0.058 423	2818.6	3081.5	6.5153	350	0.051 969	2809.5	3069.3	6.4516
400	0.073 431	2920.7	3214.5	6.7714	400	0.064 772	2914.2	3205.6	6.707	400	0.057 837	2907.5	3196.7	6.6483
450	0.080 043	3011	3331.2	6.9386	450	0.070 756	3005.8	3324.2	6.877	450	0.063 323	3000.6	3317.2	6.821
500	0.086 442	3100.3	3446	7.0922	500	0.076 521	3096	3440.4	7.0323	500	0.068 583	3091.7	3434.7	6.9781
600	0.098 859	3279.4	3674.9	7.3705	600	0.087 662	3276.4	3670.9	7.3127	600	0.078 704	3273.3	3666.8	7.2605
700	0.110 98	3462.4	3906.3	7.6214	700	0.0985	3460	3903.3	7.5646	700	0.088 518	3457.7	3900.3	7.5136
800	0.122 92	3650.6	4142.3	7.8523	800	0.109 16	3648.8	4140	7.7962	800	0.098 158	3646.9	4137.7	7.7458
900	0.134 76	3844.8	4383.9	8.0674	900	0.119 72	3843.3	4382.1	8.0118	900	0.107 69	3841.8	4380.2	7.9618
1000	0.146 52	4045.1	4631.2	8.2697	1000	0.1302	4043.9	4629.8	8.2144	1000	0.117 15	4042.6	4628.3	8.1648
1100	0.158 24	4251.4	4884.4	8.4611	1100	0.140 64	4250.4	4883.2	8.406	1100	0.126 55	4249.3	4882	8.3566
1200	0.169 92	4463.5	5143.1	8.643	1200	0.151 03	4462.5	5142.2	8.588	1200	0.135 92	4461.6	5141.2	8.5388
1300	0.181 57	4680.9	5407.2	8.8164	1300	0.1614	4680.1	5406.4	8.7615	1300	0.145 27	4679.3	5405.7	8.7124
p=6.00 MPa (275.59 °C)					p=7.00 MPa (285.83 °C)					p=8.00 MPa (295.01 °C)				
Sat.	0.032 449	2589.9	2784.6	5.8902	Sat.	0.027 379	2581	2772.6	5.8148	Sat.	0.023 526	2570.5	2758.7	5.745
300	0.036 189	2668.4	2885.5	6.0703	300	0.029 492	2633.5	2839.9	5.9337	300	0.024 279	2592.3	2786.5	5.7937
350	0.042 251	2790.4	3043.9	6.3357	350	0.035 262	2770.1	3016.9	6.2304	350	0.029 975	2748.3	2988.1	6.1321
400	0.047 419	2893.7	3178.2	6.5432	400	0.039 958	2879.5	3159.2	6.4502	400	0.034 344	2864.6	3139.4	6.3658
450	0.052 166	2989.9	3302.9	6.7219	450	0.044 187	2979	3288.3	6.6353	450	0.038 194	2967.8	3273.3	6.5579
500	0.056 671	3083.1	3423.1	6.8826	500	0.048 157	3074.3	3411.4	6.8	500	0.041 767	3065.4	3399.5	6.7266
550	0.061 021	3175.2	3541.3	7.0307	550	0.051 966	3167.9	3531.6	6.9506	550	0.045 172	3160.5	3521.8	6.8799
600	0.065 265	3267.2	3658.7	7.1693	600	0.055 665	3260.9	3650.6	7.091	600	0.048 463	3254.7	3642.4	7.0221
700	0.073 545	3453	3894.3	7.4246	700	0.062 85	3448.3	3888.2	7.3486	700	0.054 828	3443.6	3882.2	7.2821
800	0.081 648	3643.2	4133.1	7.6582	800	0.069 855	3639.5	4128.4	7.5836	800	0.061 011	3635.7	4123.8	7.5184
900	0.089 641	3838.8	4376.6	7.8751	900	0.076 75	3835.7	4373	7.8014	900	0.067 082	3832.6	4369.3	7.7371
1000	0.097 56	4040.1	4625.4	8.0786	1000	0.083 571	4037.5	4622.5	8.0055	1000	0.073 079	4035	4619.6	7.9419
1100	0.105 43	4247.1	4879.7	8.2709	1100	0.090 341	4245	4877.3	8.1981	1100	0.079 025	4242.8	4875	8.135
1200	0.113 26	4459.8	5139.3	8.4534	1200	0.097 074	4457.9	5137.4	8.381	1200	0.084 934	4456.1	5135.5	8.3181
1300	0.121 07	4677.7	5404.1	8.6272	1300	0.103 78	4676.1	5402.6	8.5551	1300	0.090 816	4674.5	5401	8.4924

Table C.5: Superheated water vapor – Pressure table (9–60MPa)

T °C	v m <sup>3</sup> /kg	u kJ/kg	h kJ/kg	s kJ/kg·K	T °C	v m <sup>3</sup> /kg	u kJ/kg	h kJ/kg	s kJ/kg·K	T °C	v m <sup>3</sup> /kg	u kJ/kg	h kJ/kg	s kJ/kg·K
p=9.00 MPa (303.35 °C)					p=10.00 MPa (311.00 °C)					p=12.50 MPa (327.81 °C)				
Sat.	0.020 491	2558.6	2743	5.6792	Sat.	0.018 03	2545.2	2725.5	5.616	Sat.	0.001 546	1492.2	1511.6	3.529
325	0.023 284	2647.6	2857.1	5.8738	325	0.019 877	2611.6	2810.3	5.7596	350	0.016 138	2624.8	2826.6	5.713
350	0.025 816	2724.9	2957.3	6.038	350	0.022 44	2699.6	2924	5.9459	400	0.020 03	2789.6	3040	6.0433
400	0.029 96	2849.2	3118.8	6.2876	400	0.026 436	2833.1	3097.4	6.2141	450	0.023 019	2913.7	3201.4	6.2749
450	0.033 524	2956.3	3258	6.4872	450	0.029 782	2944.5	3242.3	6.4219	500	0.025 63	3023.2	3343.6	6.465
500	0.036 793	3056.3	3387.4	6.6603	500	0.032 811	3047	3375.1	6.5995	550	0.028 033	3126.1	3476.5	6.6317
550	0.039 885	3153	3512	6.8164	550	0.035 654	3145.4	3502	6.7585	600	0.030 306	3225.8	3604.6	6.7828
600	0.042 861	3248.4	3634.1	6.9605	600	0.038 378	3242	3625.8	6.9045	650	0.032 49	3324.1	3730.2	6.9227
650	0.045 755	3343.4	3755.2	7.0953	650	0.041 018	3337.9	3748.1	7.0408	700	0.034 612	3422	3854.6	7.0539
700	0.048 589	3438.8	3876.1	7.2229	700	0.043 597	3434	3870	7.1693	800	0.038 724	3618.7	4102.8	7.2967
800	0.054 132	3632	4119.1	7.4606	800	0.048 629	3628.2	4114.5	7.4085	900	0.042 72	3818.9	4352.9	7.5194
900	0.059 562	3829.6	4365.7	7.6802	900	0.053 547	3826.5	4362	7.629	1000	0.046 641	4023.5	4606.5	7.7269
1000	0.064 918	4032.4	4616.7	7.8855	1000	0.058 39	4029.9	4613.8	7.8349	1100	0.050 51	4233.1	4864.5	7.9219
1100	0.070 224	4240.6	4872.7	8.079	1100	0.063 183	4238.5	4870.3	8.0288	1200	0.054 342	4447.7	5127	8.1065
1200	0.075 492	4454.2	5133.6	8.2625	1200	0.067 938	4452.3	5131.7	8.2126	1300	0.058 147	4667.3	5394.1	8.2819
1300	0.080 733	4672.9	5399.5	8.437	1300	0.072 667	4671.3	5397.9	8.3874					
p=15.00 MPa (342.16 °C)					p=17.50 MPa (354.67 °C)					p=20.00 MPa (365.75 °C)				
Sat.	0.010 339	2455.7	2610.8	5.3107	Sat.	0.007 93	2390.6	2529.4	5.1433	Sat.	0.005 865 5	2295.1	2412.4	4.9315
350	0.011 481	2520.9	2693.1	5.4437	400	0.012 463	2684.3	2902.4	5.7211	400	0.009 950 3	2617.9	2816.9	5.5525
400	0.015 671	2740.6	2975.7	5.8819	450	0.015 204	2845.4	3111.4	6.0212	450	0.012 721	2807.2	3061.7	5.9043
450	0.018 477	2880.7	3157.9	6.1434	500	0.017 385	2972.4	3276.7	6.2424	500	0.014 793	2945.3	3241.2	6.1446
500	0.020 827	2998.4	3310.8	6.348	550	0.019 305	3085.8	3423.6	6.4266	550	0.016 571	3064.7	3396.1	6.3389
550	0.022 945	3106.2	3450.4	6.523	600	0.021 073	3192.5	3561.3	6.589	600	0.018 185	3175.3	3539	6.5075
600	0.024 921	3209.3	3583.1	6.6796	650	0.022 742	3295.8	3693.8	6.7366	650	0.019 695	3281.4	3675.3	6.6593
650	0.026 804	3310.1	3712.1	6.8233	700	0.024 342	3397.5	3823.5	6.8734	700	0.021 133	3385.1	3807.8	6.799
700	0.028 621	3409.8	3839.1	6.9572	800	0.027 405	3599.7	4079.3	7.1236	800	0.023 869	3590.1	4067.5	7.0531
800	0.032 121	3609.2	4091.1	7.2037	900	0.030 348	3803.4	4334.5	7.3511	900	0.026 483	3795.7	4325.4	7.2829
900	0.035 503	3811.2	4343.7	7.4288	1000	0.033 215	4010.7	4592	7.5616	1000	0.029 02	4004.3	4584.7	7.495
1000	0.038 808	4017.1	4599.2	7.6378	1100	0.036 029	4222.3	4852.8	7.7588	1100	0.031 504	4216.9	4846.9	7.6933
1100	0.042 062	4227.7	4858.6	7.8339	1200	0.038 806	4438.4	5117.5	7.9449	1200	0.033 952	4433.8	5112.8	7.8802
1200	0.045 279	4443.1	5122.3	8.0192	1300	0.041 555	4659.2	5386.4	8.1215	1300	0.036 371	4655.2	5382.6	8.0574
1300	0.048 468	4663.2	5390.3	8.1952										
p=25.00 MPa					p=30.00 MPa					p=35.00 MPa				
375	0.001 978 3	1799.9	1849.4	4.0344	375	0.001 791 6	1738.1	1791.8	3.9313	375	0.001 701 4	1702.8	1762.4	3.8724
400	0.006 004 7	2428.5	2578.6	5.14	400	0.002 797 8	2068.9	2152.8	4.4757	400	0.002 105 4	1914.9	1988.6	4.2143
425	0.007 885 9	2607.8	2805	5.4707	425	0.005 298 6	2452.8	2611.8	5.1473	425	0.003 433 9	2253.3	2373.4	4.7751
450	0.009 176 3	2721.2	2950.6	5.6759	450	0.006 737 3	2618.9	2821	5.4421	450	0.004 957 2	2497.5	2671	5.1945
500	0.011 143	2887.3	3165.9	5.9642	500	0.008 690 4	2824	3084.7	5.7956	500	0.006 932 5	2755.3	2997.9	5.6331
550	0.012 736	3020.8	3339.2	6.1816	550	0.010 175	2974.5	3279.7	6.0402	550	0.008 347 9	2925.8	3218	5.9092
600	0.014 14	3140	3493.5	6.3637	600	0.011 445	3103.4	3446.7	6.2373	600	0.009 523 4	3065.6	3398.9	6.1228
650	0.015 43	3251.9	3637.7	6.5242	650	0.012 589	3221.7	3599.4	6.4074	650	0.010 565	3190.9	3560.7	6.303
700	0.016 643	3359.9	3776	6.6702	700	0.013 653	3334.3	3743.9	6.5598	700	0.011 523	3308.3	3711.6	6.4622
800	0.018 922	3570.7	4043.8	6.9322	800	0.015 628	3551.2	4020	6.83	800	0.013 278	3531.5	3996.3	6.7409
900	0.021 075	3780.2	4307.1	7.1668	900	0.017 473	3764.6	4288.8	7.0695	900	0.014 904	3748.9	4270.6	6.9853
1000	0.023 15	3991.5	4570.2	7.382	1000	0.019 24	3978.6	4555.8	7.288	1000	0.016 45	3965.8	4541.5	7.2069
1100	0.025 172	4206	4835.4	7.5825	1100	0.020 953	4195.2	4823.8	7.4906	1100	0.017 942	4184.4	4812.4	7.4118
1200	0.027 157	4424.6	5103.5	7.771	1200	0.022 63	4415.3	5094.2	7.6807	1200	0.019 398	4406.1	5085	7.6034
1300	0.029 115	4647.2	5375.1	7.9493	1300	0.024 279	4639.2	5367.6	7.8602	1300	0.020 827	4631.2	5360.1	7.7841
p=40.00 MPa					p=50.00 MPa					p=60.00 MPa				
375	0.001 641 2	1677	1742.6	3.829	375	0.001 559 6	1638.6	1716.6	3.7642	375	0.001 503 4	1609.7	1699.9	3.7149
400	0.001 910 8	1854.9	1931.4	4.1145	400	0.001 730 7	1787.8	1874.4	4.0029	400	0.001 632 9	1745.2	1843.2	3.9317
425	0.002 537 5	2097.5	2199	4.5044	425	0.002 009 2	1960.2	2060.7	4.2746	425	0.001 815 9	1892.9	2001.8	4.163
450	0.003 691 5	2364.2	2511.8	4.9448	450	0.002 487 3	2160.3	2284.7	4.5896	450	0.002 085 5	2055.1	2180.2	4.414
500	0.005 623 1	2681.6	2906.5	5.4744	500	0.003 89	2528.1	2722.6	5.1762	500	0.002 952 2	2393.2	2570.3	4.9356
550	0.006 984 7	2875	3154.4	5.7857	550	0.005 117 5	2769.5	3025.3	5.5563	550	0.003 955 2	2664.5	2901.9	5.3517
600	0.008 089 1	3026.8	3350.4	6.017	600	0.006 108 1	2947.1	3252.5	5.8245	600	0.004 833	2866.8	3156.8	5.6527
650	0.009 053 2	3159.5	3521.6	6.2078	650	0.006 957	3095.6	3443.4	6.0373	650	0.005 590 6	3031.3	3366.7	5.8867
700	0.009 929 7	3282	3679.1	6.374	700	0.007 716 6	3228.7	3614.6	6.2178	700	0.006 264 9	3175.4	3551.3	6.0814
800	0.011 521	3511.8	3972.6	6.6612	800	0.009 072 4	3472.2	3925.8	6.5225	800	0.007 456	3432.6	3880	6.4033
900	0.012 98	3733.3	4252.5	6.9106	900	0.010 296	3702	4216.8	6.7819	900	0.008 518 8	3670.9	4182	6.6725
1000	0.014 36	3952.9	4527.3	7.1355	1000	0.011 441	3927.3	4499.4	7.0131	1000	0.009 504 3	3901.9	4472.2	6.9099
1100	0.015 686	4173.7	4801.1	7.3425	1100	0.012 534	4152.2	4778.9	7.2244	1100	0.010 439	4130.9	4757.3	7.1255
1200	0.016 976	4396.9	5075.9	7.5357	1200	0.013 59	4378.6	5058.1	7.4207	1200	0.011 339	4360.4	5040.8	7.3248
1300	0.018 239	4623.3	5352.8	7.7175	1300	0.014 62	4607.4	5338.4	7.6048	1300	0.012 212	4591.8	5324.5	7.5111

Table C.6: Compressed liquid water – Pressure table

T °C	v m³/kg	u kJ/kg	h kJ/kg	s kJ/kg·K	T °C	v m³/kg	u kJ/kg	h kJ/kg	s kJ/kg·K	T °C	v m³/kg	u kJ/kg	h kJ/kg	s kJ/kg·K
p=0.05 MPa (81.317°C)					p=0.1 MPa (99.606°C)					p=0.25 MPa (127.41°C)				
Sat.	0.0010299	340.49	340.54	1.0912	Sat.	0.0010432	417.4	417.5	1.3028	Sat.	0.0010672	535.08	535.34	1.6072
0.01	0.0010002	0	0.05	0	0.01	0.0010002	0	0.10	0	0.01	0.0010001	0.01	0.25	0
10	0.0010003	42.02	42.07	0.1511	10	0.0010003	42.02	42.12	0.1511	10	0.0010002	42.01	42.26	0.1511
20	0.0010018	83.91	83.96	0.2965	20	0.0010018	83.91	84.01	0.2965	20	0.0010017	83.90	84.15	0.2964
30	0.0010044	125.73	125.78	0.4367	30	0.0010044	125.72	125.82	0.4367	30	0.0010043	125.71	125.96	0.4367
40	0.0010079	167.52	167.57	0.5724	40	0.0010078	167.51	167.62	0.5724	40	0.0010078	167.5	167.75	0.5723
50	0.0010121	209.32	209.37	0.7038	50	0.0010121	209.32	209.42	0.7038	50	0.001012	209.29	209.55	0.7037
60	0.0010171	251.15	251.21	0.8313	60	0.0010171	251.15	251.25	0.8313	60	0.001017	251.12	251.37	0.8312
70	0.0010228	293.03	293.08	0.9551	70	0.0010227	293.02	293.12	0.9551	70	0.0010227	292.99	293.24	0.955
80	0.0010291	334.96	335.01	1.0756	80	0.001029	334.95	335.05	1.0755	80	0.001029	334.92	335.17	1.0754
90					90	0.0010359	376.96	377.06	1.1928	90	0.0010359	376.92	377.18	1.1927
100					100					100	0.0010434	419.02	419.28	1.3071
110					110					110	0.0010515	461.23	461.49	1.4187
120					120					120	0.0010603	503.58	503.85	1.5279
p=0.5 MPa (151.83°C)					p=1 MPa (179.88°C)					p=2.5 MPa (223.95°C)				
Sat.	0.0010925	639.54	640.09	1.8604	Sat.	0.0011272	761.39	762.52	2.1381	Sat.	0.0011974	958.91	961.91	2.5543
0.01	0.0010000	0.01	0.51	0	0.01	0.0009997	0.02	1.02	0	0.01	0.0009989	0.05	2.54	0.0002
10	0.0010001	42.01	42.51	0.1510	10	0.0009999	42.00	43.00	0.151	10	0.0009992	41.96	44.46	0.1509
20	0.0010016	83.88	84.38	0.2964	20	0.0010014	83.85	84.85	0.2963	20	0.0010007	83.76	86.26	0.2960
30	0.0010042	125.68	126.19	0.4366	30	0.001004	125.64	126.64	0.4365	30	0.0010033	125.5	128.01	0.4360
40	0.0010077	167.47	167.97	0.5722	40	0.0010074	167.41	168.41	0.5720	40	0.0010068	167.22	169.74	0.5714
50	0.0010119	209.26	209.76	0.7036	50	0.0010117	209.18	210.19	0.7034	50	0.001011	208.96	211.49	0.7027
60	0.0010169	251.08	251.58	0.8310	60	0.0010167	250.99	252	0.8308	60	0.001016	250.72	253.26	0.8210
70	0.0010226	292.94	293.45	0.9549	70	0.0010223	292.84	293.86	0.9546	70	0.0010216	292.53	295.08	0.9537
80	0.0010288	334.86	335.37	1.0753	80	0.0010286	334.74	335.77	1.075	80	0.0010279	334.39	336.96	1.074
90	0.0010357	376.85	377.37	1.1926	90	0.0010355	376.72	377.76	1.1922	90	0.0010348	376.33	378.92	1.1911
100	0.0010433	418.94	419.47	1.3069	100	0.001043	418.8	419.84	1.3065	100	0.0010422	418.36	420.97	1.3053
110	0.0010514	461.15	461.67	1.4185	110	0.0010511	460.99	462.04	1.4181	110	0.0010503	460.51	463.13	1.4168
120	0.0010602	503.49	504.02	1.5276	120	0.0010599	503.32	504.38	1.5272	120	0.001059	502.78	505.43	1.5258
130	0.0010696	546	546.54	1.6344	130	0.0010693	545.81	546.88	1.6339	130	0.0010684	545.22	547.89	1.6325
140	0.0010797	588.71	589.25	1.7391	140	0.0010794	588.5	589.58	1.7386	140	0.0010784	587.85	590.55	1.737
150	0.0010905	631.65	632.19	1.8418	150	0.0010901	631.41	632.5	1.8412	150	0.0010891	630.71	633.43	1.8395
160					160	0.0011017	674.59	675.7	1.9421	160	0.0011006	673.82	676.57	1.9403
170					170	0.0011141	718.08	719.2	2.0414	170	0.0011129	717.24	720.02	2.0395
p=5.00 MPa (263.94°C)					p=10.00 MPa (311.00°C)					p=15.00 MPa (342.16°C)				
Sat.	0.0012864	1148.2	1154.6	2.9210	Sat.	0.0014526	1393.5	1408.1	3.3606	Sat.	0.0016570	1585.3	1610.2	3.6846
0.01	0.0009977	0.09	5.07	0.0003	0.01	0.0009952	0.16	10.11	0.0005	0.01	0.0009928	0.22	15.11	0.0006
20	0.0009996	83.61	88.61	0.2954	20	0.0009973	83.31	93.28	0.2944	20	0.0009951	83.007	97.934	0.2932
40	0.0010057	166.92	171.95	0.5705	40	0.0010035	166.33	176.36	0.5685	40	0.0010013	165.75	180.77	0.5666
60	0.0010149	250.29	255.36	0.8287	60	0.0010127	249.42	259.55	0.8260	60	0.0010105	248.58	263.74	0.8234
80	0.0010267	333.82	338.95	1.0723	80	0.0010244	332.69	342.94	1.0691	80	0.0010221	331.59	346.92	1.0659
100	0.001041	417.64	422.85	1.3034	100	0.0010385	416.23	426.62	1.2996	100	0.0010361	414.85	430.39	1.2958
120	0.0010576	501.9	507.19	1.5236	120	0.0010549	500.18	510.73	1.5191	120	0.0010522	498.49	514.28	1.5148
140	0.0010769	586.79	592.18	1.7344	140	0.0010738	584.71	595.45	1.7293	140	0.0010708	582.69	598.75	1.7243
160	0.0010988	672.55	678.04	1.9374	160	0.0010954	670.06	681.01	1.9315	160	0.001092	667.63	684.01	1.9259
180	0.001124	759.46	765.08	2.1338	180	0.00112	756.48	767.68	2.1271	180	0.001116	753.58	770.32	2.1206
200	0.0011531	847.91	853.68	2.3251	200	0.0011482	844.31	855.8	2.3174	200	0.0011435	840.84	857.99	2.31
220	0.0011868	938.39	944.32	2.5127	220	0.0011809	934	945.81	2.5037	220	0.0011752	929.8	947.43	2.4951
240	0.0012268	1031.6	1037.7	2.6983	240	0.0012192	1026.1	1038.3	2.6876	240	0.0012121	1021	1039.2	2.6774
260	0.0012755	1128.5	1134.9	2.8841	260	0.0012653	1121.6	1134.3	2.871	260	0.001256	1115.1	1134	2.8586
280					280	0.0013226	1221.8	1235	3.0565	280	0.0013096	1213.4	1233	3.0409
300					300	0.001398	1329.4	1343.3	3.2488	300	0.0013783	1317.6	1338.3	3.2279
320										320	0.0014733	1431.9	1454	3.4263
340										340	0.0016311	1567.9	1592.4	3.6555
p=20.00 MPa (365.75°C)					p=30.00 MPa					p=50.00 MPa				
Sat.	0.0020400	1786.4	1827.2	4.0156	0.01	0.0009857	0.33	29.9	0.0004	0.01	0.0009767	0.33	49.17	-0.0009
0.01	0.0009904	0.27	20.07	0.0006	20	0.0009887	82.11	111.77	0.2897	20	0.0009805	80.931	129.95	0.2845
20	0.0009929	82.71	102.57	0.2921	40	0.0009951	164.05	193.9	0.5607	40	0.0009872	161.9	211.25	0.5528
40	0.0009992	165.17	185.16	0.5646	60	0.0010042	246.14	276.26	0.8156	60	0.0009962	243.08	292.88	0.8055
60	0.0010084	247.75	267.92	0.8208	80	0.0010155	328.4	358.86	1.0564	80	0.0010072	324.42	374.78	1.0442
80	0.0010199	330.5	350.9	1.0627	100	0.001029	410.87	441.74	1.2847	100	0.0010201	405.93	456.94	1.2705
100	0.0010337	413.5	434.17	1.292	120	0.0010445	493.66	525	1.502	120	0.0010349	487.69	539.43	1.4859
120	0.0010496	496.85	517.84	1.5105	140	0.0010623	576.89	608.76	1.7098	140	0.0010517	569.77	622.36	1.6916
140	0.0010679	580.71	602.07	1.7194	160	0.0010823	660.74	693.21	1.9094	160	0.0010704	652.32	705.84	1.8889
160	0.0010886	665.27	687.05	1.9203	180	0.0011049	745.4	778.54	2.102	180	0.0010914	735.49	790.06	2.079
180	0.0011122	750.77	773.02	2.1143	200	0.0011304	831.1	865.02	2.2888	200	0.0011149	819.45	875.19	2.2628
200	0.001139	837.49	860.27	2.3027	220	0.0011595	918.14	952.93	2.4707	220	0.0011412	904.39	961.45	2.4414
220	0.0011697	925.77	949.16	2.4867	240	0.0011927	1006.9	1042.7	2.6491	240	0.0011708	990.55	1049.1	2.6156
240	0.0012053	1016.1	1040.2	2.6676	260	0.0012314	1097.8	1134.7	2.825	260				

Table C.7: Saturated ice–water vapor

Temp., $T$ °C	Sat. press., $P_s$ Pa	Specific volume, $\text{m}^3/\text{kg}$		Internal energy, $\text{kJ/kg}$			Enthalpy, $\text{kJ/kg}$			Entropy, $\text{kJ/kg}\cdot\text{K}$		
		Sat. ice, $v_i$	Sat. vapor, $v_g$	Sat. ice, $u_i$	Subl., $u_{ig}$	Sat. vapor, $u_g$	Sat. ice, $h_i$	Subl., $h_{ig}$	Sat. vapor, $h_g$	Sat. ice, $s_i$	Subl., $s_{ig}$	Sat. vapor, $s_g$
0.01	611.7950	0.001 091	206.06	−333.44	2708.30	2374.86	−333.44	2834.37	2500.92	−1.22	10.38	9.16
0	611.2912	0.001 091	206.23	−333.47	2708.31	2374.84	−333.47	2834.37	2500.91	−1.22	10.38	9.16
−2	517.7868	0.001 091	241.68	−337.64	2709.74	2372.10	−337.64	2834.88	2497.24	−1.24	10.46	9.22
−4	437.4986	0.001 090	283.93	−341.79	2711.15	2369.35	−341.79	2835.36	2493.57	−1.25	10.53	9.28
−6	368.7240	0.001 090	334.38	−345.91	2712.52	2366.60	−345.91	2835.81	2489.90	−1.27	10.62	9.35
−8	309.9564	0.001 089	394.80	−350.00	2713.85	2363.85	−350.00	2836.22	2486.22	−1.28	10.70	9.41
−10	259.8657	0.001 089	467.35	−354.06	2715.16	2361.09	−354.06	2836.60	2482.54	−1.30	10.78	9.48
−12	217.2804	0.001 089	554.70	−358.10	2716.43	2358.33	−358.10	2836.95	2478.86	−1.31	10.86	9.55
−14	181.1709	0.001 088	660.17	−362.10	2717.67	2355.57	−362.10	2837.27	2475.17	−1.33	10.95	9.62
−16	150.6347	0.001 088	787.86	−366.07	2718.88	2352.81	−366.07	2837.56	2471.49	−1.34	11.03	9.69
−18	124.8825	0.001 088	942.94	−370.01	2720.06	2350.04	−370.01	2837.81	2467.80	−1.36	11.12	9.76
−20	103.2260	0.001 087	1131.82	−373.93	2721.21	2347.28	−373.93	2838.04	2464.11	−1.37	11.21	9.84
−22	85.0662	0.001 087	1362.59	−377.81	2722.32	2344.51	−377.81	2838.24	2460.42	−1.39	11.30	9.91
−24	69.8835	0.001 087	1645.42	−381.67	2723.41	2341.74	−381.67	2838.40	2456.73	−1.41	11.39	9.99
−26	57.2280	0.001 087	1993.16	−385.50	2724.47	2338.97	−385.50	2838.54	2453.04	−1.42	11.49	10.06
−28	46.7118	0.001 086	2422.12	−389.29	2725.50	2336.21	−389.29	2838.64	2449.35	−1.44	11.58	10.14
−30	38.0009	0.001 086	2953.05	−393.06	2726.50	2333.44	−393.06	2838.72	2445.66	−1.45	11.67	10.22
−32	30.8088	0.001 086	3612.46	−396.80	2727.47	2330.67	−396.80	2838.77	2441.96	−1.47	11.77	10.30
−34	24.8904	0.001 085	4434.33	−400.51	2728.41	2327.90	−400.51	2838.79	2438.27	−1.48	11.87	10.39
−36	20.0369	0.001 085	5462.39	−404.19	2729.33	2325.13	−404.19	2838.78	2434.58	−1.50	11.97	10.47
−38	16.0704	0.001 085	6753.16	−407.85	2730.21	2322.37	−407.85	2838.74	2430.89	−1.51	12.07	10.56
−40	12.8406	0.001 084	8379.92	−411.47	2731.07	2319.60	−411.47	2838.67	2427.20	−1.53	12.18	10.65
−45	7.2032	0.001 084	14 617.90	−420.40	2733.09	2312.69	−420.40	2838.39	2417.99	−1.57	12.44	10.87
−50	3.9378	0.001 083	26 153.59	−429.16	2734.95	2305.80	−429.16	2837.94	2408.78	−1.61	12.72	11.11
−55	2.0942	0.001 082	48 075.71	−437.73	2736.64	2298.92	−437.73	2837.32	2399.60	−1.65	13.01	11.36
−60	1.0814	0.001 081	90 966.78	−446.12	2738.18	2292.06	−446.12	2836.55	2390.43	−1.68	13.31	11.62
−70	0.2617	0.001 080	$3.582 \times 10^5$	−462.38	2740.80	2278.42	−462.38	2834.56	2372.18	−1.76	13.95	12.19
−80	0.0548	0.001 079	$1.627 \times 10^6$	−477.94	2742.88	2264.95	−477.94	2832.03	2354.09	−1.84	14.66	12.82
−90	0.0097	0.001 078	$8.731 \times 10^6$	−492.79	2744.51	2251.72	−492.79	2829.04	2336.25	−1.92	15.45	13.53
−100	0.0014	0.001 077	$5.688 \times 10^7$	−506.96	2745.82	2238.86	−506.96	2825.73	2318.77	−2.00	16.32	14.32

### C.3 R-134a Property Tables

R-134a numerical data properties for the water saturated, superheated, and compressed tables were retrieved from the *NIST Standard Reference Database 69: NIST Chemistry WebBook*, link: <https://webbook.nist.gov/cgi/fluid.cgi?ID=C811972&Action=Page>. These thermodynamic properties are based on Ref. [1].

The reference state is the saturated liquid at 0 °C with  $h = 200 \text{ kJ/kg}$  and  $s = 1 \text{ kJ/(kg K)}$ .

### References

- [1] Reiner Tillner-Roth and Hans Dieter Baehr. 'An International Standard Formulation for the Thermodynamic Properties of 1,1,1,2-Tetrafluoroethane (HFC-134a) for Temperatures from 170 K to 455 K and Pressures up to 70 MPa'. In: *Journal of Physical and Chemical Reference Data* 23.5 (Sept. 1994), pp. 657–729 (cited on page 110).

Table C.8: Saturated R-134a – Temperature table

Temp., $T$ °C	Sat. press., $P_s$ kPa	Specific volume, $\text{m}^3/\text{kg}$		Internal energy, $\text{kJ}/\text{kg}$			Enthalpy, $\text{kJ}/\text{kg}$			Entropy, $\text{kJ}/\text{kg}\cdot\text{K}$		
		Sat. liquid, $v_f$	Sat. vapor, $v_g$	Sat. liquid, $u_f$	Evap., $u_{fg}$	Sat. vapor, $u_g$	Sat. liquid, $h_f$	Evap., $h_{fg}$	Sat. vapor, $h_g$	Sat. liquid, $s_f$	Evap., $s_{fg}$	Sat. vapor, $s_g$
−40	51.209	0.000 705	0.361 08	148.11	207.4	355.51	148.14	225.9	374.00	0.7956	0.9687	1.7643
−38	56.817	0.000 708	0.327 55	150.62	206.0	356.66	150.66	224.6	375.27	0.8063	0.9552	1.7615
−36	62.908	0.000 711	0.297 71	153.14	204.7	357.81	153.18	223.4	376.54	0.8170	0.9418	1.7588
−34	69.512	0.000 714	0.271 08	155.66	203.3	358.96	155.71	222.1	377.80	0.8276	0.9287	1.7563
−32	76.658	0.000 717	0.247 27	158.19	201.9	360.10	158.25	220.8	379.06	0.8381	0.9157	1.7538
−30	84.378	0.000 720	0.225 94	160.73	200.5	361.25	160.79	219.5	380.32	0.8486	0.9029	1.7515
−28	92.703	0.000 723	0.206 80	163.28	199.1	362.40	163.34	218.2	381.57	0.8591	0.8901	1.7492
−26	101.67	0.000 727	0.189 58	165.83	197.7	363.55	165.90	216.9	382.82	0.8694	0.8777	1.7471
−24	111.3	0.000 730	0.174 07	168.39	196.3	364.70	168.47	215.6	384.07	0.8798	0.8653	1.7451
−22	121.65	0.000 733	0.160 06	170.96	194.9	365.84	171.05	214.3	385.32	0.8900	0.8532	1.7432
−20	132.73	0.000 736	0.147 39	173.54	193.5	366.99	173.64	212.9	386.55	0.9002	0.8411	1.7413
−18	144.6	0.000 740	0.135 92	176.12	192.0	368.13	176.23	211.6	387.79	0.9104	0.8292	1.7396
−16	157.28	0.000 743	0.125 51	178.72	190.5	369.27	178.83	210.2	389.02	0.9205	0.8174	1.7379
−14	170.82	0.000 746	0.116 05	181.32	189.1	370.41	181.44	208.8	390.24	0.9306	0.8057	1.7363
−12	185.24	0.000 750	0.107 44	183.93	187.6	371.55	184.07	207.4	391.45	0.9407	0.7941	1.7348
−10	200.6	0.000 754	0.099 590	186.55	186.1	372.69	186.70	206.0	392.66	0.9506	0.7827	1.7333
−8	216.93	0.000 757	0.092 422	189.17	184.7	373.82	189.34	204.5	393.87	0.9606	0.7714	1.7320
−6	234.28	0.000 761	0.085 867	191.81	183.1	374.95	191.99	203.1	395.06	0.9705	0.7602	1.7307
−4	252.68	0.000 765	0.079 866	194.45	181.6	376.07	194.65	201.6	396.25	0.9804	0.7490	1.7294
−2	272.17	0.000 768	0.074 362	197.11	180.1	377.19	197.32	200.1	397.43	0.9902	0.7380	1.7282
0	292.8	0.000 772	0.069 309	199.77	178.5	378.31	200.00	198.6	398.60	1.0000	0.7271	1.7271
2	314.62	0.000 776	0.064 663	202.45	177.0	379.42	202.69	197.1	399.77	1.0098	0.7162	1.7260
4	337.66	0.000 780	0.060 385	205.13	175.4	380.53	205.40	195.5	400.92	1.0195	0.7055	1.7250
6	361.98	0.000 785	0.056 443	207.83	173.8	381.63	208.11	193.9	402.06	1.0292	0.6948	1.7240
8	387.61	0.000 789	0.052 804	210.53	172.2	382.73	210.84	192.4	403.20	1.0388	0.6842	1.7230
10	414.61	0.000 793	0.049 442	213.25	170.6	383.82	213.58	190.7	404.32	1.0485	0.6736	1.7221
12	443.01	0.000 797	0.046 332	215.98	168.9	384.90	216.33	189.1	405.43	1.0581	0.6631	1.7212
14	472.88	0.000 802	0.043 451	218.71	167.3	385.98	219.09	187.4	406.53	1.0677	0.6527	1.7204
16	504.25	0.000 807	0.040 780	221.46	165.6	387.05	221.87	185.7	407.61	1.0772	0.6424	1.7196
18	537.18	0.000 811	0.038 301	224.23	163.9	388.11	224.66	184.0	408.69	1.0867	0.6321	1.7188
20	571.71	0.000 816	0.035 997	227.00	162.2	389.17	227.47	182.3	409.75	1.0962	0.6218	1.7180
22	607.89	0.000 821	0.033 854	229.79	160.4	390.21	230.29	180.5	410.79	1.1057	0.6116	1.7173
24	645.78	0.000 826	0.031 858	232.59	158.7	391.25	233.12	178.7	411.82	1.1152	0.6014	1.7166
26	685.43	0.000 831	0.029 998	235.40	156.9	392.28	235.97	176.9	412.84	1.1246	0.5913	1.7159
28	726.88	0.000 837	0.028 263	238.23	155.1	393.29	238.84	175.0	413.84	1.1341	0.5811	1.7152
30	770.2	0.000 842	0.026 642	241.07	153.2	394.30	241.72	173.1	414.82	1.1435	0.5710	1.7145
32	815.43	0.000 848	0.025 126	243.93	151.4	395.29	244.62	171.2	415.78	1.1529	0.5609	1.7138
34	862.62	0.000 854	0.023 708	246.80	149.5	396.27	247.54	169.2	416.72	1.1623	0.5508	1.7131
36	911.85	0.000 860	0.022 380	249.69	147.6	397.24	250.48	167.2	417.65	1.1717	0.5407	1.7124
38	963.15	0.000 866	0.021 135	252.60	145.6	398.19	253.43	165.1	418.55	1.1811	0.5307	1.7118
40	1016.6	0.000 872	0.019 966	255.52	143.6	399.13	256.41	163.0	419.43	1.1905	0.5206	1.7111
42	1072.2	0.000 879	0.018 868	258.46	141.6	400.05	259.41	160.9	420.28	1.1999	0.5104	1.7103
44	1130.1	0.000 885	0.017 837	261.42	139.5	400.96	262.42	158.7	421.11	1.2092	0.5004	1.7096
46	1190.3	0.000 892	0.016 866	264.40	137.4	401.84	265.47	156.4	421.92	1.2186	0.4903	1.7089
48	1252.9	0.000 900	0.015 951	267.41	135.3	402.71	268.53	154.2	422.69	1.2280	0.4801	1.7081
52	1385.4	0.000 915	0.014 276	273.47	130.9	404.37	274.74	149.4	424.15	1.2469	0.4595	1.7064
56	1528.2	0.000 932	0.012 782	279.64	126.3	405.94	281.06	144.4	425.47	1.2658	0.4387	1.7045
60	1681.8	0.000 950	0.011 444	285.91	121.5	407.38	287.50	139.1	426.63	1.2848	0.4176	1.7024
65	1889.8	0.000 975	0.009 960 4	293.92	115.1	408.99	295.76	132.1	427.82	1.3088	0.3905	1.6993
70	2116.8	0.001 004	0.008 652 7	302.16	108.2	410.33	304.28	124.4	428.65	1.3332	0.3624	1.6956
75	2364.1	0.001 037	0.007 491 0	310.68	100.6	411.32	313.13	115.9	429.03	1.3580	0.3329	1.6909
80	2633.2	0.001 077	0.006 448 3	319.55	92.3	411.83	322.39	106.4	428.81	1.3836	0.3014	1.6850
85	2925.8	0.001 127	0.005 499 0	328.93	82.7	411.67	332.22	95.5	427.76	1.4104	0.2667	1.6771
90	3244.2	0.001 194	0.004 613 4	339.06	71.4	410.45	342.93	82.5	425.42	1.4390	0.2272	1.6662
95	3591.2	0.001 294	0.003 743 4	350.60	56.6	407.23	355.25	65.4	420.67	1.4715	0.1777	1.6492
100	3972.4	0.001 536	0.002 680 9	367.20	29.8	397.03	373.30	34.4	407.68	1.5188	0.0921	1.6109

Table C.9: Saturated R-134a – Pressure table

Press., <i>P</i> kPa	Sat. temp., <i>T</i> <sub>s</sub> °C	Specific volume, m <sup>3</sup> /kg		Internal energy, kJ/kg			Enthalpy, kJ/kg			Entropy, kJ/kg·K		
		Sat. liquid, <i>v</i> <sub><i>f</i></sub>	Sat. vapor, <i>v</i> <sub><i>g</i></sub>	Sat. liquid, <i>u</i> <sub><i>f</i></sub>	Evap., <i>u</i> <sub><i>fg</i></sub>	Sat. vapor, <i>u</i> <sub><i>g</i></sub>	Sat. liquid, <i>h</i> <sub><i>f</i></sub>	Evap., <i>h</i> <sub><i>fg</i></sub>	Sat. vapor, <i>h</i> <sub><i>g</i></sub>	Sat. liquid, <i>s</i> <sub><i>f</i></sub>	Evap., <i>s</i> <sub><i>fg</i></sub>	Sat. vapor, <i>s</i> <sub><i>g</i></sub>
60	−36.94	0.000 710	0.311 23	151.96	205.3	357.27	152.00	223.9	375.94	0.8120	0.9481	1.7601
70	−33.86	0.000 714	0.269 30	155.84	203.2	359.04	155.89	222.0	377.89	0.8284	0.9277	1.7561
80	−31.11	0.000 719	0.237 55	159.31	201.3	360.61	159.37	220.2	379.62	0.8428	0.9100	1.7528
90	−28.63	0.000 722	0.212 64	162.47	199.6	362.04	162.54	218.6	381.18	0.8558	0.8941	1.7499
100	−26.36	0.000 726	0.192 56	165.37	198.0	363.34	165.44	217.2	382.60	0.8676	0.8799	1.7475
120	−22.31	0.000 732	0.162 14	170.56	195.1	365.67	170.65	214.5	385.12	0.8884	0.8551	1.7435
140	−18.76	0.000 738	0.140 15	175.14	192.6	367.70	175.24	212.1	387.32	0.9066	0.8336	1.7402
160	−15.59	0.000 744	0.123 49	179.25	190.3	369.51	179.37	209.9	389.27	0.9226	0.8150	1.7376
180	−12.71	0.000 749	0.110 42	183.00	188.1	371.15	183.13	207.9	391.02	0.9371	0.7982	1.7353
200	−10.08	0.000 753	0.099 876	186.45	186.2	372.64	186.60	206.0	392.62	0.9503	0.7831	1.7334
240	−5.37	0.000 762	0.083 906	192.65	182.7	375.30	192.83	202.6	395.44	0.9736	0.7567	1.7303
280	−1.23	0.000 770	0.072 360	198.14	179.5	377.62	198.35	199.5	397.89	0.9940	0.7338	1.7278
320	2.48	0.000 777	0.063 611	203.09	176.6	379.69	203.34	196.7	400.04	1.0121	0.7136	1.7257
360	5.84	0.000 784	0.056 744	207.61	173.9	381.54	207.90	194.1	401.97	1.0284	0.6956	1.7240
400	8.93	0.000 791	0.051 207	211.79	171.5	383.24	212.11	191.6	403.72	1.0433	0.6793	1.7226
450	12.48	0.000 799	0.045 625	216.63	168.5	385.16	216.99	188.7	405.69	1.0604	0.6606	1.7210
500	15.73	0.000 806	0.041 123	221.10	165.8	386.91	221.50	186.0	407.47	1.0759	0.6438	1.7197
550	18.75	0.000 813	0.037 412	225.27	163.2	388.51	225.72	183.4	409.09	1.0903	0.6282	1.7185
600	21.57	0.000 820	0.034 300	229.19	160.8	389.99	229.68	180.9	410.57	1.1037	0.6138	1.7175
650	24.22	0.000 827	0.031 650	232.89	158.5	391.36	233.43	178.5	411.94	1.1162	0.6003	1.7165
700	26.71	0.000 833	0.029 365	236.41	156.2	392.64	236.99	176.2	413.20	1.1280	0.5876	1.7156
750	29.08	0.000 840	0.027 375	239.76	154.1	393.84	240.39	174.0	414.37	1.1392	0.5756	1.7148
800	31.33	0.000 846	0.025 624	242.97	152.0	394.96	243.65	171.8	415.46	1.1497	0.5643	1.7140
850	33.47	0.000 852	0.024 073	246.05	150.0	396.02	246.77	169.7	416.48	1.1598	0.5535	1.7133
900	35.53	0.000 858	0.022 687	249.01	148.0	397.01	249.78	167.7	417.43	1.1695	0.5431	1.7126
950	37.49	0.000 864	0.021 441	251.86	146.1	397.95	252.69	165.6	418.32	1.1787	0.5332	1.7119
1000	39.39	0.000 870	0.020 316	254.63	144.2	398.85	255.50	163.7	419.16	1.1876	0.5237	1.7113
1200	46.31	0.000 894	0.016 718	264.87	137.1	401.98	265.95	156.1	422.04	1.2201	0.4886	1.7087
1400	52.42	0.000 917	0.014 110	274.12	130.4	404.54	275.40	148.9	424.30	1.2489	0.4573	1.7062
1600	57.91	0.000 940	0.012 126	282.61	124.0	406.64	284.11	141.9	426.04	1.2748	0.4288	1.7036
1800	62.90	0.000 964	0.010 562	290.52	117.8	408.35	292.26	135.1	427.36	1.2987	0.4020	1.7007
2000	67.48	0.000 989	0.009 291 5	297.98	111.7	409.70	299.95	128.3	428.28	1.3209	0.3767	1.6976
2500	77.58	0.001 057	0.006 940 3	315.20	96.4	411.65	317.84	111.2	429.00	1.3711	0.3170	1.6881
3000	86.20	0.001 141	0.005 281 3	331.28	80.2	411.49	334.70	92.6	427.34	1.4171	0.2577	1.6748

Table C.10: Superheated R-134a vapor – Pressure table (0.06–0.4MPa)

T °C	v m <sup>3</sup> /kg	u kJ/kg	h kJ/kg	s kJ/kg·K	T °C	v m <sup>3</sup> /kg	u kJ/kg	h kJ/kg	s kJ/kg·K	T °C	v m <sup>3</sup> /kg	u kJ/kg	h kJ/kg	s kJ/kg·K
p=0.06 MPa (–36.94 °C)					p=0.10 MPa (–26.36 °C)					p=0.14 MPa (–18.76 °C)				
Sat.	0.311 23	357.27	375.94	1.7601	Sat.	0.192 56	363.34	382.6	1.7475	Sat.	0.140 15	367.70	387.32	1.7402
–20	0.336 08	368.74	388.91	1.8131	–20	0.198 41	367.81	387.65	1.7677					
–10	0.350 49	375.7	396.73	1.8433	–10	0.207 43	374.89	395.64	1.7986	–10	0.146 06	374.05	394.5	1.768
0	0.364 76	382.8	404.69	1.873	0	0.2163	382.1	403.73	1.8288	0	0.152 63	381.37	402.74	1.7987
10	0.378 93	390.07	412.81	1.9022	10	0.225 06	389.45	411.95	1.8584	10	0.159 08	388.81	411.08	1.8287
20	0.393 03	397.5	421.08	1.931	20	0.233 73	396.94	420.31	1.8874	20	0.165 44	396.37	419.53	1.858
30	0.407 05	405.1	429.52	1.9593	30	0.242 33	404.59	428.82	1.9159	30	0.171 72	404.08	428.12	1.8868
40	0.421 02	412.86	438.12	1.9872	40	0.250 88	412.4	437.49	1.9441	40	0.177 94	411.93	436.84	1.9151
50	0.434 95	420.79	446.88	2.0147	50	0.259 38	420.37	446.3	1.9718	50	0.184 12	419.94	445.72	1.943
60	0.448 84	428.88	455.81	2.0419	60	0.267 83	428.49	455.28	1.9991	60	0.190 25	428.1	454.74	1.9705
70	0.462 69	437.14	464.9	2.0688	70	0.276 26	436.78	464.41	2.0261	70	0.196 35	436.42	463.91	1.9977
80	0.476 52	445.56	474.15	2.0954	80	0.284 66	445.23	473.7	2.0528	80	0.202 42	444.9	473.24	2.0244
90	0.490 32	454.14	483.56	2.1216	90	0.293 03	453.84	483.14	2.0792	90	0.208 47	453.53	482.72	2.0509
100	0.5041	462.89	493.14	2.1477	100	0.301 38	462.61	492.74	2.1053	100	0.214 49	462.32	492.35	2.0771
p=0.18 MPa (–12.71 °C)					p=0.20 MPa (–10.08 °C)					p=0.24 MPa (–5.37 °C)				
Sat.	0.110 42	371.15	391.02	1.7353	Sat.	0.099 876	372.64	392.62	1.7334	Sat.	0.083 906	375.30	395.44	1.7303
–10	0.1119	373.17	393.31	1.7441	–10	0.099 915	372.7	392.68	1.7337					
0	0.117 22	380.62	401.72	1.7754	0	0.104 81	380.23	401.2	1.7654	0	0.086 17	379.43	400.11	1.7475
10	0.1224	388.15	410.18	1.8059	10	0.109 55	387.81	409.73	1.7961	10	0.090 261	387.13	408.79	1.7787
20	0.127 48	395.79	418.73	1.8355	20	0.114 19	395.49	418.33	1.8259	20	0.094 233	394.89	417.51	1.809
30	0.132 48	403.55	427.4	1.8646	30	0.118 74	403.29	427.04	1.8551	30	0.098 117	402.75	426.3	1.8385
40	0.137 42	411.46	436.19	1.8931	40	0.123 23	411.22	435.87	1.8838	40	0.101 93	410.74	435.2	1.8674
50	0.1423	419.51	445.12	1.9212	50	0.127 66	419.29	444.83	1.912	50	0.1057	418.86	444.22	1.8957
60	0.147 15	427.71	454.2	1.9489	60	0.132 06	427.51	453.92	1.9397	60	0.109 42	427.11	453.37	1.9236
70	0.151 96	436.06	463.41	1.9761	70	0.136 42	435.88	463.16	1.967	70	0.1131	435.51	462.65	1.9511
80	0.156 74	444.56	472.78	2.003	80	0.140 74	444.39	472.54	1.9939	80	0.116 75	444.06	472.08	1.9781
90	0.161 49	453.22	482.29	2.0296	90	0.145 05	453.06	482.07	2.0206	90	0.120 38	452.75	481.64	2.0048
100	0.166 22	462.03	491.95	2.0558	100	0.149 33	461.88	491.75	2.0468	100	0.123 98	461.59	491.35	2.0312
p=0.28 MPa (–1.23 °C)					p=0.32 MPa (2.48 °C)					p=0.40 MPa (8.93 °C)				
Sat.	0.072 360	377.62	397.89	1.7278	Sat.	0.063 612	379.69	400.04	1.7258	Sat.	0.051 207	383.24	403.72	1.7226
0	0.072 819	378.59	398.98	1.7318										
10	0.076 46	386.42	407.83	1.7636	10	0.066 088	385.68	406.83	1.7501	10	0.051 506	384.12	404.72	1.7261
20	0.079 966	394.27	416.66	1.7943	20	0.069 252	393.64	415.8	1.7812	20	0.054 214	392.32	414.01	1.7584
30	0.083 378	402.21	425.55	1.8241	30	0.072 313	401.65	424.79	1.8113	30	0.056 796	400.5	423.22	1.7893
40	0.086 719	410.25	434.53	1.8532	40	0.075 299	409.75	433.85	1.8407	40	0.059 293	408.73	432.45	1.8192
50	0.090 003	418.41	443.61	1.8818	50	0.078 226	417.96	443	1.8695	50	0.061 724	417.05	441.74	1.8484
60	0.093 242	426.71	452.81	1.9098	60	0.081 106	426.3	452.25	1.8977	60	0.064 104	425.47	451.11	1.877
70	0.096 443	435.14	462.14	1.9374	70	0.083 947	434.77	461.63	1.9254	70	0.066 443	434.01	460.58	1.905
80	0.099 612	443.71	471.61	1.9646	80	0.086 755	443.37	471.13	1.9527	80	0.068 748	442.67	470.17	1.9325
90	0.102 75	452.43	481.2	1.9914	90	0.089 535	452.11	480.77	1.9796	90	0.071 023	451.47	479.88	1.9596
100	0.105 87	461.3	490.94	2.0178	100	0.092 293	461	490.54	2.0062	100	0.073 275	460.41	489.72	1.9864
110	0.108 97	470.31	500.82	2.044	110	0.095 03	470.03	500.44	2.0324	110	0.075 505	469.48	499.68	2.0127
120	0.112 06	479.47	510.85	2.0698	120	0.097 749	479.21	510.49	2.0583	120	0.077 717	478.69	509.78	2.0387
130	0.115 12	488.78	521.01	2.0953	130	0.100 45	488.54	520.68	2.0838	130	0.079 914	488.05	520.02	2.0645
140	0.118 18	498.24	531.33	2.1206	140	0.103 14	498.01	531.02	2.1092	140	0.082 097	497.55	530.39	2.0899

Table C.11: Superheated R-134a vapor – Pressure table (0.5–1.6 MPa)

T °C	v m <sup>3</sup> /kg	u kJ/kg	h kJ/kg	s kJ/kg·K	T °C	v m <sup>3</sup> /kg	u kJ/kg	h kJ/kg	s kJ/kg·K	T °C	v m <sup>3</sup> /kg	u kJ/kg	h kJ/kg	s kJ/kg·K
p=0.50 MPa (15.73 °C)					p=0.60 MPa (21.57 °C)					p=0.70 MPa (26.71 °C)				
15.73	0.041 123	386.91	407.47	1.7197	21.57	0.034 300	389.99	410.57	1.7175	26.71	0.029 365	392.64	413.20	1.7156
20	0.042 116	390.55	411.61	1.7339	30	0.035 984	397.37	418.96	1.7455	30	0.029 966	395.62	416.6	1.7269
30	0.044 338	398.99	421.16	1.7659	40	0.037 865	406.01	428.73	1.7772	40	0.031 696	404.53	426.72	1.7598
40	0.046 456	407.4	430.63	1.7967	50	0.039 659	414.63	438.43	1.8077	50	0.033 322	413.35	436.67	1.791
50	0.048 499	415.86	440.11	1.8265	60	0.041 389	423.3	448.13	1.8373	60	0.034 875	422.16	446.57	1.8212
60	0.050 486	424.4	449.64	1.8555	70	0.043 07	432.04	457.88	1.8661	70	0.036 373	431.01	456.47	1.8505
70	0.052 427	433.04	459.25	1.8839	80	0.044 71	440.87	467.7	1.8943	80	0.037 829	439.94	466.42	1.8791
80	0.054 331	441.78	468.95	1.9118	90	0.046 319	449.82	477.61	1.922	90	0.039 25	448.97	476.44	1.907
90	0.056 205	450.65	478.75	1.9392	100	0.0479	458.88	487.62	1.9492	100	0.040 642	458.09	486.54	1.9345
100	0.058 054	459.65	488.67	1.9661	110	0.049 459	468.06	497.74	1.9759	110	0.042 01	467.34	496.74	1.9615
110	0.059 88	468.78	498.72	1.9927	120	0.050 998	477.37	507.97	2.0023	120	0.043 358	476.7	507.05	1.988
120	0.061 688	478.04	508.88	2.0189	130	0.052 52	486.82	518.33	2.0283	130	0.044 689	486.19	517.47	2.0142
130	0.063 479	487.44	519.18	2.0447	140	0.054 027	496.39	528.81	2.054	140	0.046 004	495.8	528.01	2.04
140	0.065 257	496.98	529.6	2.0703	150	0.055 522	506.11	539.42	2.0794	150	0.047 307	505.55	538.67	2.0655
150	0.067 022	506.65	540.17	2.0955	160	0.057 006	515.96	550.16	2.1045	160	0.048 598	515.44	549.45	2.0907
160	0.068 776	516.47	550.86	2.1205										
p=0.80 MPa (31.33 °C)					p=0.90 MPa (35.53 °C)					p=1.00 MPa (39.39 °C)				
Sat.	0.025 625	394.96	415.46	1.714	Sat.	0.026 323	398.87	419.93	1.7286	Sat.	0.020 316	398.85	419.16	1.7113
40	0.027 036	402.96	424.59	1.7436	40	0.027 036	402.96	424.59	1.7436	40	0.020 406	399.45	419.86	1.7135
50	0.028 547	412	434.84	1.7758	50	0.028 547	412	434.84	1.7758	50	0.021 796	409.09	430.88	1.7482
60	0.029 974	420.97	444.95	1.8067	60	0.029 974	420.97	444.95	1.8067	60	0.023 068	418.46	441.53	1.7806
70	0.031 34	429.96	455.03	1.8364	70	0.031 34	429.96	455.03	1.8364	70	0.024 261	427.74	452	1.8116
80	0.032 659	438.99	465.12	1.8654	80	0.032 659	438.99	465.12	1.8654	80	0.025 399	437	462.4	1.8414
90	0.033 941	448.1	475.25	1.8937	90	0.033 941	448.1	475.25	1.8937	90	0.026 493	446.3	472.79	1.8705
100	0.035 193	457.3	485.45	1.9214	100	0.035 193	457.3	485.45	1.9214	100	0.027 552	455.65	483.2	1.8988
110	0.036 42	466.6	495.73	1.9486	110	0.036 42	466.6	495.73	1.9486	110	0.028 584	465.09	493.67	1.9264
120	0.037 626	476.01	506.12	1.9754	120	0.037 626	476.01	506.12	1.9754	120	0.029 592	474.62	504.21	1.9536
130	0.038 813	485.55	516.6	2.0017	130	0.038 813	485.55	516.6	2.0017	130	0.030 581	484.25	514.83	1.9803
140	0.039 985	495.21	527.2	2.0277	140	0.039 985	495.21	527.2	2.0277	140	0.031 554	494	525.55	2.0065
150	0.041 144	504.99	537.91	2.0533	150	0.041 144	504.99	537.91	2.0533	150	0.032 512	503.86	536.37	2.0324
160	0.042 29	514.91	548.74	2.0786	160	0.042 29	514.91	548.74	2.0786	160	0.033 458	513.84	547.3	2.0579
170	0.043 427	524.96	559.7	2.1036	170	0.043 427	524.96	559.7	2.1036	170	0.034 392	523.95	558.34	2.0831
180	0.044 554	535.14	570.78	2.1283	180	0.044 554	535.14	570.78	2.1283	180	0.035 318	534.19	569.51	2.108
p=1.20 MPa (46.31 °C)					p=1.40 MPa (52.42 °C)					p=1.60 MPa (57.91 °C)				
Sat.	0.016 718	401.98	422.04	1.7087	Sat.	0.014 110	404.54	424.30	1.7062	Sat.	0.012 127	406.65	426.05	1.7036
50	0.017 201	405.77	426.41	1.7223	0	0	0	0	0	60	0.012 373	409.04	428.84	1.712
60	0.018 404	415.7	437.79	1.757	60	0.015 005	412.61	433.62	1.7345	70	0.013 43	419.91	441.4	1.7491
70	0.019 502	425.36	448.76	1.7895	70	0.016 06	422.77	445.25	1.7689	80	0.014 362	430.24	453.22	1.7831
80	0.020 53	434.9	459.53	1.8204	80	0.017 023	432.65	456.48	1.8012	90	0.015 216	440.32	464.66	1.815
90	0.021 506	444.41	470.22	1.8502	90	0.017 923	442.42	467.51	1.832	100	0.016 015	450.29	475.91	1.8456
100	0.022 443	453.94	480.87	1.8792	100	0.018 778	452.16	478.45	1.8617	110	0.016 773	460.22	487.06	1.8751
110	0.023 348	463.53	491.54	1.9074	110	0.019 597	461.9	489.34	1.8905	120	0.0175	470.16	498.16	1.9037
120	0.024 228	473.18	502.25	1.935	120	0.020 388	471.7	500.24	1.9186	130	0.018 201	480.15	509.27	1.9316
130	0.025 087	482.92	513.02	1.9621	130	0.021 156	481.55	511.17	1.946	140	0.018 882	490.19	520.4	1.9589
140	0.025 928	492.76	523.87	1.9886	140	0.021 904	491.49	522.16	1.973	150	0.019 546	500.32	531.59	1.9856
150	0.026 753	502.7	534.81	2.0148	150	0.022 636	501.52	533.21	1.9994	160	0.020 194	510.53	542.84	2.0119
160	0.027 566	512.76	545.84	2.0405	160	0.023 355	511.65	544.35	2.0254	170	0.020 83	520.84	554.17	2.0378
170	0.028 367	522.93	556.97	2.066	170	0.024 061	521.89	555.58	2.051	180	0.021 456	531.26	565.59	2.0632
180	0.029 158	533.23	568.22	2.091	180	0.024 758	532.25	566.91	2.0763					

# Thermodynamic Properties of Selected Ideal Gases

# D

## D.1 Air Ideal Gas Property Tables

### D.1.1 What are these tables useful for?

The concept of ideal air gas tables may seem on a first basis counterintuitive for the average student, already familiar with the key concepts and equations of ideal gases. After all, if we want to fully define the final state of a gas, after a given thermodynamic process, we need information about the full set of intensive properties  $p, v, T, \Delta u, \Delta h, \Delta s$ <sup>1</sup>.

For an ideal gas, these may be straightaway be obtained using the following sets of equations:

$$pv = RT, C_p = C_v + R \quad (\text{D.1})$$

$$du = C_v dT \quad (\text{D.2})$$

$$dh = C_p dT \quad (\text{D.3})$$

$$ds = \begin{cases} C_v \frac{dT}{T} + R \frac{dv}{v} \\ C_p \frac{dT}{T} - R \frac{dp}{p} \end{cases} \quad (\text{D.4})$$

which are analytically integrable.

So, what do we need these tables for?

The answer lies in the fact that considering the specific heat capacities for an ideal gas ( $C_v = 3/2R$  for an atomic,  $C_v = 5/2R$  for a molecular gas, recall Chapter 3 of Volume 1, to be constant may sometimes lack accuracy, given that these heat capacities may in practice have a slight temperature dependence, which typically increases as we reach higher temperatures near the 2000 K upper limit that we arbitrarily set for considering the vibrations of a molecule to be frozen.

Accordingly, if we cannot consider the specific heat of an ideal gas to be independent of temperature, it is now necessary to numerically integrate the expressions of Eqs. D.1–D.4:

$$h(T) = \int_{T_{ref}}^T C_p(T) dT + h(T_{ref})$$

$$s^\circ(T) = \int_{T_{ref}}^T \frac{C_p(T)}{T} dT + s(T_{ref}),$$

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1: we are only concerned in calculating the variations for the variables  $u, h, s$  during a thermodynamic process since the reference values for these properties are defined arbitrarily.

were one usually sets the variables  $T_{ref}$ ,  $h(T_{ref})$  and  $s^\circ(T_{ref})$  to zero.

These integrals are numerically evaluated and the results displayed in table form. Then, the variation of a property for a thermodynamic process  $1 \rightarrow 2$  is, assuming  $h(0) = 0$  and  $s^\circ(0) = 0$ :

$$\Delta h = h_2 - h_1 = \int_0^{T_2} C_p(T) dT - \int_0^{T_1} C_p(T) dT \quad (D.5)$$

$$\Delta s = s_2^\circ - s_1^\circ - R \ln \frac{p_2}{p_1} = \int_0^{T_2} \frac{C_p(T)}{T} dT - \int_0^{T_1} \frac{C_p(T)}{T} dT - R \ln \frac{p_2}{p_1} \quad (D.6)$$

### D.1.2 Isentropic Processes

For isentropic processes, the variation of entropy is zero and we may write for a constant pressure process:

$$\begin{aligned} 0 &= s_2 - s_1 = s_2^\circ - s_1^\circ - R \ln \frac{p_2}{p_1} \\ \Leftrightarrow \frac{p_2}{p_1} &= \exp \left( \frac{s_2^\circ - s_1^\circ}{R} \right) = \frac{\exp(s_2^\circ/R)}{\exp(s_1^\circ/R)} = \frac{p_{r2}}{p_{r1}} \end{aligned}$$

For a constant volume process, a slightly more convoluted expression may also be derived to express the process volume ratio  $v_2/v_1$  as a function of the dimensionless relative specific volume ratios  $v_{r2}/v_{r1}$ , itself a function of the entropy variables  $s_2^\circ$  and  $s_1^\circ$ .

The dimensionless quantities  $p_r$  and  $v_r$  are usually included in ideal air tables. These are useful for the analysis of isentropic processes ( $\Delta s = 0$ ), as they allow for the calculation of the corresponding pressure and specific volume ratios (which depend on temperature alone) according to the relations

$$\begin{aligned} \frac{p_2}{p_1} &= \frac{p_{r2}}{p_{r1}} \\ \frac{v_2}{v_1} &= \frac{v_{r2}}{v_{r1}}. \end{aligned}$$

Unfortunately, such dimensionless quantities may be confusing, as their nomenclature may mislead students into interpret these variables as pressure and volume quantities. Cengel [1] has proposed an elegant solution to this conundrum, through the introduction of an additional variable  $s^+$  such that:

$$s^+ = \int_0^T C_v(T) \frac{dT}{T},$$

this variable being related to  $s^\circ$  as

$$s^+ = s^\circ - R \ln T,$$

since  $C_v = C_p - R$  (the equality may be derived replacing one specific heat by another and developing the integral).

For an arbitrary process, one may then write the pair of equations:

$$\begin{aligned} s_2 - s_1 &= s_2^\circ - s_1^\circ - R \ln \frac{p_2}{p_1} \\ s_2 - s_1 &= s_2^+ - s_1^+ + R \ln \frac{v_2}{v_1} \end{aligned}$$

which allow determining the pressure and volume ratios  $p_2/p_1$  and  $v_2/v_1$  for isentropic processes ( $s_2 - s_1 = 0$ ) where volume or pressure information for the process is given, respectively:

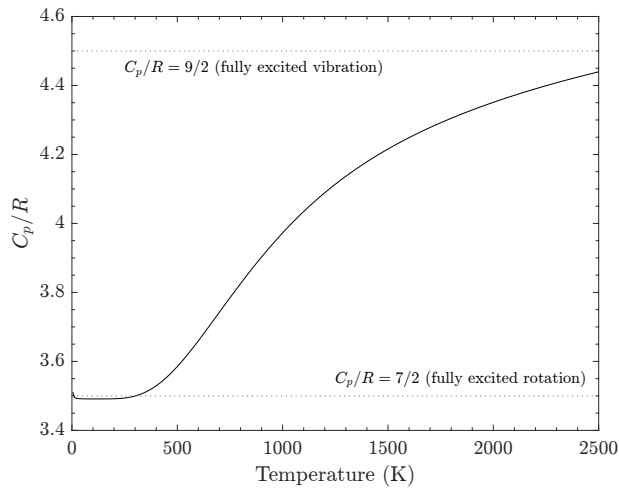
$$\begin{aligned} \exp\left(\frac{s_2^\circ - s_1^\circ}{R}\right) &= + \frac{p_2}{p_1} \\ \exp\left(\frac{s_2^+ - s_1^+}{R}\right) &= - \frac{v_2}{v_1} \end{aligned}$$

The variable pair  $s^\circ, s^+$  may therefore advantageously replace the pair  $p_r, v_r$  for calculations of isentropic processes which may be misleading for students. Additionally, this pair of variables typically requires less arithmetic operations, as demonstrated by Cengel [1].

### D.1.3 Properties used for building the tables

The fit for the air specific heat temperature dependence was taken from Ref. [2]. The fit is strictly valid in the temperature range  $T=[60-2000 \text{ K}]$ , but has been extended up to  $T=[0-2300 \text{ K}]$  for integration purposes. The terms for the fit expression which are analytically integrable have been kept, whereas those who aren't have been re-fit to an analytically integrable polynomial expansion.

Tabulated values for Eqs. D.5 and D.6 have been obtained integrating these analytical expressions for  $C_p$ . The reference entropy and enthalpy are defined as being zero for  $T = 0 \text{ K}$ , with the reference pressure for entropy being atmospheric pressure ( $p = 101\,325 \text{ Pa}$ ). Expressions for  $u$  and  $s^+$  have been derived from  $u = h - RT$  and  $s^+ = s^\circ - R \ln T$ .



**Figure D.1:** Specific heat  $C_p$  for Air

## References

- [1] Yunus A. Cengel and Mehmet Kanoğlu. 'A New Entropy Function to Analyze Isentropic Processes of Ideal Gases with Variable Specific Heats'. In: *Entropy* 24.6 (2022) (cited on pages 116, 117).
- [2] Eric W. Lemmon et al. 'Thermodynamic Properties of Air and Mixtures of Nitrogen, Argon, and Oxygen From 60 to 2000 K at Pressures to 2000 MPa'. In: *Journal of Physical and Chemical Reference Data* 29.3 (May 2000), pp. 331–385 (cited on page 117).

Table D.1: Ideal gas properties of air

$T$ K	$C_p$ kJ/kg·K	$h$ kJ/kg	$u$ kJ/kg	$s^\circ$ kJ/kg·K	$s^+$ kJ/kg·K	$T$ K	$C_p$ kJ/kg·K	$h$ kJ/kg	$u$ kJ/kg	$s^\circ$ kJ/kg·K	$s^+$ kJ/kg·K
200	1.0021	200.38	142.98	3.0025	1.4819	750	1.0861	767.67	552.42	4.3551	2.4551
210	1.0022	210.40	150.13	3.0513	1.5167	760	1.0885	778.54	560.42	4.3694	2.4657
220	1.0023	220.42	157.28	3.0979	1.5499	780	1.0932	800.36	576.50	4.3977	2.4865
230	1.0024	230.44	164.43	3.1424	1.5816	800	1.0979	822.27	592.67	4.4254	2.5069
240	1.0025	240.47	171.59	3.1850	1.6120	820	1.1025	844.27	608.93	4.4524	2.5269
250	1.0027	250.50	178.75	3.2259	1.6412	840	1.1070	866.37	625.29	4.4790	2.5465
260	1.0030	260.52	185.90	3.2652	1.6693	860	1.1114	888.55	641.73	4.5050	2.5658
270	1.0033	270.56	193.07	3.3030	1.6963	880	1.1158	910.82	658.26	4.5306	2.5847
280	1.0036	280.59	200.23	3.3395	1.7223	900	1.1201	933.18	674.88	4.5556	2.6033
285	1.0038	285.61	203.81	3.3573	1.7350	920	1.1243	955.63	691.59	4.5802	2.6216
290	1.0040	290.63	207.40	3.3747	1.7475	940	1.1284	978.15	708.37	4.6044	2.6396
295	1.0042	295.65	210.98	3.3919	1.7597	960	1.1324	1000.76	725.24	4.6281	2.6573
298	1.0043	298.66	213.13	3.4021	1.7670	980	1.1363	1023.45	742.19	4.6515	2.6747
300	1.0044	300.67	214.57	3.4088	1.7718	1000	1.1401	1046.21	759.21	4.6744	2.6919
305	1.0047	305.69	218.16	3.4254	1.7837	1020	1.1439	1069.05	776.31	4.6970	2.7088
310	1.0049	310.72	221.75	3.4418	1.7954	1040	1.1475	1091.97	793.49	4.7192	2.7254
315	1.0052	315.74	225.34	3.4578	1.8069	1060	1.1511	1114.95	810.73	4.7411	2.7418
320	1.0055	320.77	228.93	3.4737	1.8182	1080	1.1545	1138.01	828.05	4.7626	2.7580
325	1.0059	325.80	232.52	3.4893	1.8293	1100	1.1579	1161.13	845.43	4.7838	2.7739
330	1.0062	330.83	236.12	3.5047	1.8403	1120	1.1612	1184.33	862.89	4.8047	2.7896
340	1.0069	340.89	243.31	3.5348	1.8619	1140	1.1644	1207.58	880.40	4.8253	2.8051
350	1.0077	350.97	250.52	3.5640	1.8828	1160	1.1675	1230.90	897.98	4.8455	2.8204
360	1.0086	361.05	257.73	3.5925	1.9032	1180	1.1706	1254.28	915.62	4.8655	2.8355
370	1.0096	371.14	264.95	3.6202	1.9230	1200	1.1735	1277.72	933.32	4.8852	2.8504
380	1.0106	381.24	272.18	3.6472	1.9423	1220	1.1764	1301.22	951.08	4.9047	2.8651
390	1.0117	391.35	279.42	3.6735	1.9612	1240	1.1792	1324.78	968.90	4.9239	2.8796
400	1.0129	401.47	286.67	3.6992	1.9796	1260	1.1820	1348.39	986.77	4.9428	2.8939
410	1.0142	411.61	293.94	3.7243	1.9976	1280	1.1846	1372.06	1004.70	4.9614	2.9081
420	1.0156	421.76	301.22	3.7488	2.0152	1300	1.1872	1395.78	1022.68	4.9799	2.9220
430	1.0170	431.92	308.51	3.7728	2.0325	1320	1.1898	1419.55	1040.71	4.9980	2.9358
440	1.0185	442.10	315.82	3.7962	2.0493	1340	1.1922	1443.37	1058.79	5.0160	2.9495
450	1.0201	452.29	323.14	3.8192	2.0658	1360	1.1946	1467.24	1076.92	5.0337	2.9629
460	1.0217	462.50	330.48	3.8417	2.0820	1380	1.1970	1491.15	1095.09	5.0512	2.9762
470	1.0234	472.73	337.84	3.8637	2.0979	1400	1.1993	1515.12	1113.32	5.0685	2.9894
480	1.0252	482.97	345.21	3.8854	2.1135	1420	1.2015	1539.12	1131.58	5.0855	3.0024
490	1.0270	493.23	352.60	3.9066	2.1288	1440	1.2037	1563.18	1149.90	5.1024	3.0152
500	1.0289	503.51	360.01	3.9274	2.1438	1460	1.2058	1587.27	1168.25	5.1190	3.0279
510	1.0309	513.81	367.44	3.9478	2.1586	1480	1.2079	1611.41	1186.65	5.1355	3.0405
520	1.0329	524.13	374.89	3.9679	2.1731	1500	1.2099	1635.59	1205.09	5.1518	3.0529
530	1.0349	534.47	382.36	3.9876	2.1873	1520	1.2119	1659.80	1223.56	5.1678	3.0651
540	1.0370	544.82	389.84	4.0070	2.2014	1540	1.2138	1684.06	1242.08	5.1837	3.0773
550	1.0392	555.21	397.36	4.0261	2.2152	1560	1.2157	1708.36	1260.64	5.1994	3.0893
560	1.0413	565.61	404.89	4.0449	2.2288	1580	1.2176	1732.69	1279.23	5.2150	3.1012
570	1.0435	576.03	412.44	4.0634	2.2422	1600	1.2194	1757.06	1297.86	5.2303	3.1129
580	1.0458	586.48	420.02	4.0815	2.2554	1620	1.2211	1781.47	1316.53	5.2455	3.1245
590	1.0481	596.95	427.62	4.0995	2.2684	1640	1.2229	1805.91	1335.23	5.2605	3.1360
600	1.0504	607.44	435.24	4.1171	2.2812	1660	1.2246	1830.38	1353.96	5.2754	3.1474
610	1.0527	617.96	442.89	4.1345	2.2938	1680	1.2262	1854.89	1372.73	5.2900	3.1586
620	1.0550	628.49	450.55	4.1516	2.3063	1700	1.2278	1879.43	1391.53	5.3046	3.1698
630	1.0574	639.06	458.25	4.1685	2.3186	1750	1.2318	1940.92	1438.67	5.3402	3.1971
640	1.0598	649.64	465.96	4.1852	2.3307	1800	1.2355	2002.60	1486.00	5.3750	3.2238
650	1.0621	660.25	473.70	4.2016	2.3427	1850	1.2390	2064.47	1533.52	5.4089	3.2498
660	1.0645	670.88	481.46	4.2178	2.3546	1900	1.2424	2126.50	1581.20	5.4419	3.2752
670	1.0669	681.54	489.25	4.2339	2.3663	1950	1.2457	2188.71	1629.06	5.4742	3.3000
680	1.0693	692.22	497.06	4.2497	2.3778	2000	1.2488	2251.07	1677.07	5.5057	3.3243
690	1.0718	702.93	504.90	4.2653	2.3892	2050	1.2518	2313.58	1725.23	5.5366	3.3480
700	1.0742	713.66	512.76	4.2807	2.4005	2100	1.2546	2376.24	1773.54	5.5667	3.3712
710	1.0766	724.41	520.64	4.2959	2.4117	2150	1.2574	2439.04	1821.99	5.5962	3.3940
720	1.0790	735.19	528.55	4.3110	2.4227	2200	1.2600	2501.98	1870.58	5.6251	3.4163
730	1.0814	745.99	536.48	4.3258	2.4336	2250	1.2626	2565.04	1919.29	5.6534	3.4382
740	1.0838	756.82	544.44	4.3405	2.4444	2300	1.2651	2628.23	1968.13	5.6812	3.4596

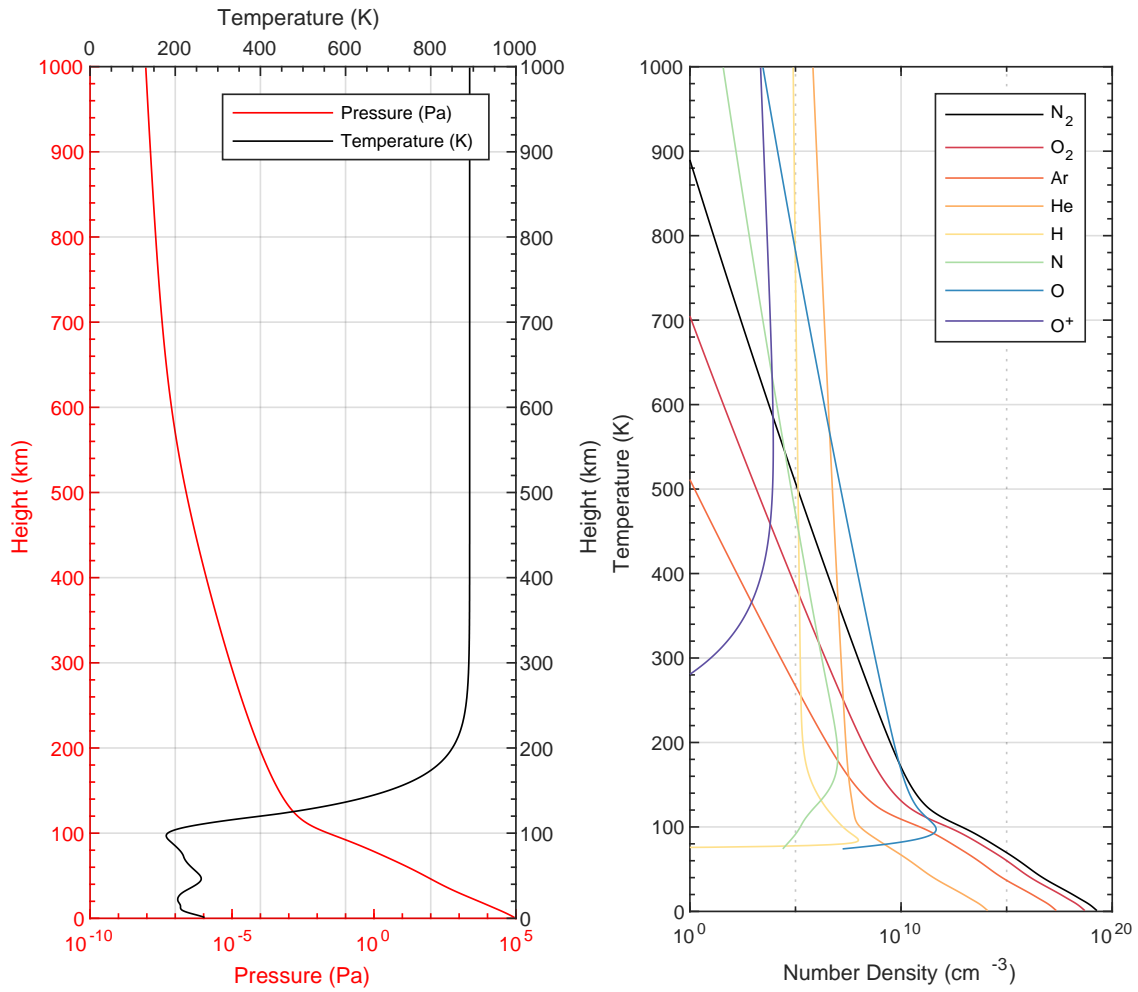
# Standard Atmosphere Properties

# E

Standard, latitude and time invariant atmospheric properties for Earth are usually obtained on the basis of the U.S. Standard Atmosphere (1976) model [1], which to date acts as a *de-facto* standard for altimeter calibrations, aircraft and rocket design and performance calculations, as well as meteorological diagrams. This model is valid up to an 84 km altitude.

More recent models, with higher altitude ranges extending well into Earth's exoatmosphere have been proposed with applications for Space weather or satellite drag applications. We present the standard atmospheric properties of Earth based on the NRLMSISE-00 model of Earth's atmosphere from ground to Space, developed by the US Naval Research Laboratory [2]. This model was developed using actual satellite drag data for the upper exoatmospheric layers of Earth, and extends up to  $h = 1000$  km. A primary use of this model is to aid predictions of satellite orbital decay due to atmospheric drag. The COSPAR International Reference Atmosphere (CIRA) 2012 and the ISO 14222 Earth Atmosphere Density standard both recommend NRLMSISE-00 for composition uses.

The model proposes Temperature  $T$ , density  $\rho$ , and chemical composition ( $N_2$ ,  $O_2$ , Ar, He, H, N, O,  $O^+$ ) in terms of number densities. It is fully parametrizable in terms of latitude, longitude, time, solar flux variability, and magnetic index data. The tables have been produced considering equatorial ( $0^\circ$ ) latitude, and averaged times and solar fluxes, with default values for other parameters (F107, F107A: 150; APH: 4). The pressure  $p$  is retrieved using the ideal gas equation  $p = \rho R_{spe} T$ . Plots for these variables (except  $\rho$ ) are presented in Figure E.1, and the altitude-dependent properties are presented in Table E.1 for  $h = 1 - 100$  km and Table E.2 for  $h = 100 - 1000$  km.



**Figure E.1:** Properties for Earth's Atmosphere: Pressure & Temperature vs. Height (left); Chemical Species Number Density (right).

## References

- [1] National Aeronautics and Space Administration (NASA). *US Standard Atmosphere*, 1976. Tech. rep. NASA-TM-X-74335. US Government Printing Office, Washington, DC, 1976 (cited on page 120).
- [2] J. M. Picone et al. 'NRLMSISE-00 empirical model of the atmosphere: Statistical comparisons and scientific issues'. In: *Journal of Geophysical Research: Space Physics* 107.A12 (2002), SIA 15-1-SIA 15-16 (cited on page 120).

Table E.1: NRLMISE-00 Atmosphere Model (0–100 km)

h km	p Pa	$\rho$ g/cm <sup>3</sup>	T K	N <sub>2</sub> 1/cm <sup>3</sup>	O <sub>2</sub> 1/cm <sup>3</sup>	Ar 1/cm <sup>3</sup>	H 1/cm <sup>3</sup>	He 1/cm <sup>3</sup>	N 1/cm <sup>3</sup>	O 1/cm <sup>3</sup>	O <sup>+</sup> 1/cm <sup>3</sup>
0.0	1.000e+5	1.159e-3	300.7	1.883e+19	5.052e+18	2.253e+17	0.000	1.264e+14	0.000	0.000	0.000
0.1	9.891e+4	1.149e-3	299.9	1.867e+19	5.008e+18	2.233e+17	0.000	1.253e+14	0.000	0.000	0.000
0.2	9.779e+4	1.138e-3	299.2	1.850e+19	4.964e+18	2.213e+17	0.000	1.242e+14	0.000	0.000	0.000
0.3	9.668e+4	1.128e-3	298.4	1.834e+19	4.920e+18	2.194e+17	0.000	1.231e+14	0.000	0.000	0.000
0.4	9.558e+4	1.118e-3	297.7	1.818e+19	4.876e+18	2.174e+17	0.000	1.220e+14	0.000	0.000	0.000
0.5	9.449e+4	1.108e-3	297.0	1.801e+19	4.832e+18	2.155e+17	0.000	1.209e+14	0.000	0.000	0.000
0.6	9.341e+4	1.098e-3	296.3	1.785e+19	4.788e+18	2.135e+17	0.000	1.198e+14	0.000	0.000	0.000
0.7	9.235e+4	1.088e-3	295.6	1.769e+19	4.744e+18	2.115e+17	0.000	1.187e+14	0.000	0.000	0.000
0.8	9.129e+4	1.078e-3	295.0	1.752e+19	4.701e+18	2.096e+17	0.000	1.176e+14	0.000	0.000	0.000
0.9	9.024e+4	1.068e-3	294.3	1.736e+19	4.657e+18	2.076e+17	0.000	1.165e+14	0.000	0.000	0.000
1.0	8.920e+4	1.058e-3	293.7	1.720e+19	4.613e+18	2.057e+17	0.000	1.154e+14	0.000	0.000	0.000
1.2	8.715e+4	1.038e-3	292.4	1.687e+19	4.526e+18	2.018e+17	0.000	1.132e+14	0.000	0.000	0.000
1.4	8.514e+4	1.018e-3	291.2	1.655e+19	4.440e+18	1.980e+17	0.000	1.111e+14	0.000	0.000	0.000
1.6	8.317e+4	9.98e-4	290.1	1.623e+19	4.354e+18	1.942e+17	0.000	1.089e+14	0.000	0.000	0.000
1.8	8.124e+4	9.79e-4	289.0	1.592e+19	4.270e+18	1.904e+17	0.000	1.068e+14	0.000	0.000	0.000
2.0	7.934e+4	9.59e-4	287.9	1.560e+19	4.186e+18	1.866e+17	0.000	1.047e+14	0.000	0.000	0.000
2.5	7.477e+4	9.127e-4	285.3	1.484e+19	3.980e+18	1.775e+17	0.000	9.955e+13	0.000	0.000	0.000
3.0	7.042e+4	8.671e-4	282.9	1.409e+19	3.781e+18	1.686e+17	0.000	9.458e+13	0.000	0.000	0.000
3.5	6.630e+4	8.231e-4	280.5	1.338e+19	3.589e+18	1.600e+17	0.000	8.978e+13	0.000	0.000	0.000
4	6.238e+4	7.810e-4	278.2	1.269e+19	3.406e+18	1.518e+17	0.000	8.519e+13	0.000	0.000	0.000
5	5.814e+4	7.025e-4	273.4	1.142e+19	3.063e+18	1.366e+17	0.000	7.663e+13	0.000	0.000	0.000
6	4.864e+4	6.317e-4	268.2	1.027e+19	2.754e+18	1.228e+17	0.000	6.890e+13	0.000	0.000	0.000
7	4.279e+4	5.683e-4	262.3	9.237e+18	2.478e+18	1.105e+17	0.000	6.199e+13	0.000	0.000	0.000
8	3.753e+4	5.118e-4	255.4	8.318e+18	2.232e+18	9.950e+16	0.000	5.582e+13	0.000	0.000	0.000
9	3.279e+4	4.614e-4	247.5	7.500e+18	2.012e+18	8.971e+16	0.000	5.033e+13	0.000	0.000	0.000
10	2.851e+4	4.165e-4	238.4	6.770e+18	1.816e+18	8.098e+16	0.000	4.543e+13	0.000	0.000	0.000
11	2.466e+4	3.758e-4	228.5	6.108e+18	1.639e+18	7.306e+16	0.000	4.099e+13	0.000	0.000	0.000
12	2.118e+4	3.372e-4	218.8	5.481e+18	1.470e+18	6.556e+16	0.000	3.678e+13	0.000	0.000	0.000
13	1.808e+4	2.996e-4	210.2	4.870e+18	1.306e+18	5.825e+16	0.000	3.268e+13	0.000	0.000	0.000
14	1.535e+4	2.627e-4	203.5	4.270e+18	1.146e+18	5.108e+16	0.000	2.866e+13	0.000	0.000	0.000
15	1.296e+4	2.269e-4	199.0	3.688e+18	9.894e+17	4.411e+16	0.000	2.475e+13	0.000	0.000	0.000
16	1.092e+4	1.930e-4	197.2	3.136e+18	8.414e+17	3.752e+16	0.000	2.105e+13	0.000	0.000	0.000
17	9.199e+3	1.623e-4	197.4	2.638e+18	7.077e+17	3.155e+16	0.000	1.770e+13	0.000	0.000	0.000
18	7.754e+3	1.356e-4	199.2	2.204e+18	5.913e+17	2.636e+16	0.000	1.479e+13	0.000	0.000	0.000
19	6.549e+3	1.130e-4	201.8	1.837e+18	4.928e+17	2.197e+16	0.000	1.233e+13	0.000	0.000	0.000
20	5.545e+3	9.427e-5	204.9	1.532e+18	4.111e+17	1.833e+16	0.000	1.028e+13	0.000	0.000	0.000
21	4.706e+3	7.891e-5	207.7	1.283e+18	3.441e+17	1.534e+16	0.000	8.607e+12	0.000	0.000	0.000
22	4.003e+3	6.629e-5	210.3	1.077e+18	2.891e+17	1.289e+16	0.000	7.231e+12	0.000	0.000	0.000
23	3.412e+3	5.586e-5	212.7	9.079e+17	2.436e+17	1.086e+16	0.000	6.093e+12	0.000	0.000	0.000
24	2.913e+3	4.720e-5	215.0	7.671e+17	2.058e+17	9.176e+15	0.000	5.148e+12	0.000	0.000	0.000
25	2.491e+3	3.997e-5	217.1	6.496e+17	1.743e+17	7.771e+15	0.000	4.359e+12	0.000	0.000	0.000
26	2.134e+3	3.391e-5	219.2	5.512e+17	1.479e+17	6.593e+15	0.000	3.699e+12	0.000	0.000	0.000
27	1.830e+3	2.882e-5	221.2	4.684e+17	1.257e+17	5.603e+15	0.000	3.143e+12	0.000	0.000	0.000
28	1.572e+3	2.452e-5	223.3	3.986e+17	1.069e+17	4.767e+15	0.000	2.675e+12	0.000	0.000	0.000
29	1.353e+3	2.089e-5	225.6	3.395e+17	9.107e+16	4.061e+15	0.000	2.278e+12	0.000	0.000	0.000
30	1.166e+3	1.780e-5	228.1	2.894e+17	7.763e+16	3.462e+15	0.000	1.942e+12	0.000	0.000	0.000
31	1.006e+3	1.519e-5	230.8	2.468e+17	6.622e+16	2.953e+15	0.000	1.656e+12	0.000	0.000	0.000
32	8.705e+2	1.296e-5	234.0	2.106e+17	5.651e+16	2.519e+15	0.000	1.413e+12	0.000	0.000	0.000
33	7.545e+2	1.107e-5	237.5	1.799e+17	4.825e+16	2.151e+15	0.000	1.207e+12	0.000	0.000	0.000
34	6.554e+2	9.477e-6	240.9	1.540e+17	4.132e+16	1.842e+15	0.000	1.034e+12	0.000	0.000	0.000
35	5.704e+2	8.142e-6	244.0	1.323e+17	3.550e+16	1.583e+15	0.000	8.881e+11	0.000	0.000	0.000
36	4.972e+2	7.016e-6	246.9	1.140e+17	3.059e+16	1.364e+15	0.000	7.652e+11	0.000	0.000	0.000
37	4.342e+2	6.061e-6	249.5	9.852e+16	2.643e+16	1.178e+15	0.000	6.611e+11	0.000	0.000	0.000
38	3.796e+2	5.249e-6	251.9	8.532e+16	2.289e+16	1.021e+15	0.000	5.726e+11	0.000	0.000	0.000
39	3.324e+2	4.556e-6	254.1	7.406e+16	1.987e+16	8.858e+14	0.000	4.970e+11	0.000	0.000	0.000
40	2.913e+2	3.963e-6	256.0	6.441e+16	1.728e+16	7.705e+14	0.000	4.322e+11	0.000	0.000	0.000
41	2.556e+2	3.453e-6	257.8	5.612e+16	1.506e+16	6.713e+14	0.000	3.766e+11	0.000	0.000	0.000
42	2.244e+2	3.013e-6	259.4	4.898e+16	1.314e+16	5.859e+14	0.000	3.287e+11	0.000	0.000	0.000
43	1.972e+2	2.634e-6	260.8	4.280e+16	1.148e+16	5.120e+14	0.000	2.872e+11	0.000	0.000	0.000
44	1.734e+2	2.304e-6	262.2	3.746e+16	1.005e+16	4.480e+14	0.000	2.514e+11	0.000	0.000	0.000
45	1.526e+2	2.019e-6	263.4	3.281e+16	8.802e+15	3.925e+14	0.000	2.202e+11	0.000	0.000	0.000
46	1.344e+2	1.770e-6	264.5	2.877e+16	7.718e+15	3.441e+14	0.000	1.931e+11	0.000	0.000	0.000
47	1.184e+2	1.554e-6	265.5	2.525e+16	6.774e+15	3.020e+14	0.000	1.695e+11	0.000	0.000	0.000
48	1.044e+2	1.365e-6	266.3	2.219e+16	5.953e+15	2.654e+14	0.000	1.489e+11	0.000	0.000	0.000
49	9.205e+1	1.201e-6	266.9	1.952e+16	5.237e+15	2.335e+14	0.000	1.310e+11	0.000	0.000	0.000
50	8.119e+1	1.058e-6	267.2	1.720e+16	4.614e+15	2.057e+14	0.000	1.154e+11	0.000	0.000	0.000
51	7.162e+1	9.335e-7	267.2	1.517e+16	4.070e+15	1.815e+14	0.000	1.018e+11	0.000	0.000	0.000
52	6.317e+1	8.245e-7	266.9	1.340e+16	3.595e+15	1.603e+14	0.000	8.993e+10	0.000	0.000	0.000
53	5.571e+1	7.293e-7	266.1	1.185e+16	3.180e+15	1.418e+14	0.000	7.954e+10	0.000	0.000	0.000
54	4.911e+1	6.458e-7	264.9	1.050e+16	2.816e+15	1.256e+14	0.000	7.044e+10	0.000	0.000	0.000
55	4.326e+1	5.726e-7	263.2	9.307e+15	2.497e+15	1.113e+14	0.000	6.246e+10	0.000	0.000	0.000
56	3.808e+1	5.082e-7	261.0	8.260e+15	2.216e+15	9.880e+13	0.000	5.543e+10	0.000	0.000	0.000
57	3.347e+1	4.513e-7	258.4	7.335e+15	1.968e+15	8.774e+13	0.000	4.922e+10	0.000	0.000	0.000
58	2.939e+1	4.008e-7	255.4	6.514e+15	1.748e+15	7.792e+13	0.000	4.371e+10	0.000	0.000	0.000
59	2.576e+1	3.558e-7	252.2	5.783e+15	1.551e+15	6.917e+13	0.000	3.881e+10	0.000	0.000	0.000
60	2.254e+1	3.156e-7	248.8	5.129e+15	1.376e+15	6.136e+13	0.000	3.442e+10	0.000	0.000	0.000
61	1.969e+1	2.796e-7	245.3	4.545e+15	1.219e+15	5.436e+13	0.000	3.050e+10	0.000	0.000	0.000
62	1.717e+1	2.474e-7	241.8	4.020e+15	1.079e+15	4.809e+13	0.000	2.698e+10	0.000	0.000	0.000
63	1.493e+1	2.184e-7	238.2	3.550e+15	9.510e+14	4.246e+13	0.000	2.384e+10	0.000	0.000	0.000
64	1.296e+1	1.923e-7	234.7	3.129e+15	8.356e+14	3.741e+13	0.000	2.102e+10	0.000	0.000	0.000
65	1.122e+1	1.690e-7	231.3	2.751e+15	7.325e+14	3.289e+13	0.000	1.850e+10	0.000	0.000	0.000
66	9.701	1.481e-7	228.1	2.413e+15	6.406e+14	2.884e+13	0.000	1.624e+10	0.000	0.000	0.000
67	8.368	1.295e-7	225.0	2.111e+15	5.588e+14	2.522e+13	0.000	1.423e+10	0.000	0.000	0.000
68	7.205	1.129e-7	222.2	1.842e+15	4.861e+14	2.200e+13	0.000	1.242e+10	0.00		

Table E.2: NRLMSISE-00 Atmosphere Model (100–1000 km)

h km	p Pa	$\rho$ g/cm <sup>3</sup>	T K	N <sub>2</sub> 1/cm <sup>3</sup>	O <sub>2</sub> 1/cm <sup>3</sup>	Ar 1/cm <sup>3</sup>	H 1/cm <sup>3</sup>	He 1/cm <sup>3</sup>	N 1/cm <sup>3</sup>	O 1/cm <sup>3</sup>	O <sup>+</sup> 1/cm <sup>3</sup>
100	3.902e-2	7.687e-10	173.0	1.294e+13	2.689e+12	1.324e+11	2.448e+7	1.423e+8	4.017e+5	5.834e+11	3.727e-37
110	7.130e-3	1.088e-10	215.6	1.850e+12	2.923e+11	1.277e+10	9.996e+6	5.136e+7	7.292e+5	2.415e+11	1.212e-31
120	2.256e-3	2.112e-11	337.1	3.560e+11	4.080e+10	1.506e+9	4.229e+6	2.537e+7	1.469e+6	8.667e+10	2.418e-27
130	1.096e-3	6.980e-12	478.5	1.150e+11	9.942e+9	3.355e+8	1.937e+6	1.665e+7	3.248e+6	4.074e+10	3.349e-23
140	6.446e-4	3.250e-12	586.3	5.190e+10	3.580e+9	1.142e+8	1.002e+6	1.312e+7	6.809e+6	2.408e+10	3.783e-19
150	4.173e-4	1.794e-12	668.8	2.756e+10	1.604e+9	4.799e+7	5.789e+5	1.116e+7	1.208e+7	1.598e+10	1.345e-15
160	2.861e-4	1.095e-12	732.1	1.609e+10	8.229e+8	2.282e+7	3.748e+5	9.868e+6	1.793e+7	1.136e+10	1.736e-12
170	2.038e-4	7.138e-13	780.7	9.983e+9	4.609e+8	1.174e+7	2.699e+5	8.929e+6	2.275e+7	8.432e+9	9.211e-10
180	1.491e-4	4.867e-13	818.0	6.452e+9	2.735e+8	6.374e+6	2.125e+5	8.199e+6	2.549e+7	6.447e+9	2.243e-7
190	1.113e-4	3.430e-13	846.7	4.291e+9	1.690e+8	3.593e+6	1.793e+5	7.606e+6	2.600e+7	5.033e+9	2.760e-5
200	8.446e-5	2.479e-13	868.7	2.914e+9	1.074e+8	2.081e+6	1.591e+5	7.104e+6	2.479e+7	3.991e+9	1.866e-3
210	6.490e-5	1.828e-13	885.7	2.010e+9	6.966e+7	1.231e+6	1.462e+5	6.675e+6	2.258e+7	3.200e+9	7.465e-2
220	5.040e-5	1.369e-13	898.8	1.402e+9	4.589e+7	7.388e+5	1.376e+5	6.296e+6	1.994e+7	2.589e+9	1.885
230	3.950e-5	1.040e-13	908.8	9.873e+8	3.059e+7	4.488e+5	1.316e+5	5.956e+6	1.726e+7	2.108e+9	3.179e+1
240	3.120e-5	7.985e-14	916.6	6.999e+8	2.058e+7	2.752e+5	1.273e+5	5.648e+6	1.477e+7	1.725e+9	3.766e+2
250	2.481e-5	6.193e-14	922.6	4.989e+8	1.394e+7	1.700e+5	1.240e+5	5.364e+6	1.255e+7	1.418e+9	3.271e+3
260	1.985e-5	4.844e-14	927.3	3.572e+8	9.501e+6	1.056e+5	1.213e+5	5.102e+6	1.062e+7	1.169e+9	2.165e+4
280	1.291e-5	3.030e-14	933.6	1.850e+8	4.469e+6	4.133e+4	1.172e+5	4.629e+6	7.585e+6	8.001e+8	4.781e+5
290	1.048e-5	2.419e-14	935.8	1.337e+8	3.080e+6	2.600e+4	1.155e+5	4.414e+6	6.414e+6	6.638e+8	1.685e+6
300	8.549e-6	1.942e-14	937.5	9.680e+7	2.128e+6	1.640e+4	1.139e+5	4.212e+6	5.425e+6	5.516e+8	5.060e+6
310	7.001e-6	1.566e-14	938.8	7.022e+7	1.474e+6	1.037e+4	1.124e+5	4.020e+6	4.599e+6	4.588e+8	1.320e+7
320	5.761e-6	1.269e-14	939.8	5.103e+7	1.023e+6	6.575e+3	1.111e+5	3.839e+6	3.904e+6	3.821e+8	3.042e+7
330	4.764e-6	1.033e-14	940.5	3.713e+7	7.114e+5	4.173e+3	1.097e+5	3.667e+6	3.318e+6	3.186e+8	6.295e+7
340	3.957e-6	8.433e-15	941.2	2.706e+7	4.954e+5	2.658e+3	1.084e+5	3.504e+6	2.823e+6	2.658e+8	1.185e+8
350	3.292e-6	6.909e-15	941.6	1.975e+7	3.455e+5	1.695e+3	1.071e+5	3.349e+6	2.405e+6	2.219e+8	2.052e+8
360	2.737e-6	5.678e-15	942.0	1.442e+7	2.413e+5	1.082e+3	1.059e+5	3.202e+6	2.051e+6	1.855e+8	3.305e+8
370	2.273e-6	4.679e-15	942.3	1.055e+7	1.688e+5	6.922e+2	1.047e+5	3.061e+6	1.675e+6	1.551e+8	4.993e+8
380	1.886e-6	3.866e-15	942.5	7.724e+6	1.182e+5	4.434e+2	1.036e+5	2.928e+6	1.495e+6	1.298e+8	7.133e+8
390	1.566e-6	3.201e-15	942.7	5.661e+6	8.285e+4	2.845e+2	1.024e+5	2.800e+6	1.278e+6	1.086e+8	9.703e+8
400	1.301e-6	2.657e-15	942.8	4.153e+6	5.815e+4	1.828e+2	1.013e+5	2.679e+6	1.094e+6	9.101e+7	1.264e+9
410	1.082e-6	2.209e-15	942.9	3.050e+6	4.086e+4	1.176e+2	1.001e+5	2.563e+6	9.366e+5	7.629e+7	1.587e+9
420	9.020e-7	1.840e-15	943.0	2.242e+6	2.874e+4	7.576e+1	9.905e+4	2.453e+6	8.025e+5	6.398e+7	1.927e+9
430	7.528e-7	1.536e-15	943.1	1.650e+6	2.024e+4	4.888e+1	9.796e+4	2.347e+6	6.880e+5	5.369e+7	2.273e+9
440	6.293e-7	1.283e-15	943.1	1.215e+6	1.427e+4	3.158e+1	9.690e+4	2.247e+6	5.902e+5	4.508e+7	2.614e+9
450	5.268e-7	1.074e-15	943.2	8.956e+5	1.007e+4	2.042e+1	9.585e+4	2.151e+6	5.065e+5	3.787e+7	2.940e+9
460	4.416e-7	9.005e-16	943.2	6.608e+5	7.116e+3	1.323e+1	9.481e+4	2.060e+6	4.345e+5	3.183e+7	3.243e+9
470	3.707e-7	7.559e-16	943.2	4.880e+5	5.032e+3	8.580	9.379e+4	1.972e+6	3.734e+5	2.677e+7	3.518e+9
480	3.116e-7	6.354e-16	943.2	3.607e+5	3.563e+3	5.572	9.278e+4	1.889e+6	3.211e+5	2.252e+7	3.758e+9
490	2.622e-7	5.348e-16	943.3	2.669e+5	2.525e+3	3.623	9.178e+4	1.809e+6	2.761e+5	1.896e+7	3.964e+9
500	2.210e-7	4.507e-16	943.3	1.976e+5	1.791e+3	2.359	9.080e+4	1.733e+6	2.376e+5	1.597e+7	4.132e+9
510	1.865e-7	3.803e-16	943.3	1.465e+5	1.272e+3	1.538	8.984e+4	1.661e+6	2.046e+5	1.346e+7	4.266e+9
520	1.576e-7	3.214e-16	943.3	1.087e+5	9.040e+2	1.004	8.888e+4	1.591e+6	1.762e+5	1.135e+7	4.365e+9
530	1.333e-7	2.719e-16	943.3	8.067e+4	6.432e+2	6.558e-1	8.794e+4	1.525e+6	1.518e+5	9.570e+6	4.432e+9
540	1.129e-7	2.303e-16	943.3	5.995e+4	4.581e+2	4.291e-1	8.702e+4	1.462e+6	1.309e+5	8.077e+6	4.470e+9
550	9.581e-8	1.954e-16	943.3	4.459e+4	3.266e+2	2.611e-1	8.610e+4	1.401e+6	1.129e+5	6.820e+6	4.482e+9
560	8.140e-8	1.660e-16	943.3	3.319e+4	2.331e+2	1.844e-1	8.520e+4	1.343e+6	9.738e+4	5.761e+6	4.471e+9
570	6.926e-8	1.412e-16	943.3	2.473e+4	1.665e+2	1.211e-1	8.431e+4	1.288e+6	8.403e+4	4.869e+6	4.440e+9
580	5.902e-8	1.204e-16	943.3	1.844e+4	1.191e+2	7.962e-2	8.343e+4	1.235e+6	7.258e+4	4.117e+6	4.392e+9
590	5.037e-8	1.027e-16	943.3	1.376e+4	8.522e+1	5.242e-2	8.256e+4	1.185e+6	6.270e+4	3.483e+6	4.330e+9
600	4.307e-8	8.784e-17	943.3	1.028e+4	6.105e+1	3.455e-2	8.170e+4	1.136e+6	5.419e+4	2.948e+6	4.256e+9
610	3.689e-8	7.524e-17	943.3	7.683e+3	4.378e+1	2.280e-2	8.086e+4	1.090e+6	4.685e+4	2.497e+6	4.172e+9
620	3.166e-8	6.458e-17	943.3	5.748e+3	3.143e+1	1.506e-2	8.003e+4	1.046e+6	4.053e+4	2.115e+6	4.080e+9
630	2.723e-8	5.554e-17	943.3	4.304e+3	2.258e+1	9.964e-3	7.920e+4	1.003e+6	3.507e+4	1.793e+6	3.983e+9
640	2.347e-8	4.787e-17	943.3	3.226e+3	1.624e+1	6.599e-3	7.839e+4	9.629e+5	3.036e+4	1.521e+6	3.882e+9
650	2.028e-8	4.135e-17	943.3	2.420e+3	1.169e+1	4.376e-3	7.759e+4	9.241e+5	2.629e+4	1.290e+6	3.777e+9
660	1.756e-8	3.581e-17	943.3	1.816e+3	8.423	2.905e-3	7.680e+4	8.870e+5	2.278e+4	1.095e+6	3.670e+9
670	1.525e-8	3.110e-17	943.3	1.364e+3	6.074	1.931e-3	7.602e+4	8.515e+5	1.974e+4	9.300e+5	3.563e+9
680	1.328e-8	2.708e-17	943.3	1.026e+3	4.385	1.285e-3	7.525e+4	8.175e+5	1.712e+4	7.901e+5	3.455e+9
690	1.159e-8	2.364e-17	943.3	7.720e+2	3.168	8.557e-4	7.449e+4	7.850e+5	1.485e+4	6.716e+5	3.347e+9
700	1.015e-8	2.071e-17	943.3	5.814e+2	2.291	5.707e-4	7.374e+4	7.538e+5	1.289e+4	5.712e+5	3.241e+9
710	8.923e-9	1.820e-17	943.3	4.382e+2	1.659	3.811e-4	7.300e+4	7.240e+5	1.119e+4	4.860e+5	3.136e+9
720	7.867e-9	1.605e-17	943.3	3.306e+2	1.202	2.547e-4	7.227e+4	6.954e+5	9.718e+3	4.136e+5	3.033e+9
730	6.961e-9	1.420e-17	943.3	2.495e+2	8.715e-1	1.705e-4	7.154e+4	6.680e+5	8.444e+3	3.523e+5	2.932e+9
740	6.180e-9	1.260e-17	943.3	1.885e+2	6.326e-1	1.142e-4	7.083e+4	6.418e+5	7.339e+3	3.001e+5	2.833e+9
750	5.508e-9	1.123e-17	943.3	1.426e+2	4.596e-1	7.661e-5	7.013e+4	6.167e+5	6.382e+3	2.558e+5	2.737e+9
760	4.927e-9	1.005e-17	943.3	1.079e+2	3.342e-1	5.144e-5	6.943e+4	5.926e+5	5.552e+3	2.181e+5	2.643e+9
770	4.424e-9	9.022e-18	943.3	8.169e+1	2.432e-1	3.458e-5	6.875e+4	5.695e+5	4.831e+3	1.861e+5	2.552e+9
780	3.987e-9	8.133e-18	943.3	6.192e+1	1.772e-1	2.327e-5	6.807e+4	5.474e+5	4.206e+3	1.588e+5	2.463e+9
790	3.608e-9	7.359e-18	943.3	4.696e+1	1.292e-1	1.568e-5	6.740e+4	5.262e+5	3.663e+3	1.356e+5	2.377e+9
800	3.277e-9	6.684e-18	943.3	3.565e+1	9.428e-2	1.058e-5	6.674e+4	5.059e+5	3.191e+3	1.159e+5	2.294e+9
810	2.988e-9	6.094e-18	943.3	2.708e+1	6.887e-2	7.142e-6	6.609e+4	4.864e+5	2.782e+3	9.902e+4	2.214e+9
820	2.735e-9	5.577e-18	943.3	2.059e+1	5.034e-2	4.828e-6	6.544e+4	4.677e+5	2.425e+3	8.466e+4	2.136e+9
830	2.512e-9	5.123e-18	943.3	1.566e+1	3.684e-2	3.267e-6	6.481e+4	4.498e+5	2.115e+3	7.242e+4	2.061e+9
840	2.315e-9	4.722e-18	943.3	1.193e+1	2.698e-2	2.213e-6	6.418e+4	4.326e+5	1.846e+3	6.197e+4	1.989e+9
850	2.141e-9	4.367e-18	943.3	9.088	1.977e-2	1.501e-6	6.356e+4	4.162e+5	1.611e+3	5.306e+4	1.919e+9
860	1.987e-9	4.052e-18	943.3	6.930	1.451e-2	1.019e-6	6.295e+4	4.004e+5	1.407e+3	4.545e+4	1.851e+9
870	1.849e-9	3.772e-18	943.3	5.289	1.065e-2	6.926e-7	6.234e+4	3.852e+5	1.229e+3	3.894e+4	1.786e+9
880	1.726e-9	3.521e-18	943.3	4.039	7.826e-3	4.713e-7	6.175e+4	3.706e+5	1.074e+3	3.338e+4	1.723e+9
890	1.616e-9	3.296e-18	943.3	3.087	5.756e-3	3.210e-7					

# Human Body Energy Expenditure

# F

This chapter describes the energy expenditure necessary to support human life and the energy expenditure associated to human activities.

## F.1 Metabolic Rate

The metabolic rate is the amount of energy used by an human per unit of time. About 70% of a human's daily energy use is spent to sustain the basal life processes within the organs of the body. 10% is spent for digestion of food after eating, and only the remainder 20% energy use is spent to sustain physical activity and other high level activities [1].

We may accordingly define the following parameters:

- ▶ **Basal Metabolic Rate (BMR):** The minimum amount of energy required to maintain the basic body functions
- ▶ **Resting Metabolic Rate (RMR):** The amount of energy used daily by humans at rest.
- ▶ **Metabolic Equivalent of Task (MET):** The ratio of the work metabolic rate to the resting metabolic rate for a given task.

### F.1.1 Basal Metabolic Rate (BMR)

The Basal Metabolic Rate refers to the minimum energy consumption required to sustain basic physiological functions at rest. It represents the energy needed for essential processes such as breathing, circulation, and maintaining body temperature. The Harris-Benedict formula [2] is commonly used to estimate BMR:

- ▶ **For females:**  
$$\text{BMR [kcal]} = 655 + (9.6 \times \text{weight [kg]}) + (1.8 \times \text{height [cm]}) - (4.7 \times \text{age [yr]})$$
- ▶ **For males:**  
$$\text{BMR [kcal]} = 66 + (13.7 \times \text{weight [kg]}) + (5 \times \text{height [cm]}) - (6.8 \times \text{age [yr]})$$

### F.1.2 Resting Metabolic Rate (RMR)

The Resting Metabolic Rate is similar to BMR but takes into account energy expenditure under resting conditions, such as sleep, lying down, or sitting. BMR instead corresponds to a body being at perfect rest, without any activities at all, like digesting nutrients, generating body waste, or light movements. One analogy would be a person being in a coma on a hospital bed. RMR accounts for light activities such as eating, using the restroom, or small movements. In keeping with the previous analogy

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this corresponds to being confined in a hospital bed while conscious.

RMR is typically 10%-20% higher than BMR.

### F.1.3 Metabolic Equivalent of Task (MET)

The Metabolic Equivalent of Task is the ratio of the work metabolic rate to the resting metabolic rate. One MET is defined as 1 kcal/(kg h) and is roughly equivalent to the energy cost of sitting quietly. A MET also is defined as oxygen uptake in mL min/kg with one MET equal to the oxygen cost of sitting quietly, equivalent to 3.5 mL/(kg min).

#### Definition Based on oxygen utilization and body mass

The original definition of metabolic equivalent of task is the oxygen used by a person in mL/(kg min) body mass divided by 3.5.

Other definitions which roughly produce the same numbers have been devised, such as:

$$1 \text{ MET} = 1 \frac{\text{kcal}}{\text{kg} \times \text{h}} = 4.184 \frac{\text{kJ}}{\text{kg} \times \text{h}} = 1.162 \frac{\text{W}}{\text{kg}}$$

#### Definition based on watts produced and body surface area

Another definition is based on the body surface area, BSA, and energy itself, where the BSA is expressed in m<sup>2</sup>:

$$1 \text{ MET} = 58.2 \frac{\text{W}}{\text{m}^2}$$

which is equal to the rate of energy produced per unit surface area of an average person seated at rest. The metabolic rate is usually expressed in terms of the unit area of the total body surface (ANSI/ASHRAE Standard 55 [3]). One may then multiply the BSA of the person under consideration to get the equivalence between MET and energy expenditure rate (in W). Note that recent studies have found that this conversion value systematically overestimates the metabolic rate by 20% [4], therefore this value may be re-evaluated in future revisions of the standard. Coincidentally, using the standard weight and heights of standard persons presented in the next section yields a MET value, as defined by body area 20% higher than the per weight definition.

## F.2 Standard Person Body Surface Area and Weight

In physiology and medicine, the body surface area (BSA) is the measured or calculated surface area of a human body. For many clinical purposes, BSA is a better indicator of metabolic mass than body weight because it is less affected by abnormal adipose mass.

The most widely used is the Du Bois formula,[5, 6] which has been shown to be equally as effective in estimating body fat in obese and non-obese patients, something the Body mass index fails to do.[7]

$$BSA = 0.007184 \times W^{0.425} \times H^{0.725},$$

Where  $W$  is the person weight and  $H$  the person height.

### F.2.1 Latest standards for human BSA and Weight

As humans evolve through generations and their developmental/living conditions generally increase over time, so will the standard BSA values.

There was an average BSA of  $1.73 \text{ m}^2$  for 3,000 cancer patients from 1990 to 1998 in a European Organisation for Research and Treatment of Cancer (EORTC) database [8]. During 2005 there was an average BSA of  $1.79 \text{ m}^2$  for 3,613 adult cancer patients in the UK. Among them the average BSA for men was  $1.91 \text{ m}^2$  and for women was  $1.71 \text{ m}^2$  [9].

Finally an ISO standard for male/female person physiological properties has been defined in 2004. The standard relevant parameters are reproduced in table F.1.

	Male	Female
Height (m)	1.7	1.6
Weight (kg)	70	60
Surface area ( $\text{m}^2$ )	1.8	1.6
Age (yrs)	35	35
Basal metabolic rate ( $\text{W}/\text{m}^2$ )	44	41

**Table F.1:** Standard persons used when estimating values of metabolic rate presented in ISO 8996(2004)

## F.3 Energy Expenditure for Human Activities

Energy expenditure can be measured using direct and indirect calorimetry. Direct calorimetry is rarely used due to its technical and financial demands. Indirect calorimetry, on the other hand, is more common and involves estimating energy expenditure based on oxygen intake ( $\text{VO}_2$ ) measurements.

- ▶ Basal Metabolic Rate (BMR) ranges between individuals from approximately 1,200 to 2,400 kcal (5,000 to 10,000 kJ) per day.
- ▶ Resting Metabolic Rate (RMR) is about 10–20% higher than BMR.
- ▶ Energy expenditure during routine daily activities ranges from 1,800 to 3,000 kcal (7,600 to 12,600 kJ) per day, depending on factors like sex, age, body build, and fitness level.
- ▶ Energy expenditure during specific activities and sports can vary widely, ranging from 290 kJ/h (sleeping, reading) to 1000 kJ/h or more (running, swimming, basketball, etc.).

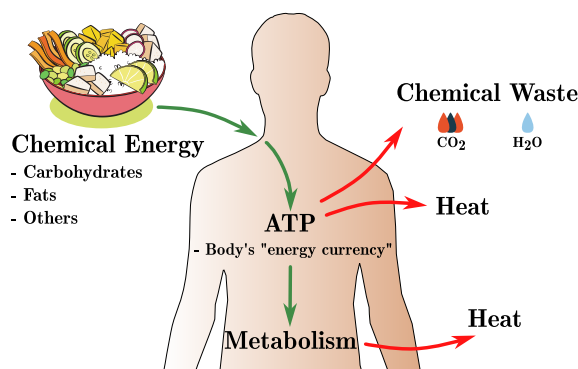
These values are estimates and can vary among individuals. Factors such as body composition, genetics, and overall health can influence energy expenditure. To provide a precise measurement of an individual's daily energy expenditure, it would be necessary to perform direct or indirect calorimetry tests. These tests would provide more accurate data tailored to the specific individual.

### F.3.1 Human Metabolism

The metabolism of a human being acts to sustain its activities through the input of energy in the form of food (eating) and oxygen (breathing) and a cascade of chemical reactions to substances that allow providing work and heat that sustain the basic and high-level functions of our body.

The human body (as any living thing) therefore is a thermodynamic system that consumes chemical potential energy by breaking chemical bonds and releasing energy. This energy is available for work (such as mechanical work) or for other processes (such as chemical synthesis and anabolic processes in growth), when weak bonds are broken and stronger bonds are made. The production of stronger bonds allows release of usable energy.

The mediator for these processes, Adenosine triphosphate (ATP) is metabolically synthesized from available chemical materials, transported, and broken down by biological processes into adenosine diphosphate (ADP), releasing energy.



#### ATP: The Fuel of Life

Adenosine triphosphate (ATP,  $\text{C}_{10}\text{H}_{16}\text{N}_5\text{O}_{13}\text{P}_3$ ) is a molecule that all living things produce to store energy and to move around to the cells

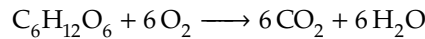
that require energy for living processes. This includes the synthesis of macromolecules such as DNA and RNA. ATP also helps macromolecules get across cell membranes. When ATP breaks a bond with a phosphate group and becomes ADP, energy is released since this is an exothermic reaction. In a cell, the ratio of ATP to ADP concentrations is known as the "energy charge" of the cell.

### Efficiency of Metabolic Processes

Food is processed by the digestive system into sugar glucose that provides the predominant source of energy in animal cells. The digestive system is very efficient at processing food into raw chemicals. Total tract energy digestibility (capability for processing the food nutrients) will depend on the kind of food ingested, and is estimated to range from 92% (High-Fibre diet) to 96% (Pectin/Low-Fibre diet) [10]. The energy absorbed by the digestive tract is referenced as Metabolizable Energy (ME).

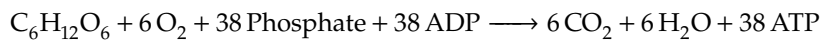
Energy has to be spent to digest these nutrients, and the amount of the required energy will depend on the food type being digested. It is estimated that proteins require the equivalent of 20–30% of their energy for digestion processes, carbohydrates 5–10% and fats 0–3% [11].

Glucose enters the body circulatory system and is then combined with oxygen to yield several key chemical reactions essential for sustaining life processes in our body. The reaction of this substance with oxygen in standard condition yields:



with the reaction energy  $\Delta G^0 = -686 \text{ kcal/mol}$

In living cells, the above reaction is coupled to the synthesis of ATP from ADP in the resulting reaction



This is the most efficient reaction in biologic cells, producing  $N = 38$  ATP molecules, and instead more modest ATP yields estimated around  $N = 29 - 30$  are obtained due to different loss factors [12].

The above reaction is exothermic and some energy is lost as heat. We can determine the efficiency of ATP production by comparing the ratio of the energy that was spent to create ATP by the energy released by the simple reaction where glucose is simply "burned" to yield waste heat and combustion products:

$$\eta = \frac{N \times -E_{\text{ATP}}}{E_{\text{react}}}$$

where  $N$  is the number of ATP molecules formed,  $E_{\text{react}}$  is the free energy from the oxidation of glucose into water and carbon dioxide, and  $E_{\text{ATP}}$  is the free energy when ADP and phosphate reacts to form ATP (ADP +

phosphate  $\longrightarrow$  ATP). The reaction energy for this latter endothermic reaction is, under standard conditions:  $\Delta G^{0'} = 7.3 \text{ kcal/mol}$ .

The efficiency may then be rewritten as:

$$\eta = N \times \frac{-E_{\text{ATP}}}{E_{\text{react}}} = N \times \frac{7.3}{686} = 0.0106 \times N$$

Therefore, the maximum conversion efficiency of glucose into ATP is 40.4% at maximum (with  $N = 38$ ) or 31.4% in normal conditions (with  $N = 29 - 30$ ).

Finally, the efficiency of ATP reactions converting chemical potential energy into work is around 20-25% with the rest of the energy being lost as waste heat [13].

Compounding all those efficiencies, and accounting that only 20% of our daily energy is used to sustain physical activity, yields a meagre efficiency of about 1.5% into converting the chemical potential energy of food into actual work, which is lower than the typical efficiency of a car engine (about 20–25%). However this is an over-simplistic view since the human body is a very sophisticated machine capable of simultaneously performing several high-level functions, which a car is incapable of performing. For example, a human performing manual work will also be performing high-level cognitive activities which require their own energy too. Furthermore, one may also need to take into consideration that the waste heat of the chain of thermodynamic processes examined here is not really wasted since it will contribute to compensate the heat losses of our body through conduction, convection and radiation, allowing the body temperature to be maintained.

In short, 3.7 billions of years of life evolution have yielded humans as a very sophisticated machine with a remarkable versatility and extremely efficient thermodynamic energy conversion processes, with a level of cognitive performance sufficiently high to design and manufacture machines that offer even better efficiencies and versatility at performing several tasks. That these dedicated machines will be more efficient than humans at performing their assigned tasks is hardly surprising: If they would be less efficient than the human body, they wouldn't have been invented in the first place.

### Thermodynamics and Life

This general topic of nonequilibrium thermodynamics and life, which is beyond the scope of this booklet, is a very rich subset of the discipline of thermodynamics, with its own subset of fundamental laws. These have been formulated by Bauer [14] and Dobroborsky [15]:

- *“All and only living systems are never in equilibrium and perform constant work at the expense of their free energy against the equilibrium required by the laws of physics and chemistry under existing external conditions”*

- “The stability of the nonequilibrium thermodynamic state of biological systems is ensured by the continuous alternation of phases of energy consumption and release through controlled reactions of synthesis and cleavage of ATP”

### F.3.2 Measuring Activity Metabolism

Activity metabolism refers to the energy expenditure during various activities, including both daily routines and sports activities. The energy expenditure for different activities is measured in kilojoules per hour (kJ/h) or expressed as a multiple of the Basal Metabolic Rate (e.g., 100% indicates the same energy expenditure as BMR).

A “Compendium of Physical Activities” has been created by a team of specialists to provide a standardized way of measuring and classifying physical activities, allowing the assessment of the energy expenditures associated with different activities [16].

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**Table F.2:** Energy expenditure of common physical activities, curated from the “Compendium of Physical Activities”, 2024 edition

Activity	Code	MET			70kg person W
		kcal/kg/h		W/kg	
Sitting, watching TV	7020	1		1.16	81
Sitting, computer work	11 582	1.3		1.51	106
Sitting, playing video game	22 120	1.5		1.74	122
Sitting in class	9065	1.8		2.09	146
Automobile driving	16 010	2		2.32	163
Yoga	2175	2.3		2.67	187
Sex	14 020	3		3.49	244
Cleaning	5010	3.3		3.84	268
Dancing	3070	3.8		4.42	309
Walking, 5km/h	16 060	3.5		4.07	285
Descending stairs	17 070	3.5		4.07	285
Climbing stairs	17 131	6.8		7.90	553
Surfing, Bodyboard	18 220	3		3.49	244
Bowling	15 090	3		3.49	244
Volleyball	15 710	4		4.65	325
Canoeing	18 020	4		4.65	325
Golf	15 255	4.5		5.23	366
Skateboarding	15 580	5		5.81	407
Horseback riding	15 370	5.5		6.39	447
Tennis, casual	15 675	6.8		7.90	553
Tennis, competitive	15 676	8		9.30	651
Basketball	15 055	7.5		8.72	610
Bicycling	1014	7		8.14	569
Soccer	15 610	7		8.14	569
Skiing	19 075	7		8.14	569
Swimming, casual	18 240	5.8		6.74	472
Swimming fast	18 230	9.8		11.39	797
Running, 9km/h	12 045	9		10.46	732
Judo	15 433	11.3		13.13	919
Handball	15 320	12		13.95	976

Source: Stephen D. Herrmann et al. “2024 Adult Compendium of Physical Activities: A Third Update of the Energy Costs of Human Activities”. In: *Journal of Sport and Health Science*, Vol. 13, No. 1 (2024), pp. 6–12.

**Table F.3:** Energy content of common foods (in Cal)

Apple (1)	72	Beans (130g)	120
Banana (1)	105	Cucumber (1)	30
Grapes (20)	69	Carrots (1)	25
Kiwi (1)	46	Garden salad (250ml)	47
Orange (1)	62	Olives (4)	23
Orange juice (125ml)	58	Olive oil (15ml)	121
Peach (1)	38	Oil and vinegar (15ml)	73
Strawberries (7)	27	Onions (60ml)	17
		Potato, baked (1)	161
		Rice (125ml)	109
		Tomatoes (1)	22
Boiled egg (1) large	78	Salmon (75g)	155
Cheese omelet (1)	273	Tuna, canned with oil (75g)	149
Cheese, Camembert (50g)	150	Chicken (75g)	153
Milk, skimmed (250ml)	88	Beef (75g)	144
Milk, whole (250ml)	155		
Yogourt, plain (175ml)	114	Cheesecake (1 part)	321
Bread, white (1slice)	93	Croissant (1)	231
Butter (5ml)	34	Chocolate and caramel bar (1)	245
		Vanilla Ice cream cone (1)	266
Chicken sandwich & vegetables (1)	335	Cola (250ml)	110
Pizza, cheese & pepperoni (1 slice)	219	Beer (1 bottle)	140
Spaghetti Bolognese (250ml)	401	Red Wine (125ml)	90
Hot-dog (1)	242	White Wine (125ml)	85
Cheeseburger (1)	319	Coffee (250ml)	3
Hamburger, double (1)	576	Tea (250ml)	3
Hamburger, single (1)	275		
French fries (20-25)	236		
Potato chips (1 small bag)	202		
Ketchup (15ml)	15		

Source: Institute of Medicine. “Dietary Reference Intakes for Energy, Carbohydrate, Fiber, Fat, Fatty Acids, Cholesterol, Protein, and Amino Acids”. Washington, DC: *The National Academies Press*, 2005.

## F.4 The ISO 8996 Standard for Metabolic Heat Production of Professional Activities

The ISO 8996 standard provides a standard for metabolic heat production of human activities. These values are then used to determine the heating rates of workplaces and the requirements for ventilation/air condition systems.

The range of metabolic rates (minimum, average, and maximum, in  $\text{W}/\text{m}^2$ ) are reported in Tables F.4 to F.6.

**Table F.4:** Metabolic rate for various occupations [ISO8996(2004)]

Sector	Professional occupation	Metabolic rate ( $\text{W}/\text{m}^2$ )	
		Min	Max
Mining industry	Haulage operator	70	85
	Coal hewer	110	110
	Cokeoven worker	115	175
Iron and steel industry	Blast furnace worker	170	220
	Electric furnace worker	125	145
	Hand moulder	140	240
	Machine moulder	105	165
	Foundry man	140	240
Iron and metal-working industry	Smith	90	200
	Welder	75	125
	Turner	75	125
	Drilling machine operator	80	140
	Precision mechanic	70	110
Agriculture	Gardener	115	190
	Tractor driver	85	110
Craftsmen	Bricklayer	110	160
	Carpenter	110	175
	Glazier	90	125
	Painter	100	130
	Baker	110	140
	Butcher	105	140
	Clock and watch repairer	55	70
Traffic	Car driver	70	100
	Bus driver	75	125
	Tramway driver	80	115
	Crane operator	65	145
	Sedentary work	55	65
Office work	Clerical work	65	100
	Janitor	80	115
	Hand compositor	70	95
Graphic occupations	Book-binder	75	100
	Laboratory assistant	85	100
Various occupations	Teacher	85	100
	Shop assistant	100	120
	Secretary	70	85

Source: ISO 8996; "Ergonomics of the Thermal Environment–Determination of Metabolic Rate". 2nd ed. ISO: Geneva, Switzerland, 2004.

**Table F.5:** Classification of metabolic rate by category [ISO8996(2004)]

Class		Metabolic rate (W/m <sup>2</sup> )			
		Min.	Avg.	Max.	
0	Resting	55	65	70	Resting, sitting at ease
1	Low metabolic rate	70	100	130	Light manual work (writing, typing, drawing, sewing, book-keeping); hand and arm work (small bench tools, inspection); assembly or sorting of light materials; arm and leg work (driving vehicle in normal conditions, operating foot switch or pedal).
2	Moderate metabolic rate	130	165	200	Standing drilling (small parts); milling machine (small parts); coil winding; small armature winding; machining with low power tools; casual walking (speed up to 2.5 km/h). Sustained hand and arm work (hammering in nails, filing); arm and leg work (off-road operation of lorries, tractors or construction equipment); arm and trunk work (work with pneumatic hammer, tractor assembly, plastering, intermittent handling of moderately heavy material, weeding, hoeing, picking fruits or vegetables, pushing or pulling lightweight carts or wheelbarrows, walking at a speed of 2.5 km/h to 5.5 km/h, forging).
3	High metabolic rate	200	230	260	Intense arm and trunk work; carrying heavy material; shovelling; sledgehammer work; sawing; planning or chiselling hard wood; hand mowing; digging; walking at a speed of 5.5 km/h to 7 km/h. Pushing or pulling heavily loaded hand carts or wheelbarrows; chipping castings; concrete block laying.
4	Very High metabolic rate	260	290	++	Very intense activity at fast to maximum pace; working with an axe; intense shovelling or digging; climbing stairs, ramp or ladder; walking quickly with small steps; running; walking at a speed greater than 7 km/h.

Source: ISO 8996; "Ergonomics of the Thermal Environment—Determination of Metabolic Rate". 2nd ed. ISO: Geneva, Switzerland, 2004.

**Table F.6:** Metabolic rate for specific activities [ISO8996(2004)]

Activity			Metabolic rate (W/m <sup>2</sup> )
Sleeping			40
Reclining			45
At rest, sitting			55
At rest, standing			70
Walking on the level, even path, solid	1. without load	at 2 km/h	110
		at 3 km/h	140
		at 4 km/h	165
		at 5 km/h	200
		10 kg, 4 km/h	185
Walking uphill, even path, solid	2. with load	30 kg, 4 km/h	250
		5° inclination, 4 km/h	180
		15° inclination, 3 km/h	210
		25° inclination, 3 km/h	300
		15° inclination, 4 km/h	270
Walking downhill at	2. with a 20 kg load	25° inclination, 4 km/h	410
		5 km/h, without load	135
		5° inclination	140
Ladder at 70°, climbing at a rate of 11.2 m/min	5 km/h, without load	15° inclination	180
		25° inclination	290
		without load	360
Pushing or pulling a tip-wagon, 3.6 km/h, even path, solid	with a 20 kg load	pushing force: 12 kg	290
		pulling force: 16 kg	375
Pushing a wheelbarrow, even path, 4.5 km/h, rubber tyres, 100 kg load			230
Filing iron			100
Work with a hammer, 2 hands, mass of the hammer 4.4 kg, 15 strokes/min			190
Carpentry work hand sawing			290
machine sawing			220
hand planing			100
Brick-laying, 5 bricks/min			300
Screw driving			170
Digging a trench			100
Sedentary activity (office, dwelling, school, laboratory)			290
Standing, light activity (shopping, laboratory, light industry)			70
Standing, medium activity (shop assistant, domestic work, machine work)			95
Work on a machine tool			115
Work with a hand tool	Light (adjusting, assembling)	Medium (loading)	100
		Heavy	140
		Light (light polishing)	210
		Medium (polishing)	100
		Heavy (heavy drilling)	160
			230

Source: ISO 8996; "Ergonomics of the Thermal Environment—Determination of Metabolic Rate". 2nd ed. ISO: Geneva, Switzerland, 2004.