Quasi-Synchronous Orbits and Preliminary Mission Analysis for Phobos Observation and Access Orbits

Paulo J. S. Gil

Instituto Superior Técnico

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Outline

- Introduction
- Distant Satellite Orbits
- Quasi-Satellite Orbits around Phobos
- Phobos Mission Analysis Issues
- Future Work
Introduction – Target: Phobos

- Mars 1st and largest Moon
- Orbital major axis $a = 9377 \text{ km}$, with sidereal period $T = 0.32 \text{ d}$
- Almost, but not exactly, circular equatorial orbit $e = 0.0151$, $i \sim 1^\circ$
- Small, $m \sim 10^{16}$, irregular shape
  - Ellipsoidal shape with mean radius $11.32 \text{ km}$; huge crater: Stickney
  - Very small gravity at surface $g \sim 10^{-3} \text{ m/s}^2$
- Tidally locked to Mars
- Particularly interesting for a sample return mission
  - Possibly a captured asteroid
  - Studies of the minor bodies of the solar system

http://www.esa.int/SPECIALS/Mars_Express/SEM21TVJD1E_0.html
Missions to Phobos

■ Past Missions
  ▪ Phobos 1 & 2 launched in 1988 by Soviet Union - Failed

■ Future Missions
  ▪ Phobos Grunt - Sample Return Mission to Phobos
    ◦ To be launched in 2012?
  ▪ Future ESA mission?

■ Challenges when approaching Phobos
  ▪ Phobos: small mass… – impossible to orbit it a keplerian way
    ◦ There is a need to orbit it somehow
  ▪ …but not negligible – orbit not Martian
  ▪ Irregular gravitational field
  ▪ Ephemeris not well known
Challenge: Force Field at Phobos and 3BP

- In the case of a larger body e.g. the Moon, there is no problem orbiting it
  - The region of influence is sufficiently large to allow keplerian-type orbits, where the Earth is a small perturbation
- The Hill sphere, where the Lagrange points are located, is large enough
- Problems appear when the “moon” is smaller and smaller – the Hill’s problem, when the mass ratio of the primaries goes to zero in a certain way

Case of Moon

Hill’s sphere
Region of influence

0

0.5

-0.5

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Challenge: Force Field at Phobos and 3BP

- Ellipsoidal model of Phobos
  - The thinnest axis is represented by the solid line

- Hill’s sphere just above Phobos (outer dashed line)

\[ r_H = \left( \frac{\mu_{Ph}}{3\mu_M} \right)^{1/3} \approx 16.6 \text{ km} \]

- Region of influence below the surface (inner dashed line): usual orbits impossible

\[ r_{\text{inf}} = \left( \frac{\mu_{Ph}}{\mu_M} \right)^{2/5} \approx 7.2 \text{ km} \]

- But mass is not negligible...

How to orbit Phobos?

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Equilibrium of Forces

- Delicate equilibrium of forces
- Mars and centrifugal force tend to cancel
- Gravitational field of Phobos plays a role
- Eccentricity is very important; cannot be discarded
- Phobos J2 and other higher order terms are important at small distances

Orbit is not completely determined by Mars
Families of distant orbits in the 3BP, stable or quasi-stable

- Tadpole orbits (elongated shapes around $L_{4,5}$)...

- ...Horseshoe orbits (light blue)...

- ... and Quasi-satellite (or quasi-synchronous) orbits (next slide)
Relatively extensive literature about QSO orbits

- Root on a problem stated by Hill
- Stability, movement of the guiding center, small values of $\mu$, problem in 2D

All figures from Benest (1976)
Quasi-Synchronous Orbits (QSO)

- Quasi-stable orbits around Phobos also called Quasi-Satellite Orbits
- Appear in the context of the 3BP, existing beyond the region of influence of the $M_2$
- Motion is dominated by Mars gravity but the gravitational field of Phobos plays a role
- (quasi) stable orbits circumventing Phobos, observation and preparation for landing becomes possible

QSO in the synodic or LVLH ref. frame
Quasi-Synchronous Orbits (QSO)

- QSO and 2-body orbits (neglecting Phobos attraction) are not too different.

- In reality:
  - Phobos has to be taken into account.
  - QSO must be used to address the problem.

- QSO are more stable due to the restoring force of Phobos.
QSO and Mars Orbits

- QSO are still orbits around Mars
- Almost indistinguishable from “normal” orbits
- How orbital elements vary?
Variation of Orbital Elements of QSO I

Example - QSO1 and an orbit with no Phobos gravity but same initial conditions; differences between both are striking.
Variation of Orbital Elements of QSO II

Eccentricity

S/C with Phobos (green)
S/C without Phobos (blue)
Phobos (red)

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Variation of Orbital Elements of QSO III

Orbital inclination and longitude of ascending node present practically no variation; the same is not true for Arg. Perimartem:

![Graph showing variation of Argument of Perimartem over time](image)

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Historical Developments

- Henon, Benest (1970’s) – identification of QSO in the context of the 3BP
- Kogan (1987,1990), others - First order perturbation methods and averaging techniques; Constants of motion of approximate equations
  - Lie perturbation method applied to the study of QSO (very complicated)
- Difficulties and Limitations (Lie method)
  - Relative order of magnitude of the several parameters ($\mu, e$, etc.) appearing in the problem is huge, making the theory of restrictive application – in particular in the case of Phobos; higher order gravity terms not considered.
- Wiesel (1993) - 2D model including eccentricity, Mars oblateness, ellipsoidal model for Phobos,
  - Zero eccentricity model as stepping stone to the more complex case – from periodic orbits when $e = 0$ at any distance from Phobos to resonant orbits and non-periodic orbits in the $e \neq 0$ case
  - Floquet theory used after a periodic orbit has been found to determine the Poincaré exponents
  - Numerical exploration of the phase space
Geometry description of QSOs

- Kogan (1987, 1990); approximate solution in terms of parameters

Fig: 3D geometry of the problem, with its natural parameters (from Kogan, 1990)
Wiesel’s approach

- 25 day integrations
- Assessment of “mortality rate” of quasi-orbits with successive longer integrations
- $V_y$ must be controlled to within a fraction of a m/s to establish stable orbits
- No assessment of $V_x$ in this work

Resonant periodic orbits ($e\neq 0$)

Fig. from Wiesel, 1993
Phobos ‘Grunt’ Approach

- Tuchin et al, Akim et al, 2002+
- Planar elliptical 3BP, no J2
- Case $e = 0, \mu = 0$ used for insight and zero order solution
- Linearized equations for analytical simplified solution
  - Const. of solution from const. Motion of model and initial conditions
  - Phase space scan – general behavior of QSO
- Chosen solutions checked against full numerical model
- Semi-numerical approach seems the best for solving practical problems

\[
\frac{d^2\hat{\xi}}{d\phi^2} = 2 \frac{d\hat{\eta}}{d\phi} + (3\rho - k\rho)\hat{\xi}
\]

\[
\frac{d^2\hat{\eta}}{d\phi^2} = -2 \frac{d\hat{\xi}}{d\phi} - \rho k\hat{\eta}
\]

Linear equations if \( \rho \) considered const.

QSO classification scheme
Objectives

- Acquire capabilities and experience in QSO for the possibility of a future ESA sample return mission to Phobos
- Knowledge and experience in dealing with QSO
- Mission design capabilities in problems involving QSO; applications to Phobos and possibly other minor bodies
- Search for better method to describe QSOs (e.g. 3D case) and search for enough stable solutions for practical problems ($e \neq 0$)
- First step: full numerical simulations of QSO around Phobos; assessment of how to search for (quasi-)stable solutions; mission design issues
2D QSO quasi-stable solutions

- Stable = stable for at least 30 days
- Easy to generate stable QSO
- Points at equal times in fig:
- QSO easily obtained in the $x$-$y$ plane
  - $127\times72$, $103\times62$, $61\times44$, $42\times34$, ...
- Demonstration 3D QSO (next slides):
  - $(x,y,z) = (0,-100,40)$ [km]
  - $(v_x,v_y,v_z) = (-20,0,10)$ [m/s]
3 day simulation

x-y view of a test QSO with 
\[(x, y, z) = (0, -100, 40) \text{ [km]}, (v_x, v_y, v_z) = (-20, 0, 10) \text{ [m/s]}\]

y-z view of a test QSO with 
\[(x, y, z) = (0, -100, 40) \text{ [km]}, (v_x, v_y, v_z) = (-20, 0, 10) \text{ [m/s]}\]
3 day simulation (cont’d)

x-z view of a test QSO with 
\((x, y, z) = (0, -100, 40) \text{ [km]}; (v_x, v_y, v_z) = (-20, 0, 10) \text{ [m/s]}\)

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30 day simulation

\[
x, y, z \text{ view of a test QSO with} \\
(x, y, z) = (0, -100, 40) \text{ [km]}, \ (v_x, v_y, v_z) = (20, 0, 10) \text{ [m/s]}
\]

\[
y, z \text{ view of a test QSO with} \\
(x, y, z) = (0, -100, 40) \text{ [km]}, \ (v_x, v_y, v_z) = (20, 0, 10) \text{ [m/s]}
\]

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30 day simulation (cont’d)

\( (x,y,z) = (0,-100,40) \) [km], \( (v_x,v_y,v_z) = (-20,0,10) \) [m/s]

\( x\text{-}z \) view of a test QSO with

\( (x,y,z) = (0,-100,40) \) [km], \( (v_x,v_y,v_z) = (-20,0,10) \) [m/s]

\( x\text{-}y\text{-}z \) view of a test QSO with

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Preliminary phase space exploration

- Assess and test the search for stable QSO;
- Point \((x,y,z) = (0,-100,0)\) [km] seems much more forgiving than the x axis chosen by Wiesel
- Search for stable QSO varying the velocity components
- Start with a broad search and fine tune latter
- \((v_x,v_y,v_z) = (10i,j,10k),\) \(i,j,k = -4,...,4\) [m/s] – a 1st broad exploration

Starting point
1day simulation

- Variation in $v_z$, then $v_y$ and then $v_x$ with max and min altitudes
1day simulation

- Order of variation of velocities gives information: $v_y$ most crucial
Another order of variation of velocities
1day simulation

- Less higher “frequencies” are easier to analyze since variation is slower
1 day simulation - detail

- Analysis to prepare a refinement
7 day simulation

- Extending the simulation more and more instabilities grow and less QSO remain stable

Variation in vel. at (0,-100km,0) - 7 day simulation
30 day simulation

- Extending the simulation more and more instabilities grow and less QSO remain stable
- 30 days provide a good margin for correcting trajectories
30 day simulation

- Detail
Refinement of Simulations

- \( \mathbf{V} = (-36+2i, 0.1j, 2k), i=[0,8], j=[-10,5], k=[-4,4]; \) \( v_y \) is the most critical parameter; must be within an interval of \(~1\ m/s\)
Detail

Variation in vel. at (0,-100km,0) - 30 day simulation

Max. and Min. Altitudes [km]

Vx, Vy, 10*Vz [m/s]

Simul. #
Another Simulation Example

- Other types of simulations possible e.g. Variation of $|v|$, Az, El at the initial point
- Example: variation of $y$ and $v_x$ with $(0,y,0), (v_x,0,0), y, v_x < 0$
- Stable $v_x$ negative since at the initial point $\vec{\omega} \times \vec{r} \approx -22.8$ m/s
- Include additional information about QSO
  - Average dimensions in x and y
  - QSO period, normalized by Phobos period of revolution
- Interesting features:
  - Relation between the period and velocity, more than with distance
  - Relation between max and min altitudes and axes of the QSO ellipse (distances)
  - Results need to be interpreted
Total of simulation
Detail – some interesting observations

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Mission Design Include

- Observation of Phobos surface and choice of QSO
- Illumination of observed surface
- Occurrence of eclipses and Earth occultation
- Insertion into a QSO for observation and approach of Phobos
  - Geometry
  - The best choice for minimizing possible insertion errors
- Trajectory for landing
Eclipse and Occultation

- Using the QSO example
Ground Track

- Motion relative to the surface: cf. QSO initial cond. (rel. inc.)

Phobos Longitude-Latitude coverage of a test QSO with 
\((x, y, z) = (0, -100, 40)\) [km], \((v_x, v_y, v_z) = (-20, 0, 10)\) [m/s]
Sun's elevation varies a lot... and eclipses can take time
Eclipse at ground track point can last for days

Eclipse for a long time; sun too low

Whole simulation
Open questions

- **QSO insertion strategy**
  - Phobos ephemeris not well known
  - Need for prior observation
  - Small delta-V but high precision required

  Fig. from Akim et al, 1993

- **Approach and Landing**
  - Avoid crash => small delta-V
  - Highly irregular gravity field
  - Velocity at surface from a QSO is several dozens of m/s

  Fig. 3. Transition from observation orbit on QSO

- **3D QSO**
  - Application to Saturn system and other minor bodies

Fig. from Akim et al, 1993

Fig. 3. Transition from observation orbit on QSO
Search Stability Using Fast Lyapunov Indicators

- Fast Lyapunov Indicator (FLI)
  - What: Chaoticity analysis technique
  - Who: Pioneer work by Froehlé, Lega & Gonczi (1997)
  - Objective: Distinguish regular from chaotic motion
  - Technique derived from Lyapunov Characteristics Exponents (LCEs)

FLI value has a different behavior for regular and chaotic motion.

![Graph showing FLI values for regular and chaotic trajectories over time](image-url)
FLI Maps

- FLI Maps distinguishes the stability islands (purple) from the chaotic sea (yellow/orange), Villac & Lara 2005

- Goal: generalization to the elliptical Case (in course, Gil & Cabral, 2011)
Summary

- QSOs can be used to orbit small bodies
- Lots of features and things to worry about
- Numerical application to the case of Phobos
- Mission design issues
- 3D QSOs can be interesting
  - Need for approximate solutions
  - New perturbation methods
- New techniques to
  - Search for stable QSOs in the cases with eccentricity
  - Approach and landing strategies
  - Special QSO for special purposes
Summary

- Still many open questions => Lots of further work
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