Quasi-Synchronous Orbits
and
Preliminary Mission Analysis for Phobos Observation and Access Orbits

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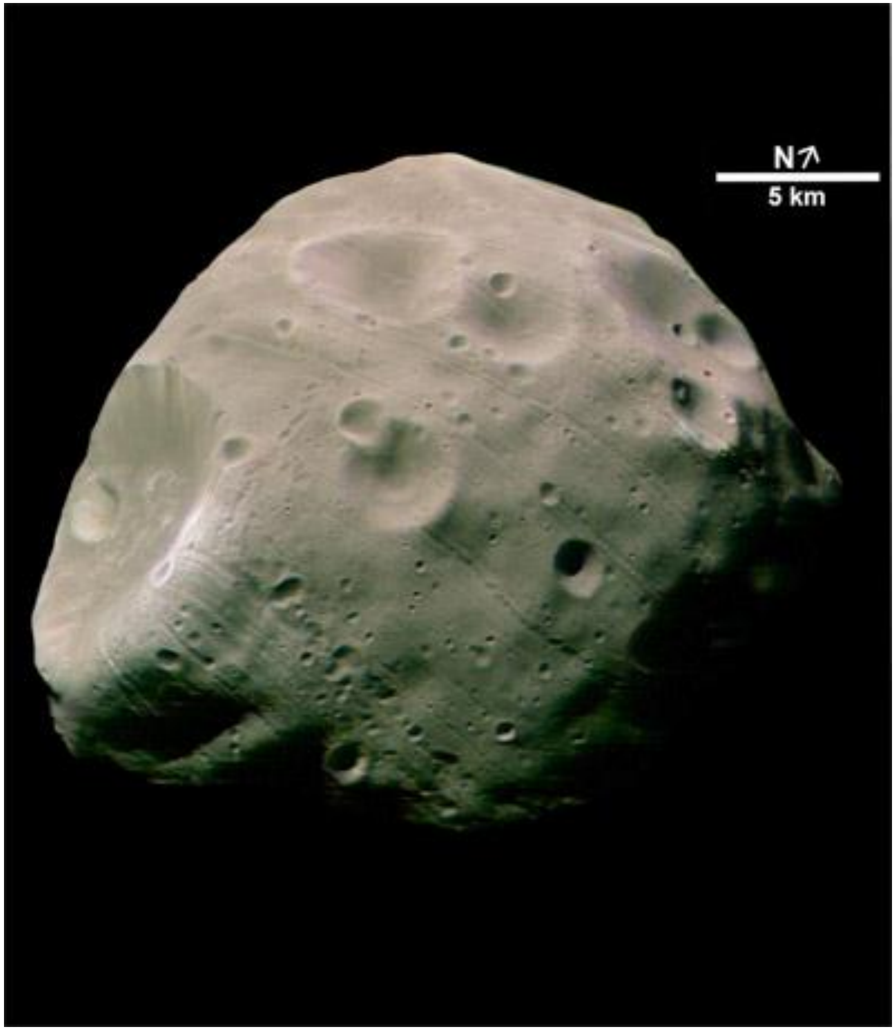
Simpósio Espaço “50 anos do 1º Voo Espacial Tripulado”
12 de Abril de 2011

Outline

- Introduction
- Distant Satellite Orbits
- Quasi-Satellite Orbits around Phobos
- Phobos Mission Analysis Issues
- Future Work

Introduction – Target: Phobos

- Mars 1st and largest Moon
- Orbital major axis $a = 9377$ km, with sidereal period $T = 0.32$ d
- Almost, but not exactly, circular equatorial orbit $e = 0.0151$, $i \sim 1^o$
- Small, $m \sim 10^{16}$, irregular shape
  - Ellipsoidal shape with mean radius 11.32 km; huge crater: Stickney
  - Very small gravity at surface $g \sim 10^{-3}$ m/s^2
- Tidally locked to Mars
- Particularly interesting for a sample return mission
  - Possibly a captured asteroid
  - Studies of the minor bodies of the solar system



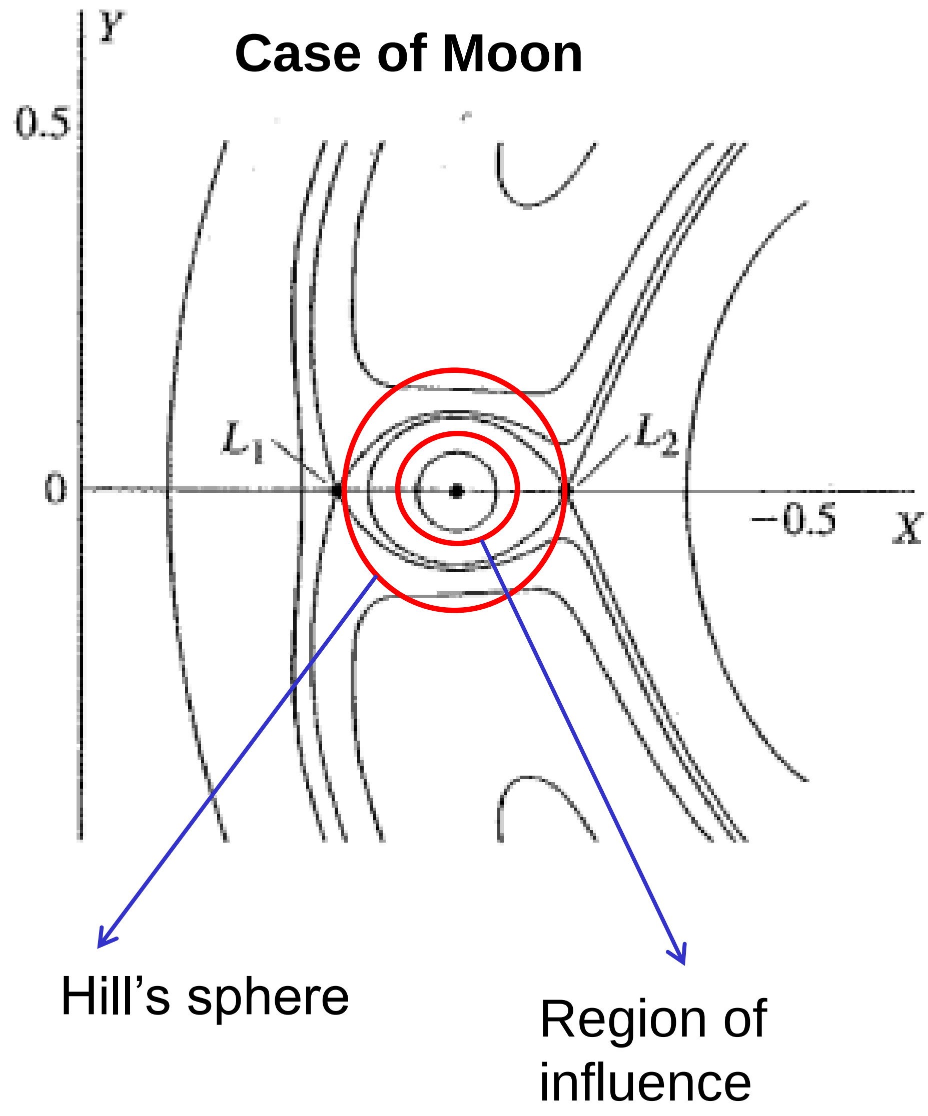
http://www.esa.int/SPECIALS/Mars_Express/SEM21TVJD1E_0.html

Missions to Phobos

- Past Missions
  - Phobos 1 & 2 launched in 1988 by Soviet Union - Failed
- Future Missions
  - Phobos Grunt - Sample Return Mission to Phobos
    - To be launched in 2012?
  - Future ESA mission?
- Challenges when approaching Phobos
  - Phobos: small mass… – impossible to orbit it a keplerian way
    - There is a need to orbit it somehow
  - …but not negligible – orbit not Martian
  - Irregular gravitational field
  - Ephemeris not well known

Challenge: Force Field at Phobos and 3BP

- In the case of a larger body e.g. the Moon, there is no problem orbiting it
  - The region of influence is sufficiently large to allow keplerian-type orbits, where the Earth is a small perturbation
- The Hill sphere, where the Lagrange points are located, is large enough
- Problems appear when the “moon” is smaller and smaller – the Hill’s problem, when the mass ratio of the primaries goes to zero in a certain way



Challenge: Force Field at Phobos and 3BP

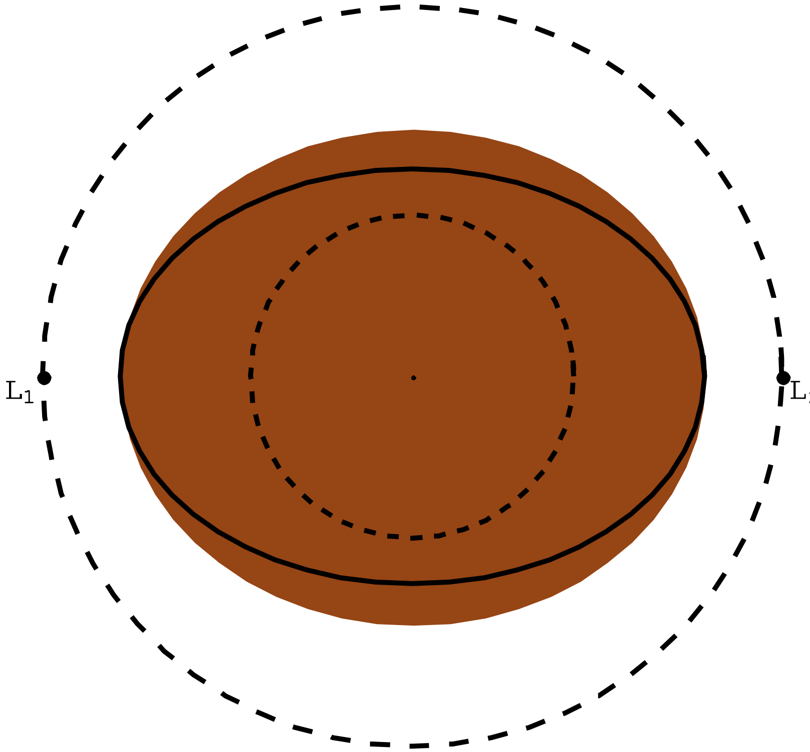
- Ellipsoidal model of Phobos
  - The thinnest axis is represented by the solid line
- Hill's sphere just above Phobos (outer dashed line)

$$r_H = \left(\frac{\mu_{Ph}}{3\mu_M} \right)^{1/3} \cong 16.6 \text{ km}$$

- Region of influence below the surface (inner dashed line): usual orbits impossible

$$r_{\inf} = \left(\frac{\mu_{Ph}}{\mu_M} \right)^{2/5} \cong 7.2 \text{ km}$$

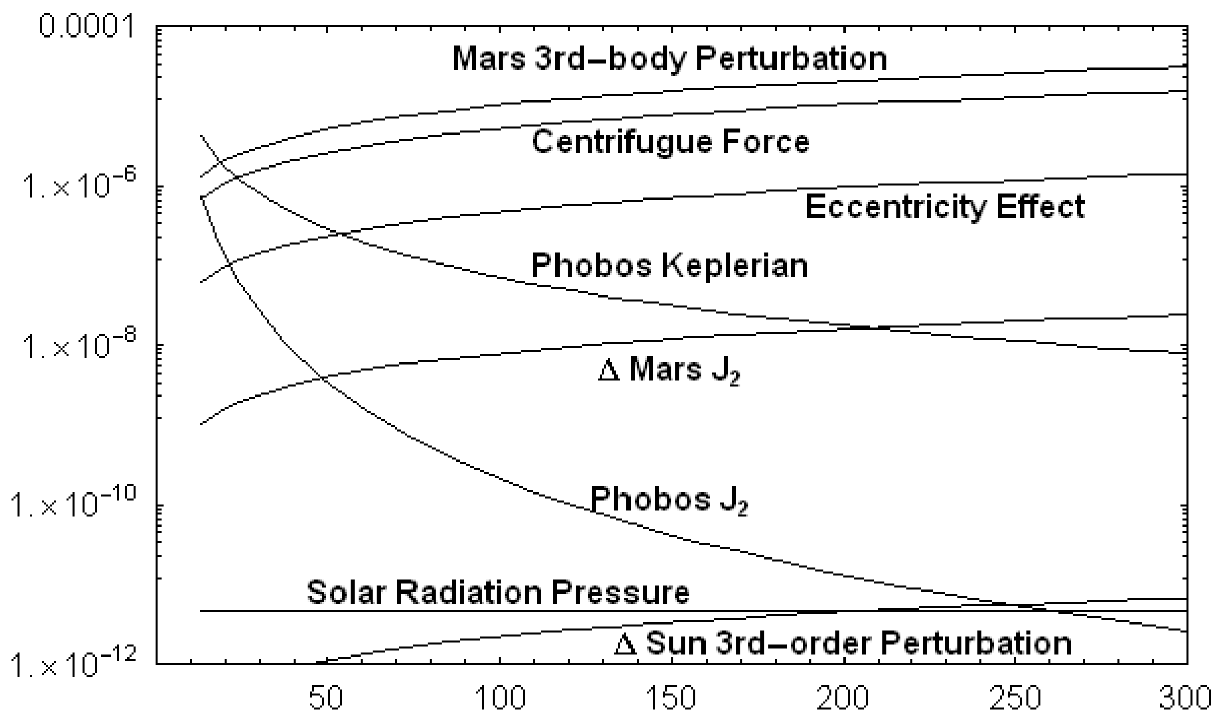
- But mass is not negligible...



How to orbit Phobos?

Equilibrium of Forces

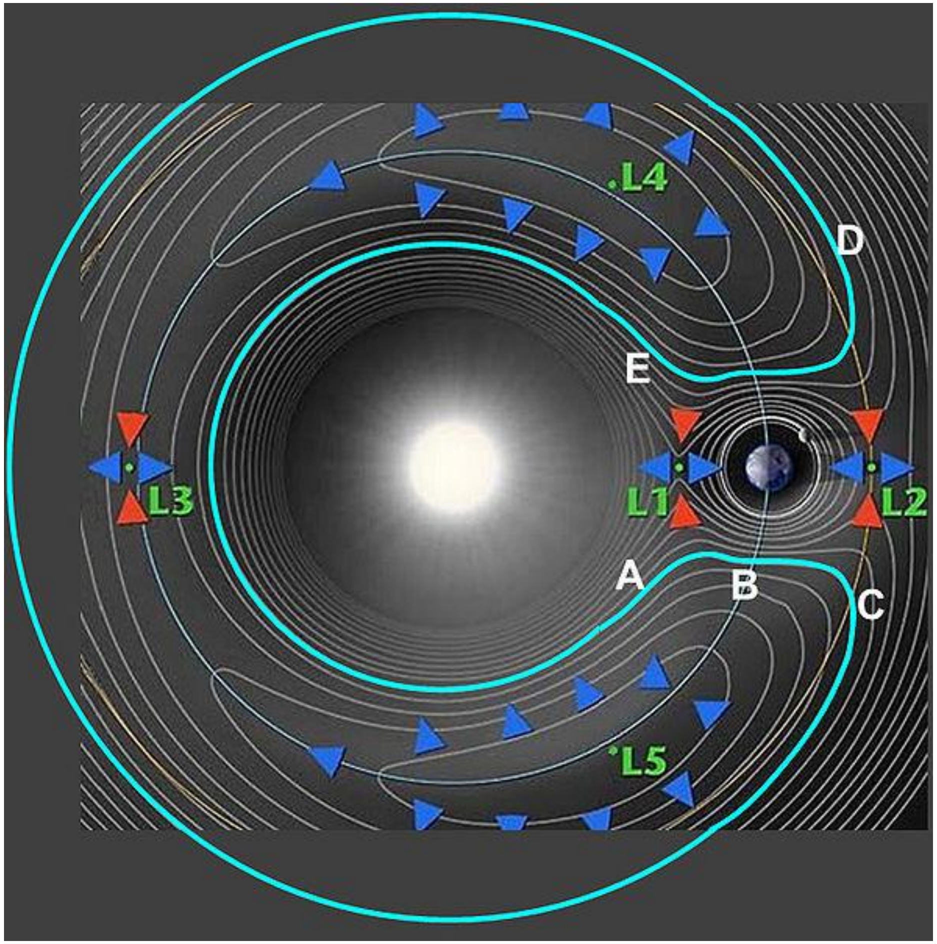
- Delicate equilibrium of forces
- Mars and centrifugal force tend to cancel
- Gravitational field of Phobos plays a role
- Eccentricity is very important; cannot be discarded
- Phobos J2 and other higher order terms are important at small distances



Orbit is not completely determined by Mars

Distant Satellite Orbits

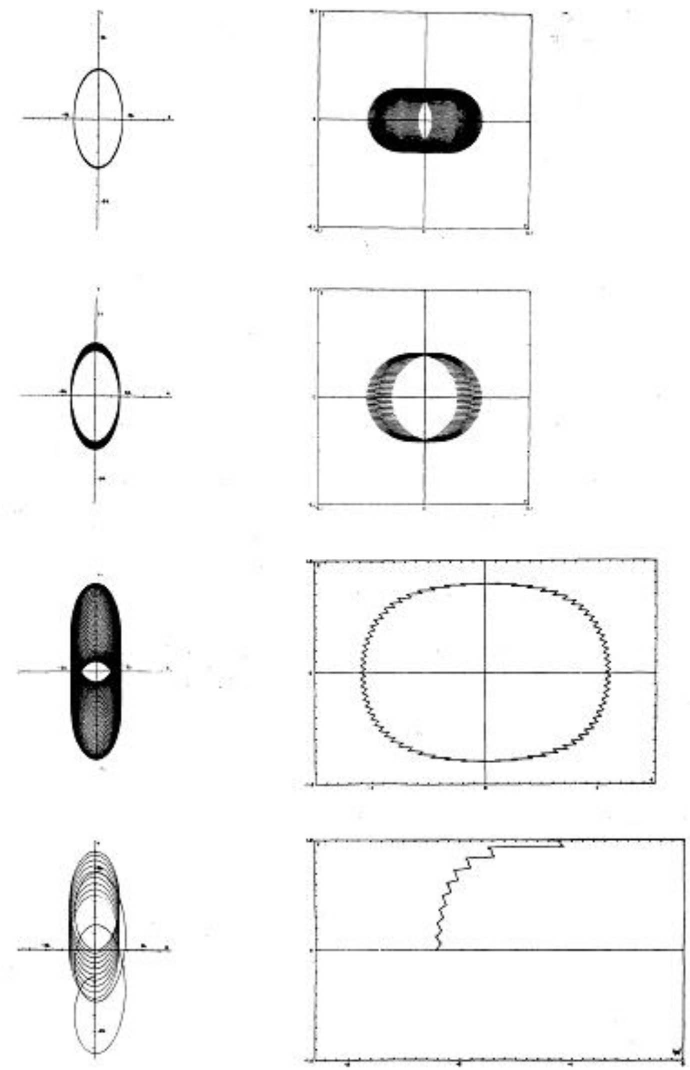
- Families of distant orbits in the 3BP, stable or quasi-stable
- Tadpole orbits (elongated shapes around $L_{4,5}$)…
- …Horseshoe orbits (light blue)…
- … and Quasi-satellite (or quasi-synchronous) orbits (next slide)

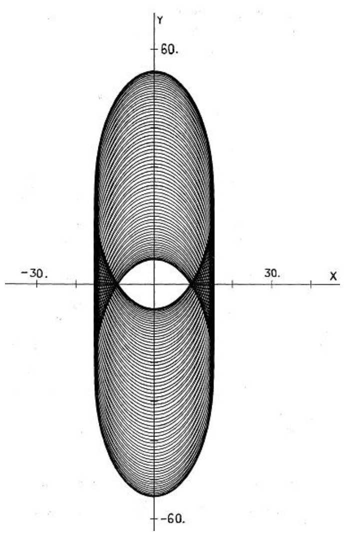


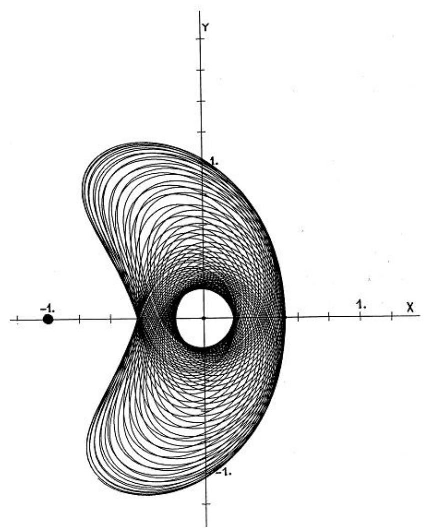
Quasi-Satellite Orbits

Relatively extensive literature about QSO orbits

- Root on a problem stated by Hill
- Studies in 1970's in the 3BP context: Hénon (1969), Benest (1974,1976)
- Stability, movement of the guiding center, small values of μ, problem in 2D



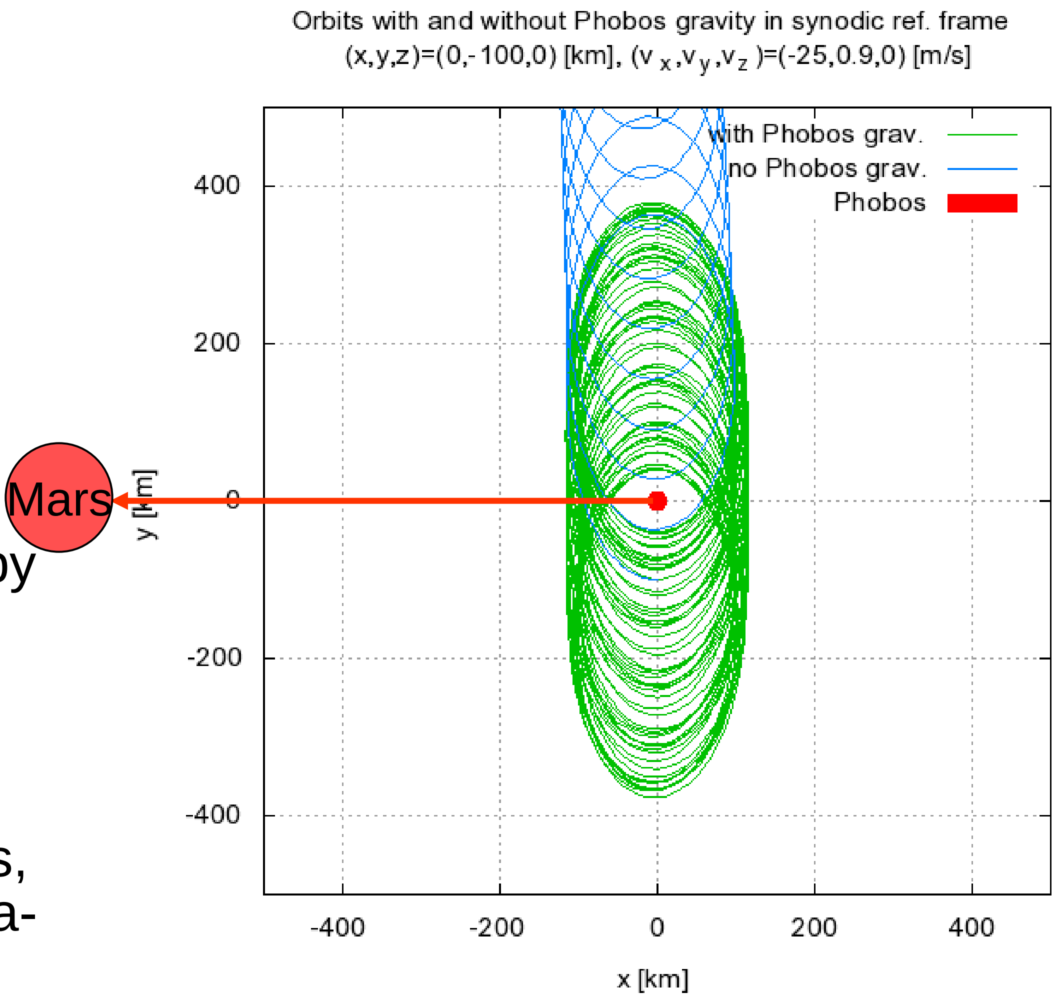




All figures from Benest (1976)

Quasi-Synchronous Orbits (QSO)

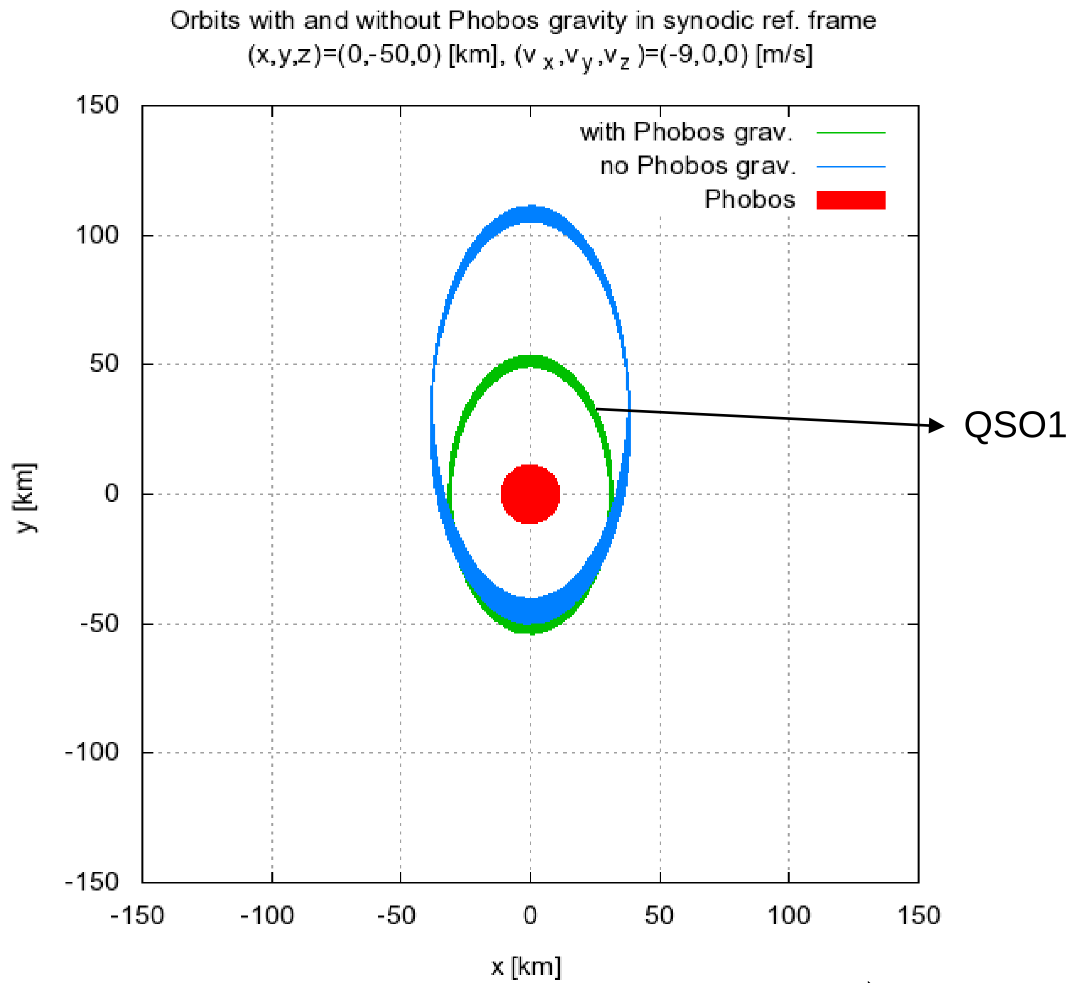
- Quasi-stable orbits around Phobos also called Quasi-Satellite Orbits
- Appear in the context of the 3BP, existing beyond the region of influence of the M_2
- Motion is dominated by Mars gravity but the gravitational field of Phobos plays a role
- (quasi) stable orbits circumventing Phobos, observation and preparation for landing becomes possible



QSO in the synodic or LVLH ref. frame

Quasi-Synchronous Orbits (QSO)

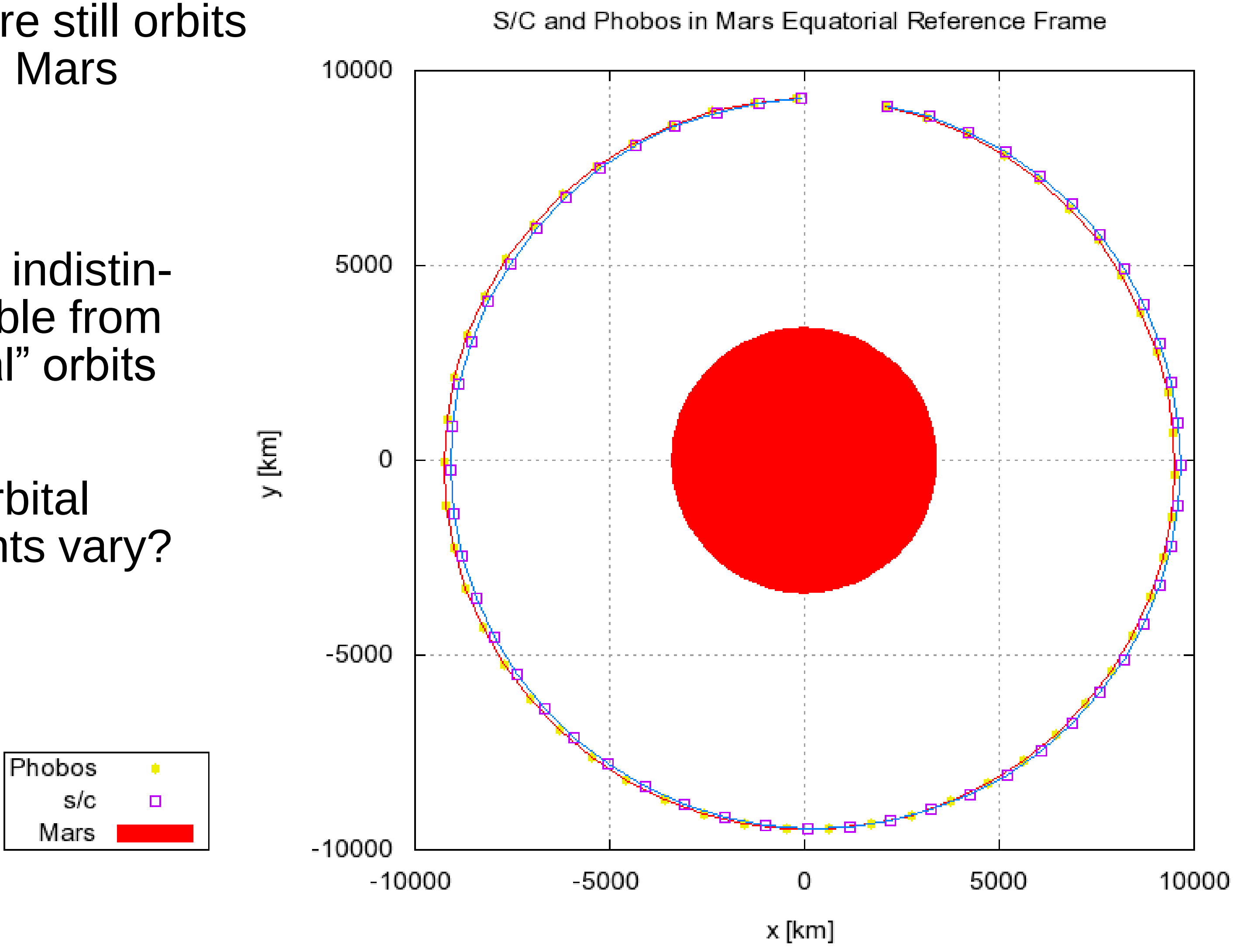
- QSO and 2-body orbits (neglecting Phobos attraction) are not too different
- In reality
  - Phobos has to be taken into account
  - QSO must be used to address the problem
- QSO are more stable due to the restoring force of Phobos



Other QSO ex

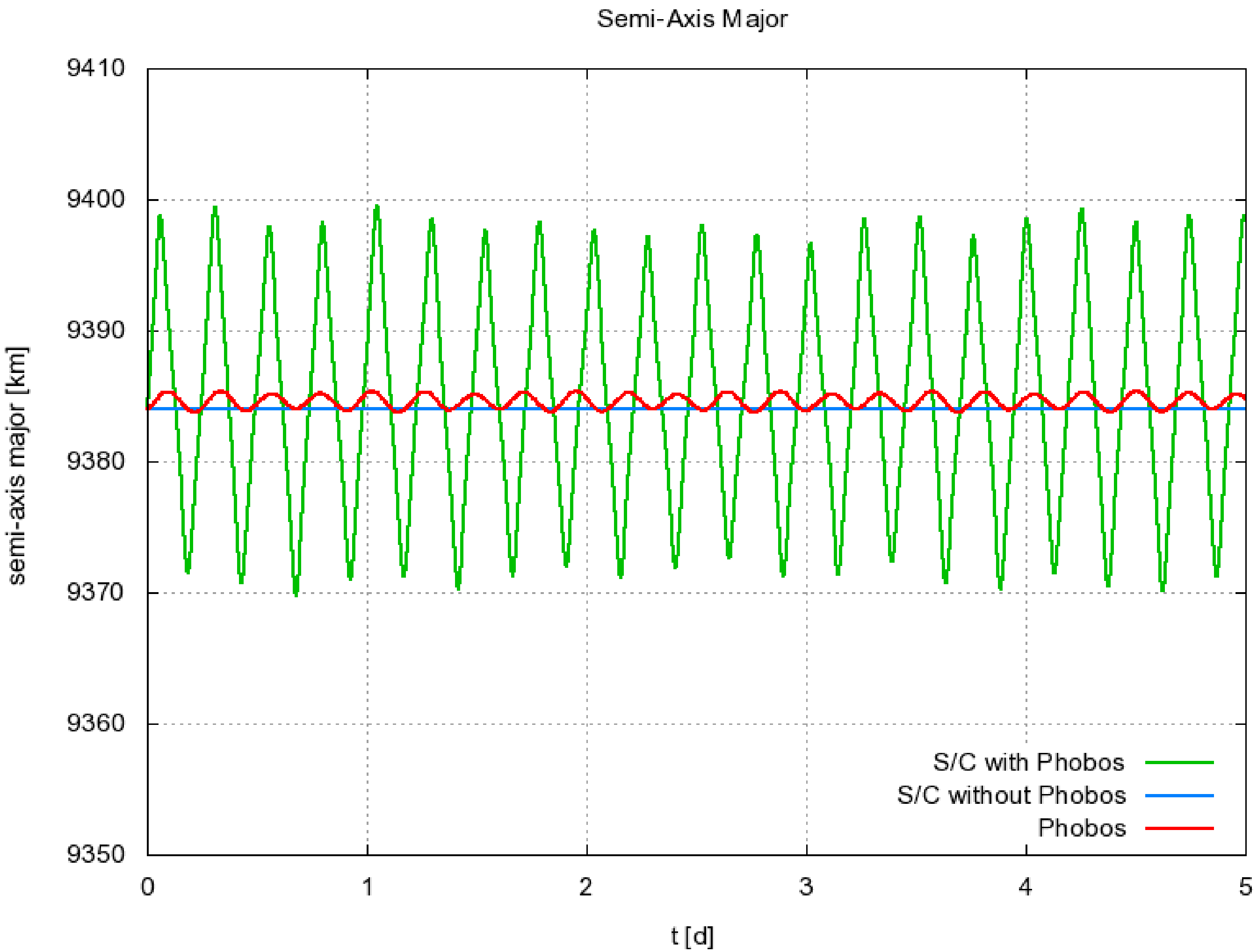
QSO and Mars Orbits

- QSO are still orbits around Mars
- Almost indistin-guishable from “normal” orbits
- How orbital elements vary?

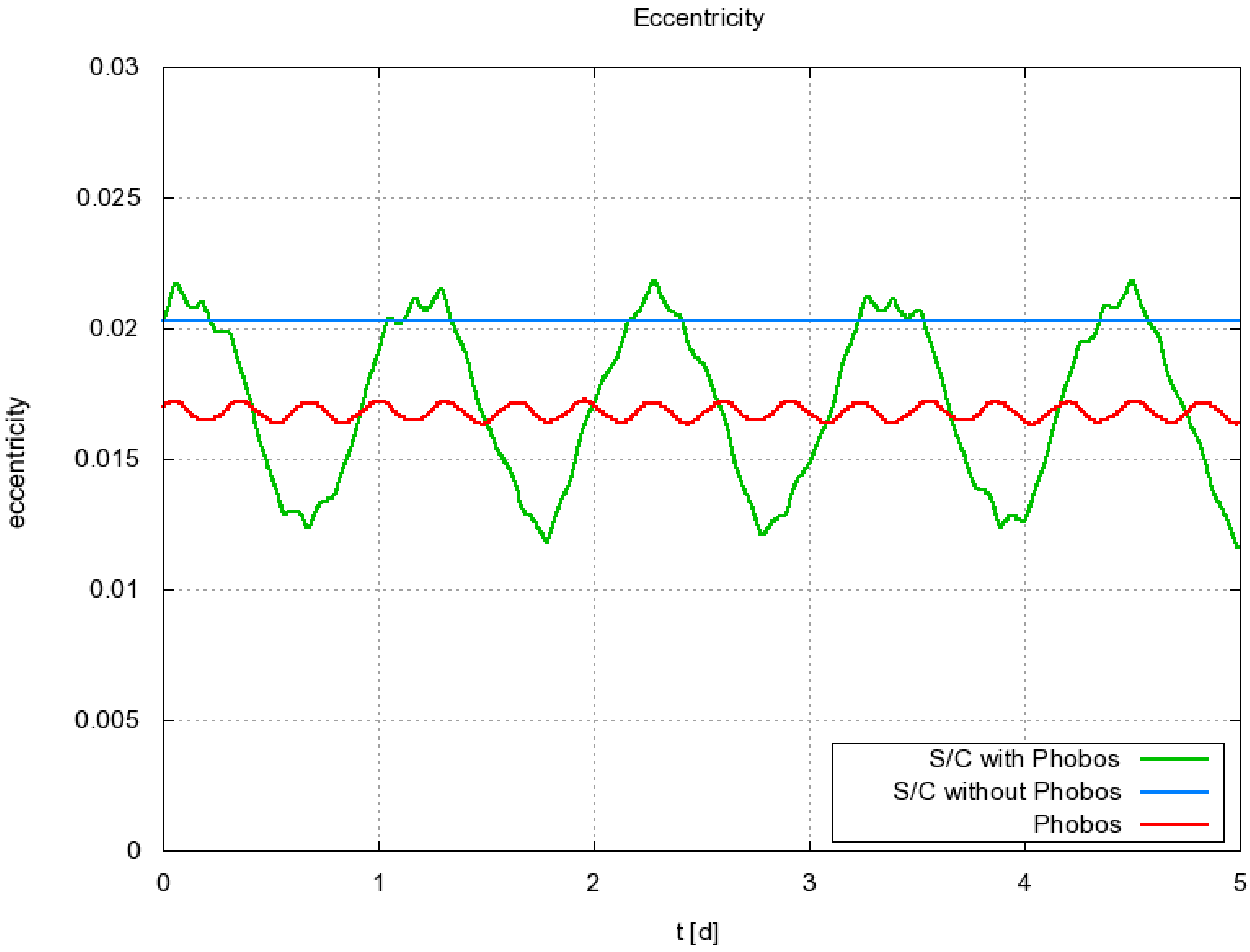


Variation of Orbital Elements of QSO I

Example - QSO1 and an orbit with no Phobos gravity but same initial conditions; differences between both are striking

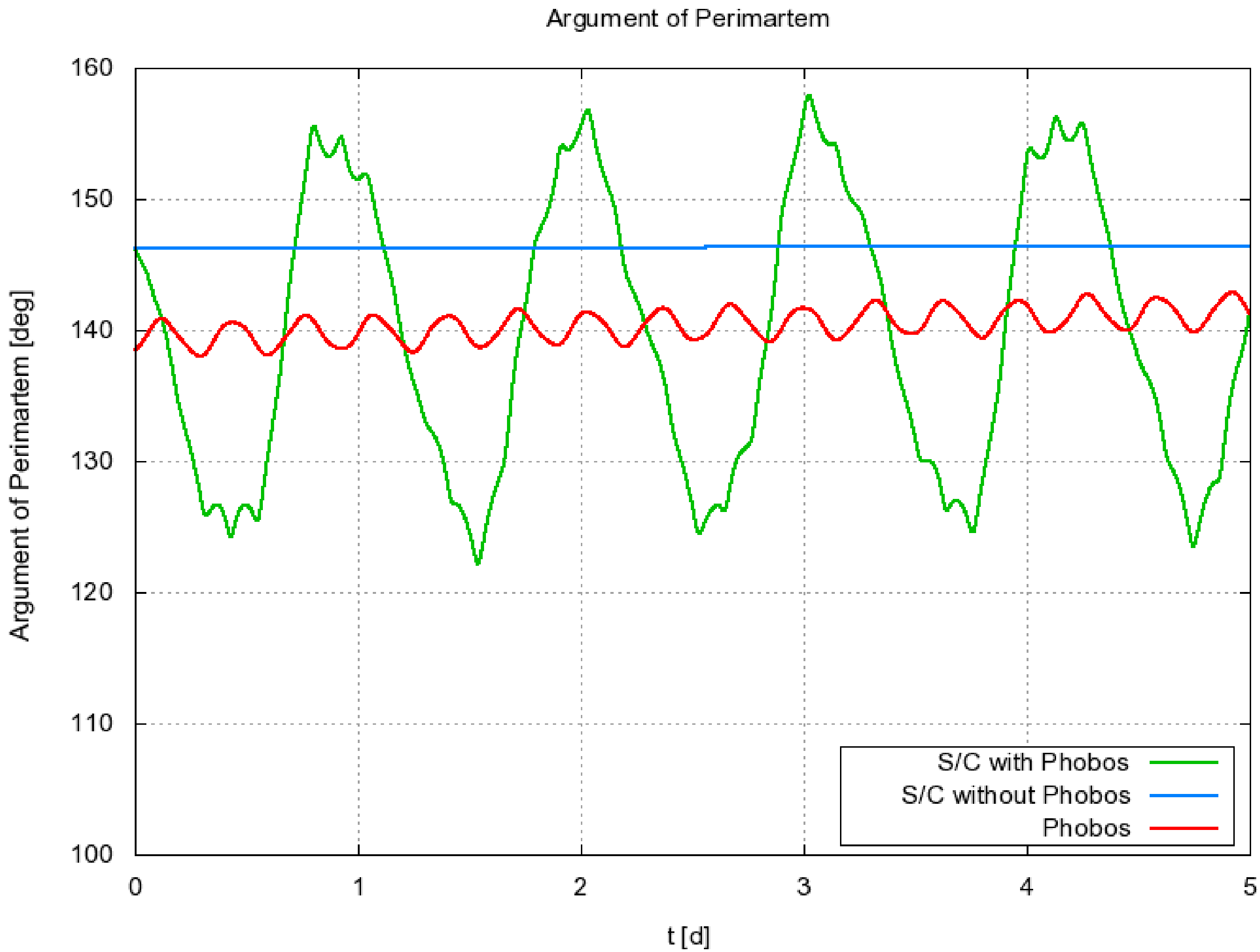


Variation of Orbital Elements of QSO II



Variation of Orbital Elements of QSO III

Orbital inclination and longitude of ascending node present practically no variation; the same is not true for Arg. Perimartem:



Historical Developments

- Henon, Benest (1970's) – identification of QSO in the context of the 3BP
- Kogan (1987,1990), others - First order perturbation methods and averaging techniques; Constants of motion of approximate equations
- Lidov & Vashkov'yak (1993,1994)
  - Lie perturbation method applied to the study of QSO (very complicated)
- Difficulties and Limitations (Lie method)
  - Relative order of magnitude of the several parameters (μ, e, etc.) appearing in the problem is huge, making the theory of restrictive application – in particular in the case of Phobos; higher order gravity terms not considered.
- Wiesel (1993) - 2D model including eccentricity, Mars oblateness, ellipsoidal model for Phobos,
  - Zero eccentricity model as stepping stone to the more complex case – from periodic orbits when e = 0 at any distance from Phobos to resonant orbits and non-periodic orbits in the e ≠ 0 case
  - Floquet theory used after a periodic orbit has been found to determine the Poincaré exponents
  - Numerical exploration of the phase space

Geometry description of QSOs

- Kogan (1987,1990); approximate solution in terms of parameters

Ellipse with semi-axes *a* and *2a* in the synodic *x* and *y* direction

Guiding center

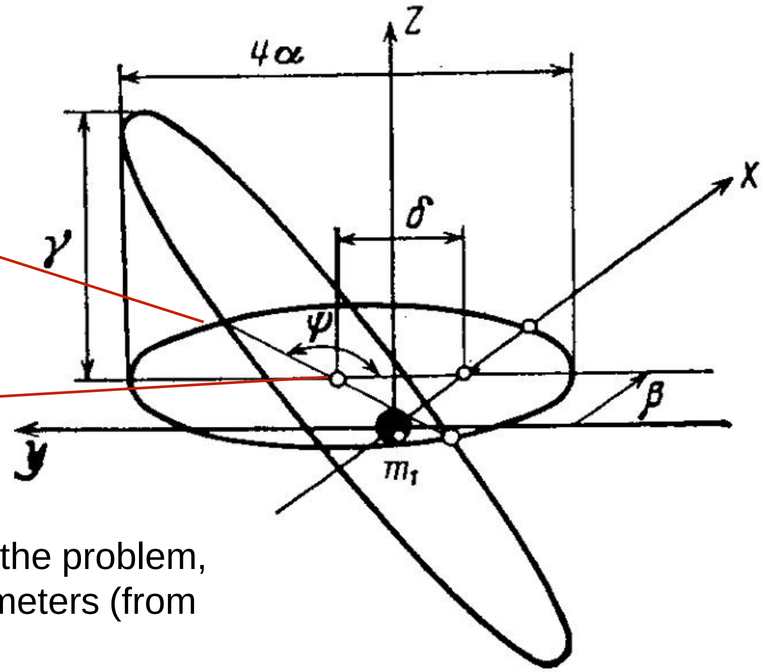
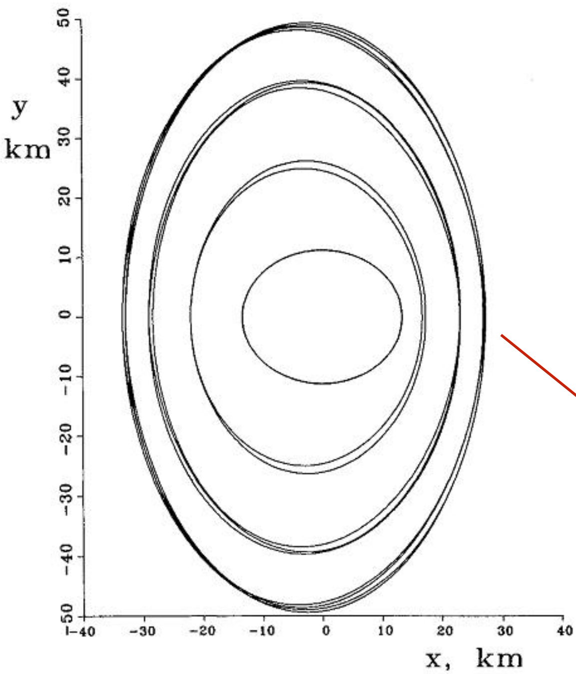


Fig: 3D geometry of the problem, with its natural parameters (from Kogan, 1990)

Wiesel's approach

- 25 day integrations
- Assessment of "mortality rate" of quasi-orbits with successive longer integrations
- V_y must be controlled to within a fraction of a m/s to establish stable orbits
- No assessment of V_x in this work



Stable points after 25d integration

Resonant periodic orbits ($e \neq 0$)

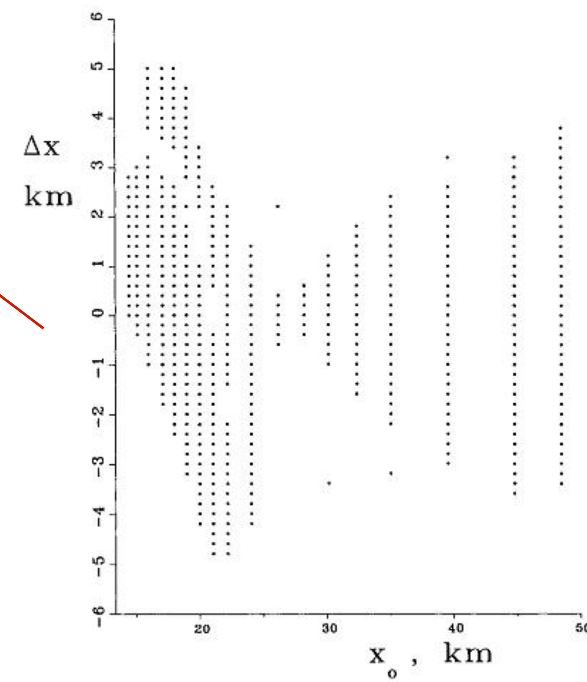


Fig. from Wiesel, 1993

Phobos 'Grunt' Approach

- Tuchin et al, Akim et al, 2002+
- Planar elliptical 3BP, no J2
- Case $e = 0, \mu = 0$ used for insight and zero order solution
- Linearized equations for analytical simplified solution
  - Const. of solution from const. Motion of model and initial conditions
  - Phase space scan – general behavior of QSO
- Chosen solutions checked against full numerical model
- Semi-numerical approach seems the best for solving practical problems

$$\frac{d^2\hat{\xi}}{d\upsilon^2} = 2\frac{d\tilde{\eta}}{d\upsilon} + (3\rho - k\rho)\hat{\xi}$$

$$\frac{d^2\tilde{\eta}}{d\upsilon^2} = -2\frac{d\hat{\xi}}{d\upsilon} - \rho k\tilde{\eta}$$

Linear equations if ρ considered const.

QSO classification scheme

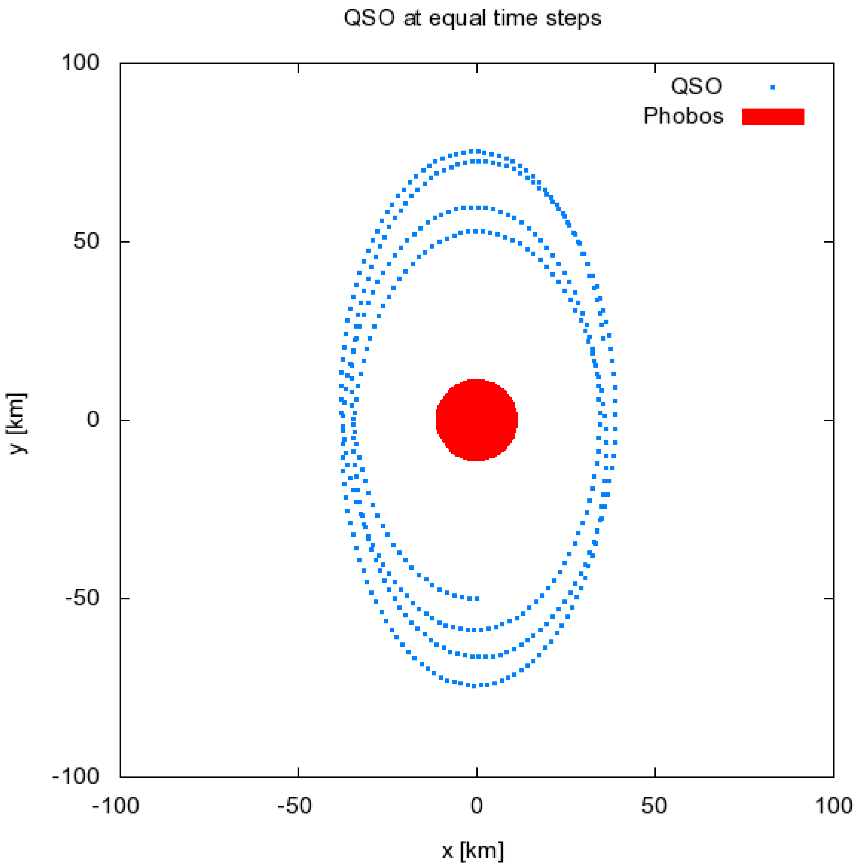
- Other refs: de Broeck, 1989, Utashima, 1993, Broucke, 1999, Namouni, 1999

Objectives

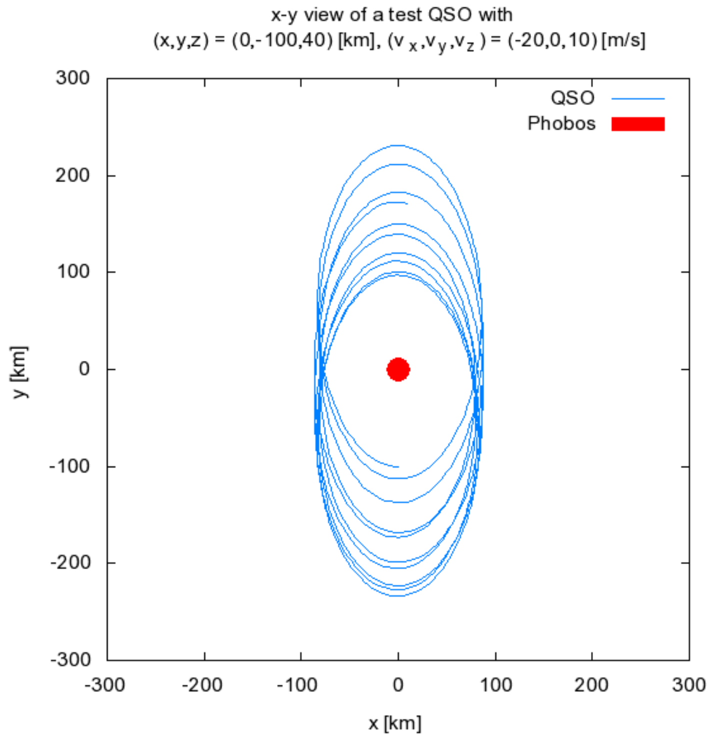
- Acquire capabilities and experience in QSO for the possibility of a future ESA sample return mission to Phobos
- Knowledge and experience in dealing with QSO
- Mission design capabilities in problems involving QSO; applications to Phobos and possibly other minor bodies
- Search for better method to describe QSOs (e.g. 3D case) and search for enough stable solutions for practical problems ($e \neq 0$)
- First step: full numerical simulations of QSO around Phobos; assessment of how to search for (quasi-)stable solutions; mission design issues

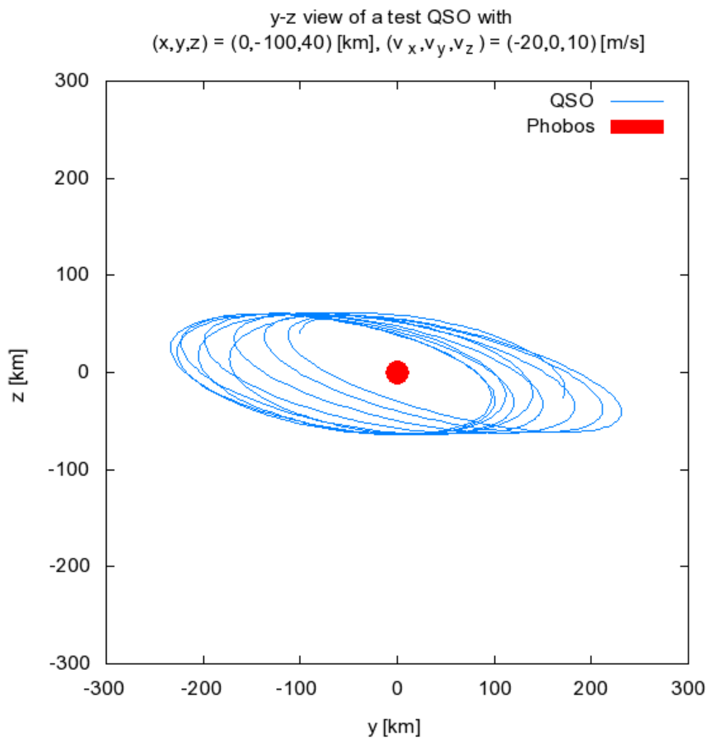
2D QSO quasi-stable solutions

- Stable = stable for at least 30 days
- Easy to generate stable QSO
- Points at equal times in fig:
- QSO easily obtained in the *x-y* plane
  - 127x72, 103x62, 61x44, 42x34, …
- Demonstration 3D QSO (next slides):
  - (x,y,z) = (0,-100,40) [km]
  - (initial relative inclination with Phobos: 21.8º)
  - (v_x,v_y,v_z) = (-20,0,10) [m/s]

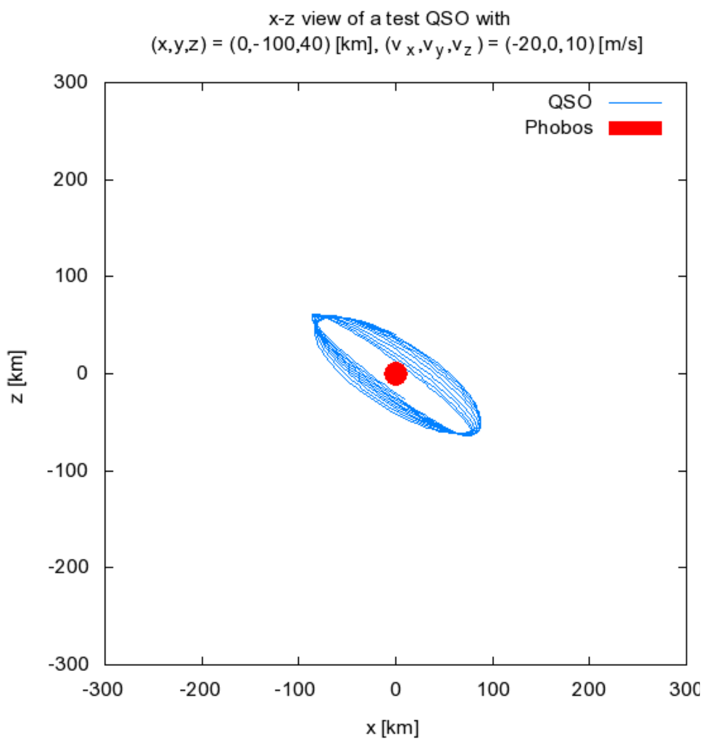


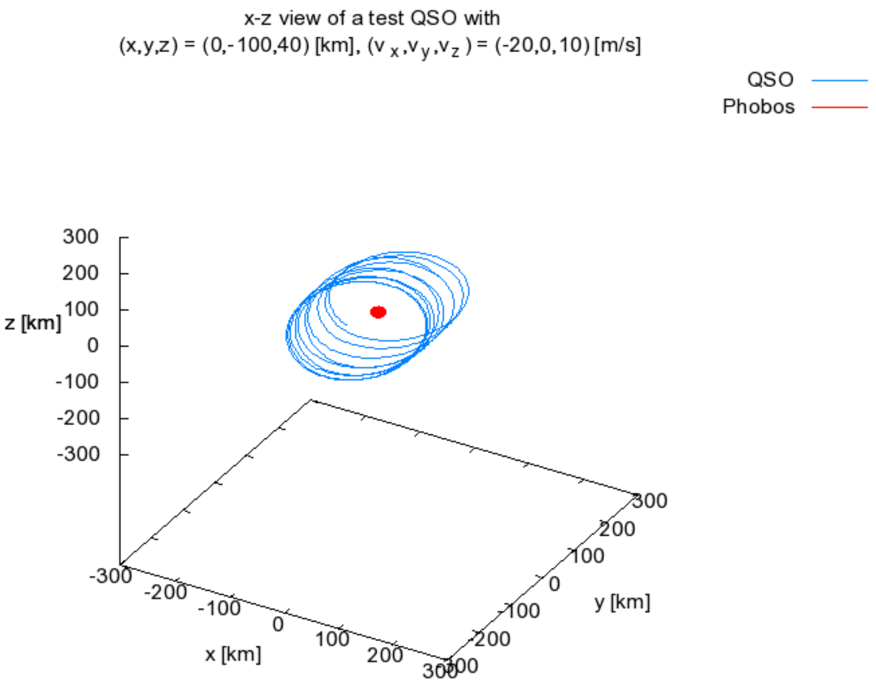
3 day simulation



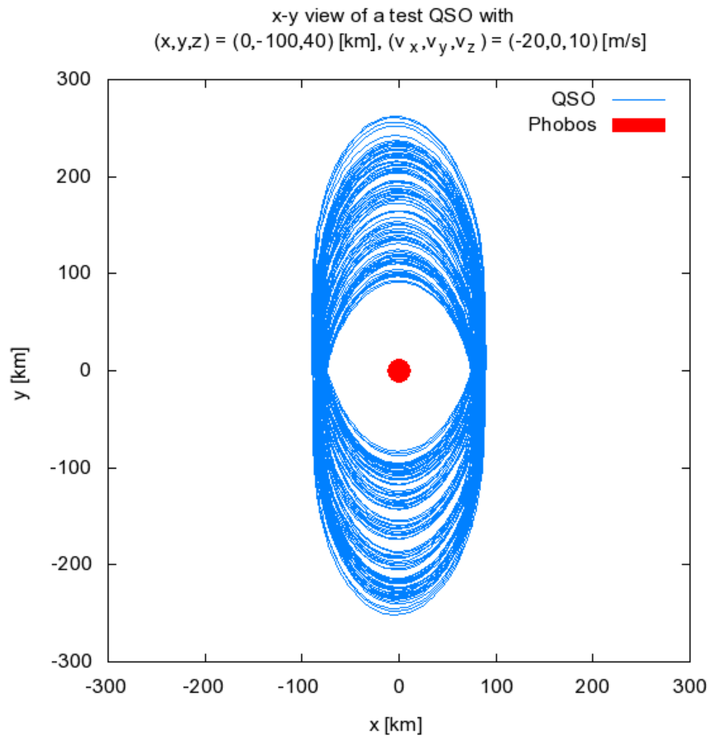


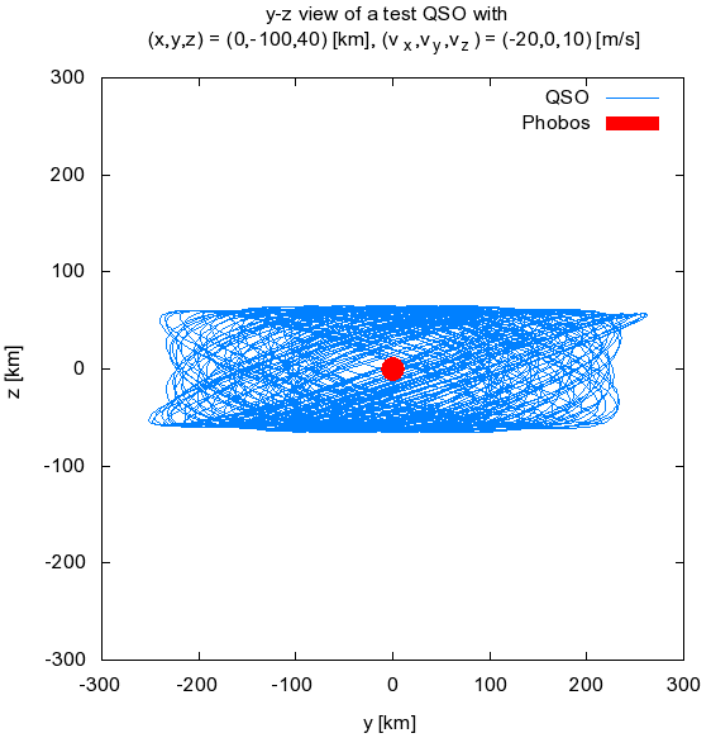
3 day simulation (cont'd)



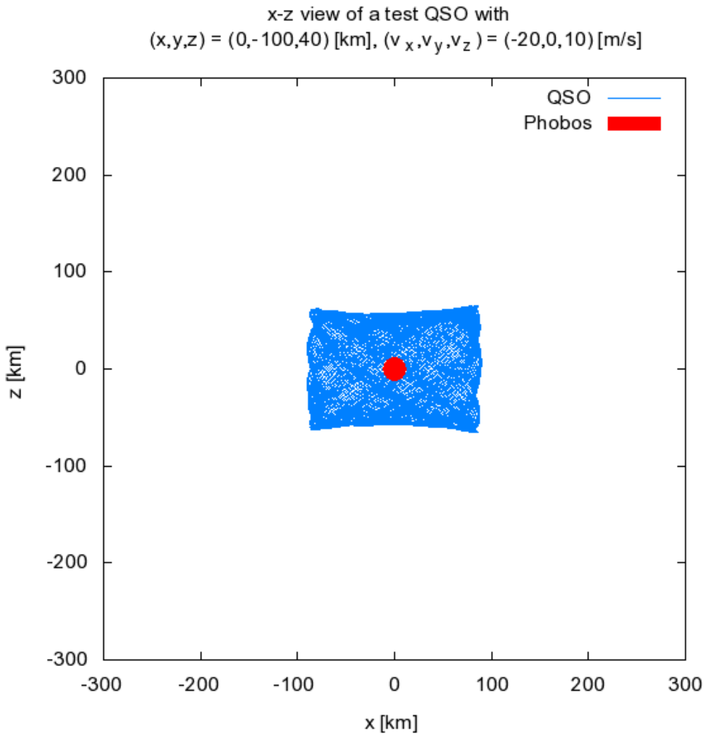


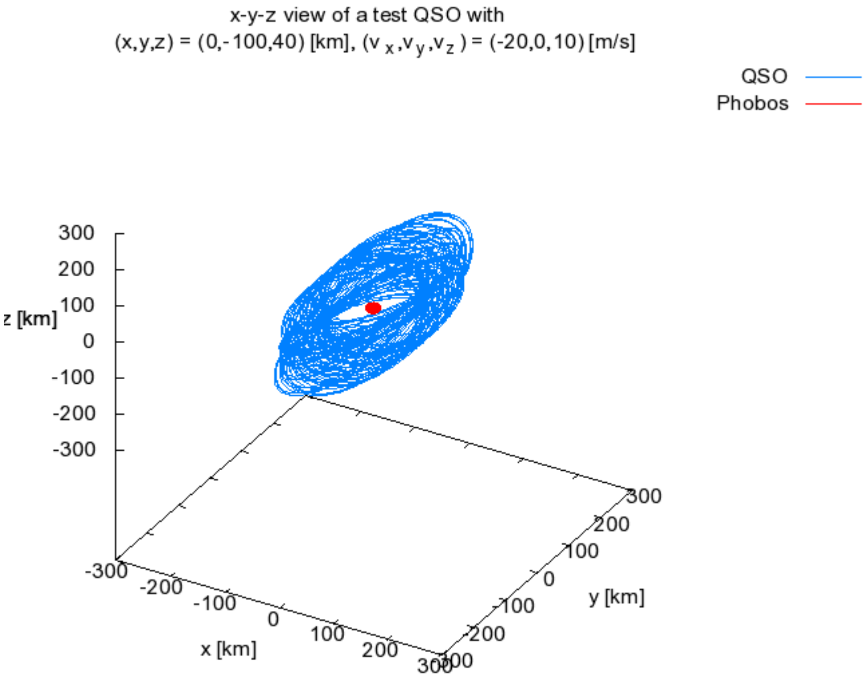
30 day simulation





30 day simulation (cont'd)

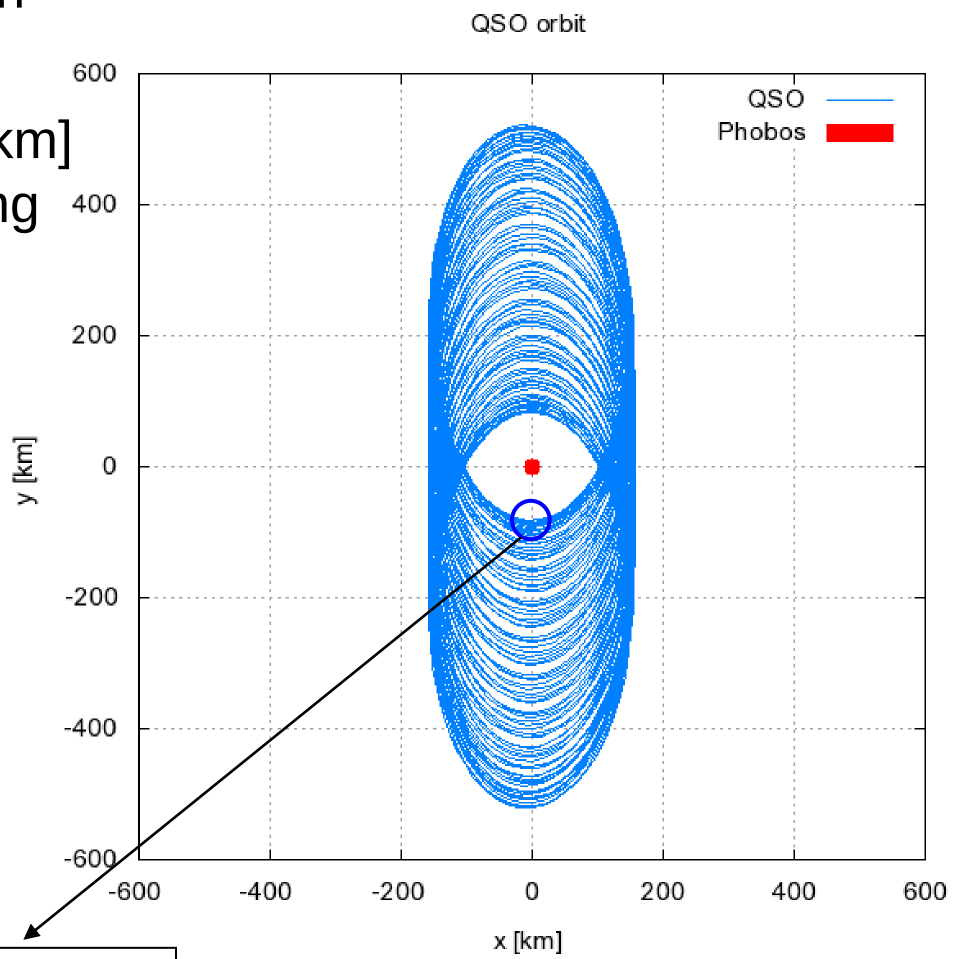




QSO movie

Preliminary phase space exploration

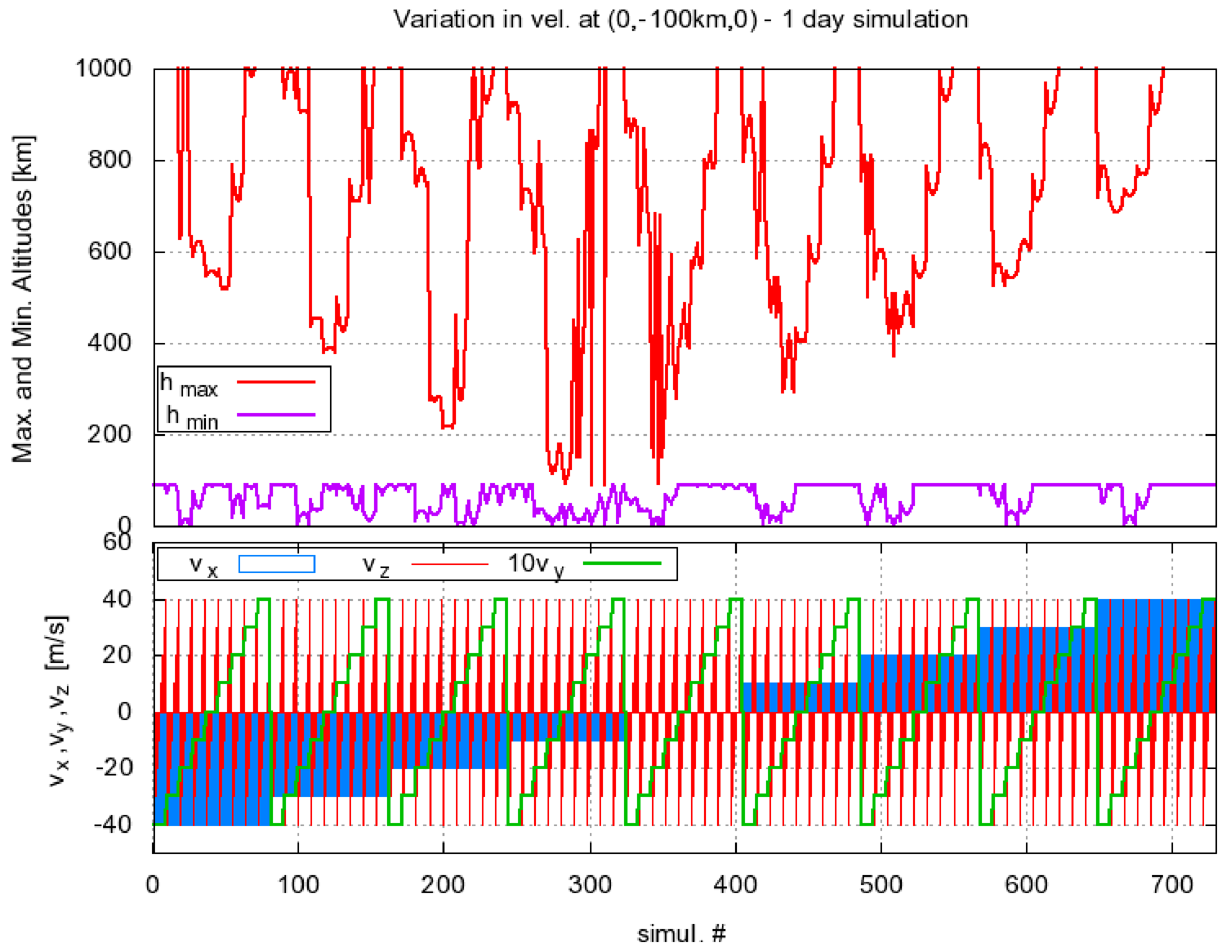
- Assess and test the search for stable QSO;
- Point (x,y,z) = (0,-100,0) [km] seems much more forgiving than the x axis chosen by Wiesel
- Search for stable QSO varying the velocity components
- Start with a broad search and fine tune latter
- *$(v_x,v_y,v_z) = (10i,j,10k)$, $i,j,k = -4,\dots,4$* [m/s] – a 1st broad exploration



Starting point

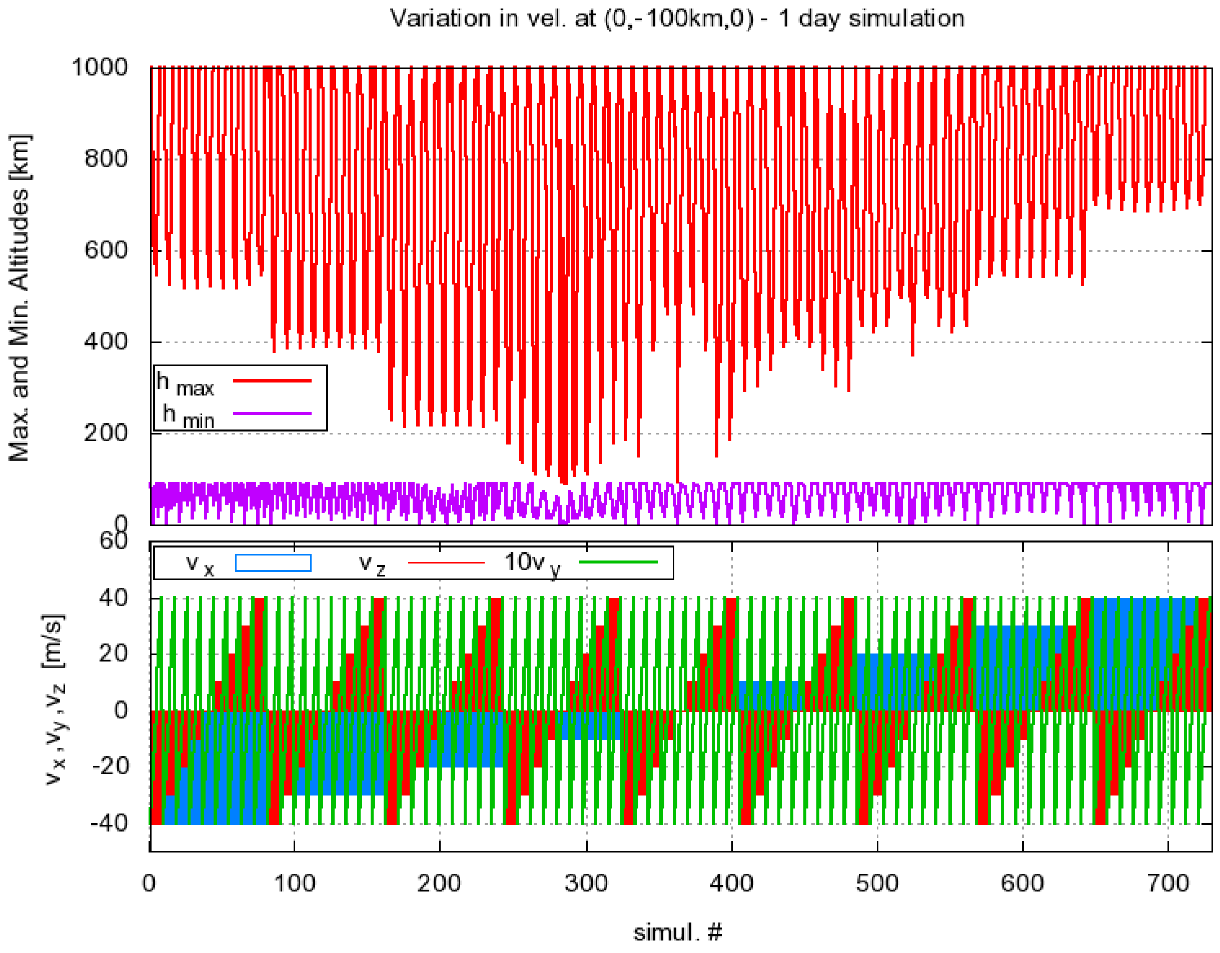
1day simulation

- Variation in v_z, then v_y and then v_x with max and min altitudes



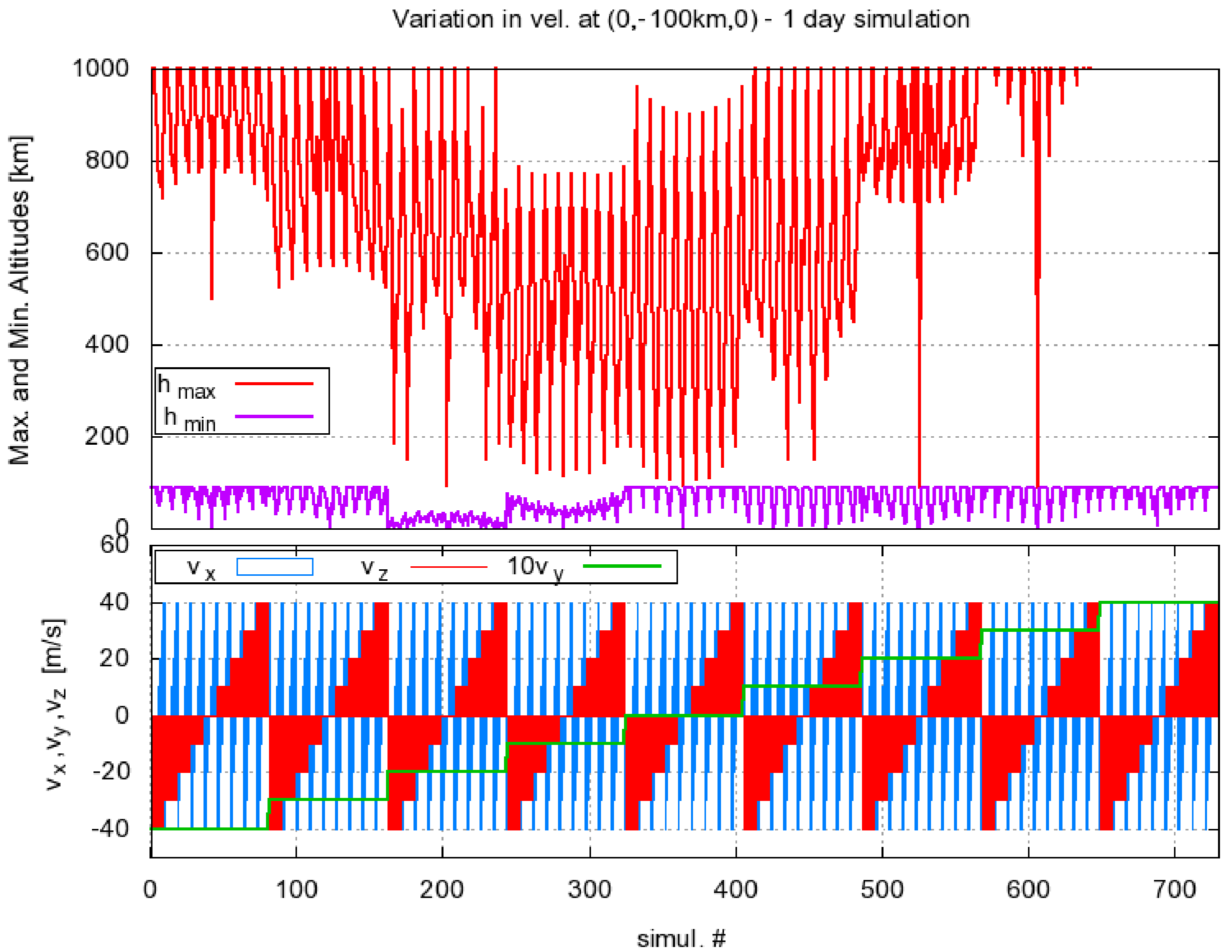
1day simulation

- Order of variation of velocities gives information: v_y most crucial



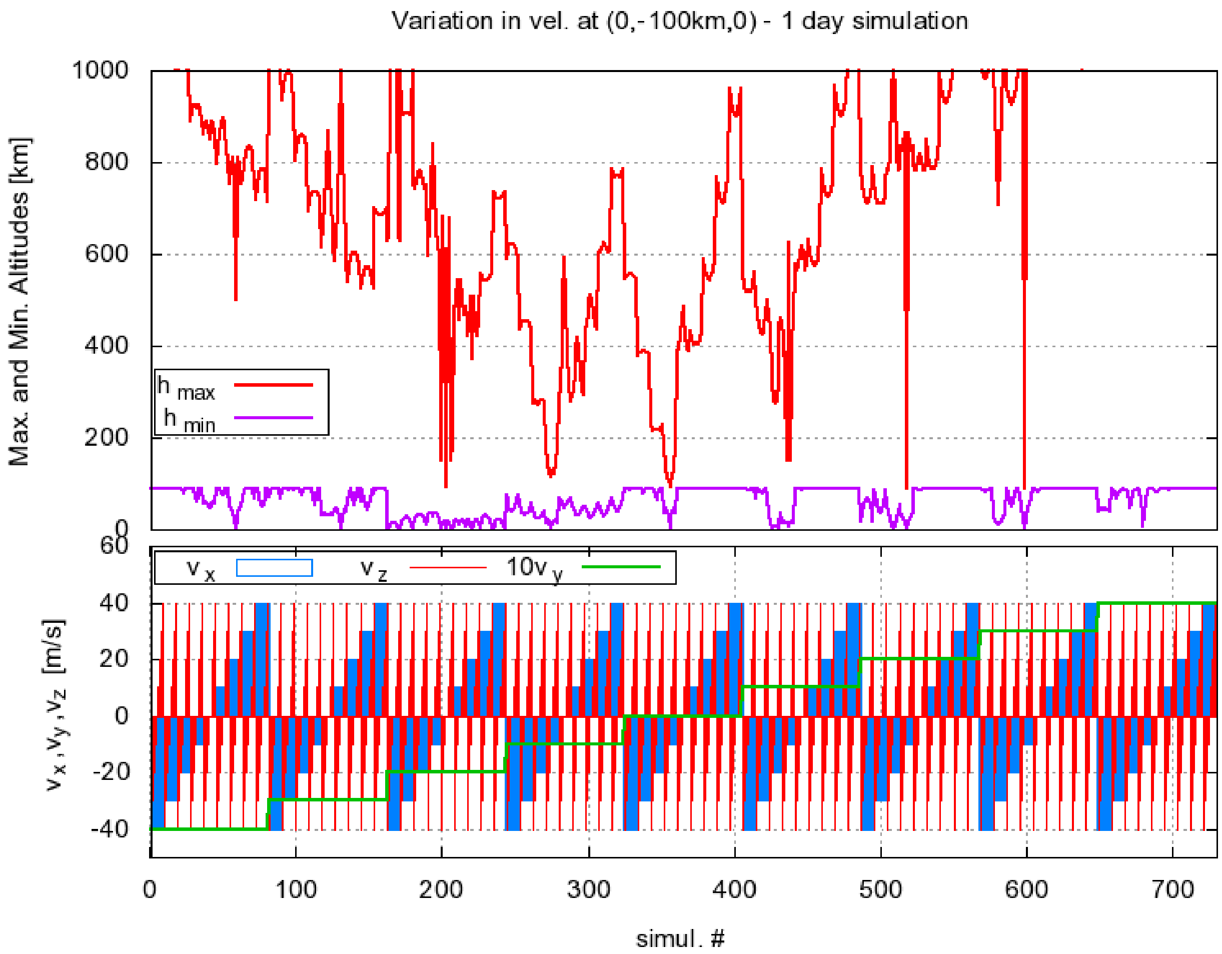
1day simulation

- Another order of variation of velocities



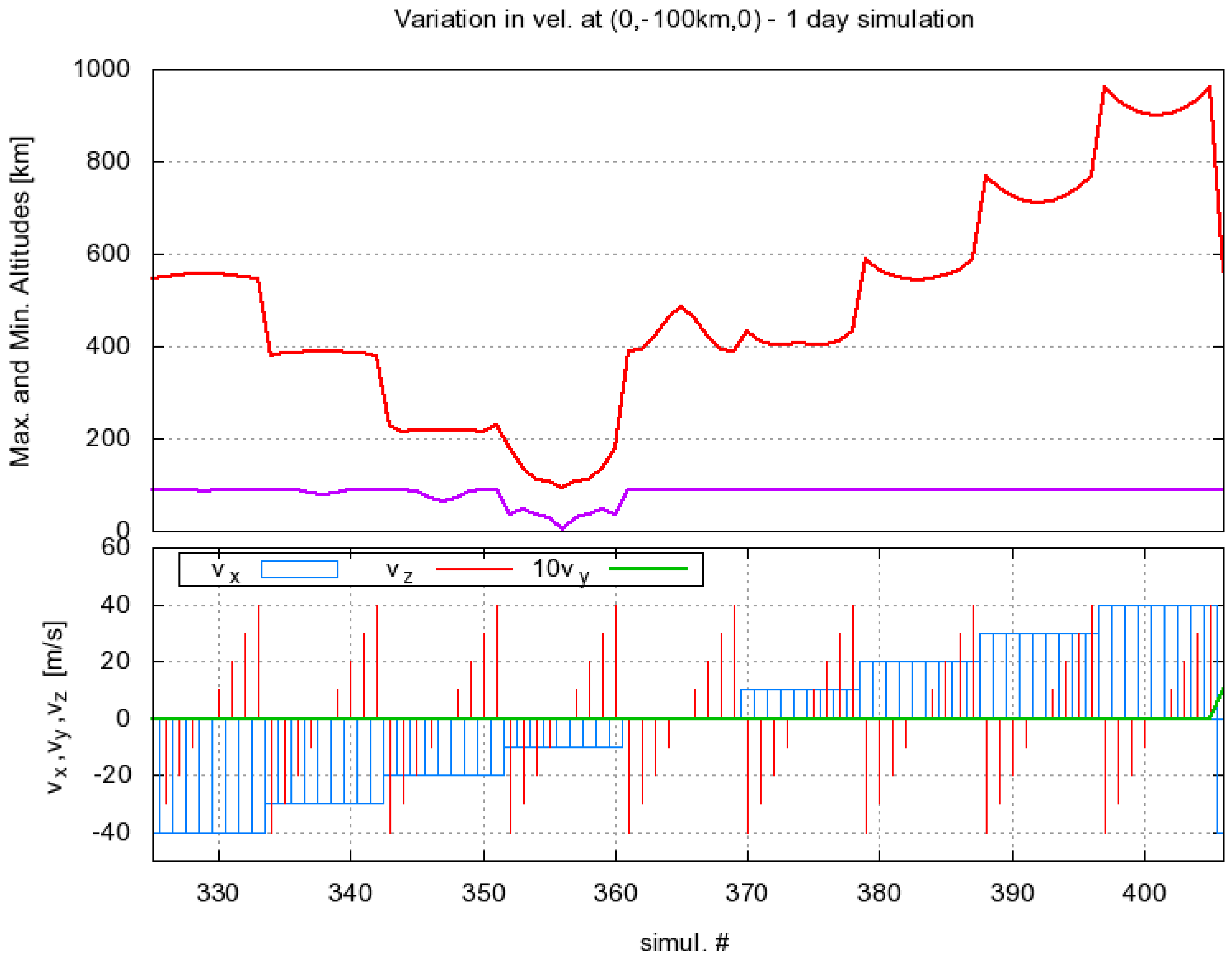
1day simulation

- Less higher “frequencies” are easier to analyze since variation is slower



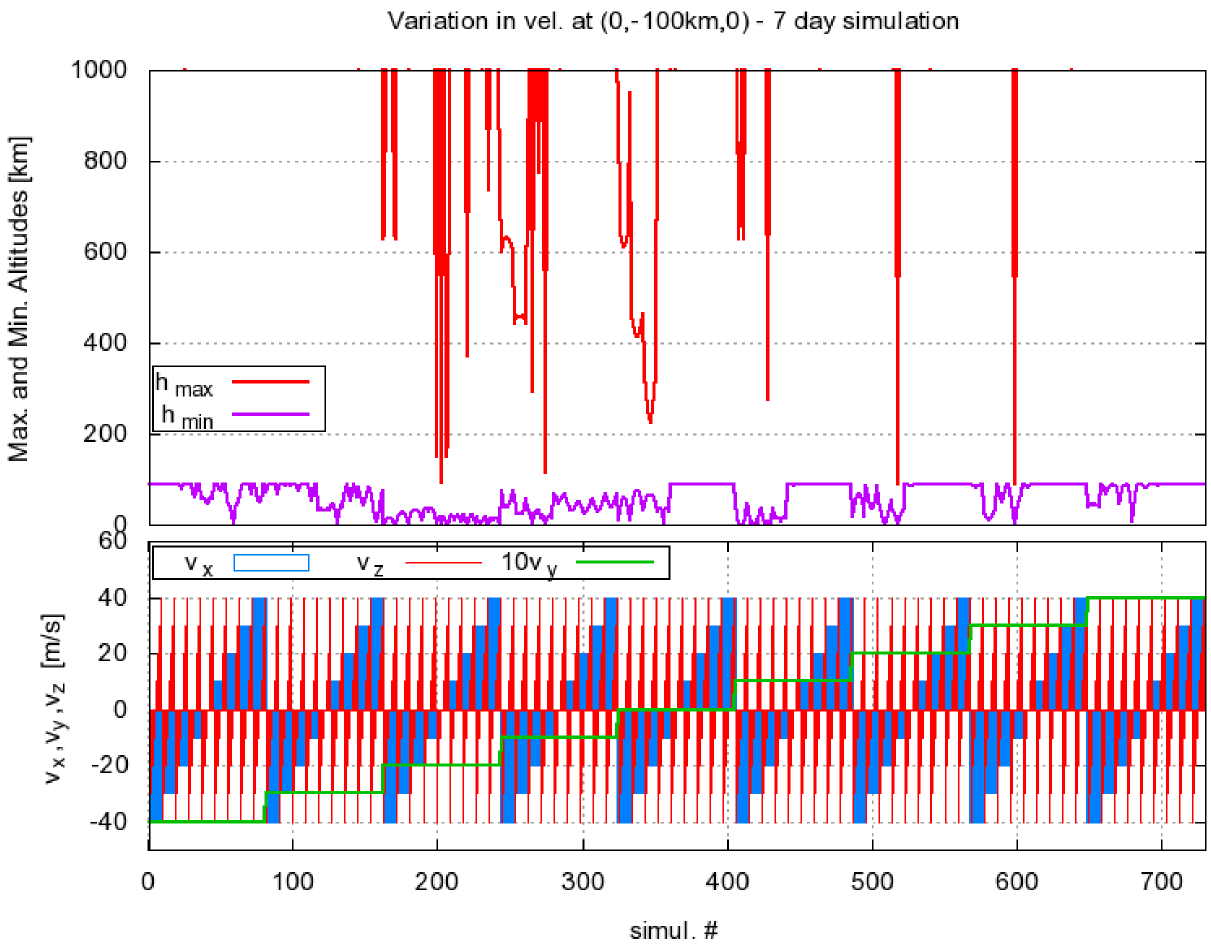
1day simulation - detail

- Analysis to prepare a refinement



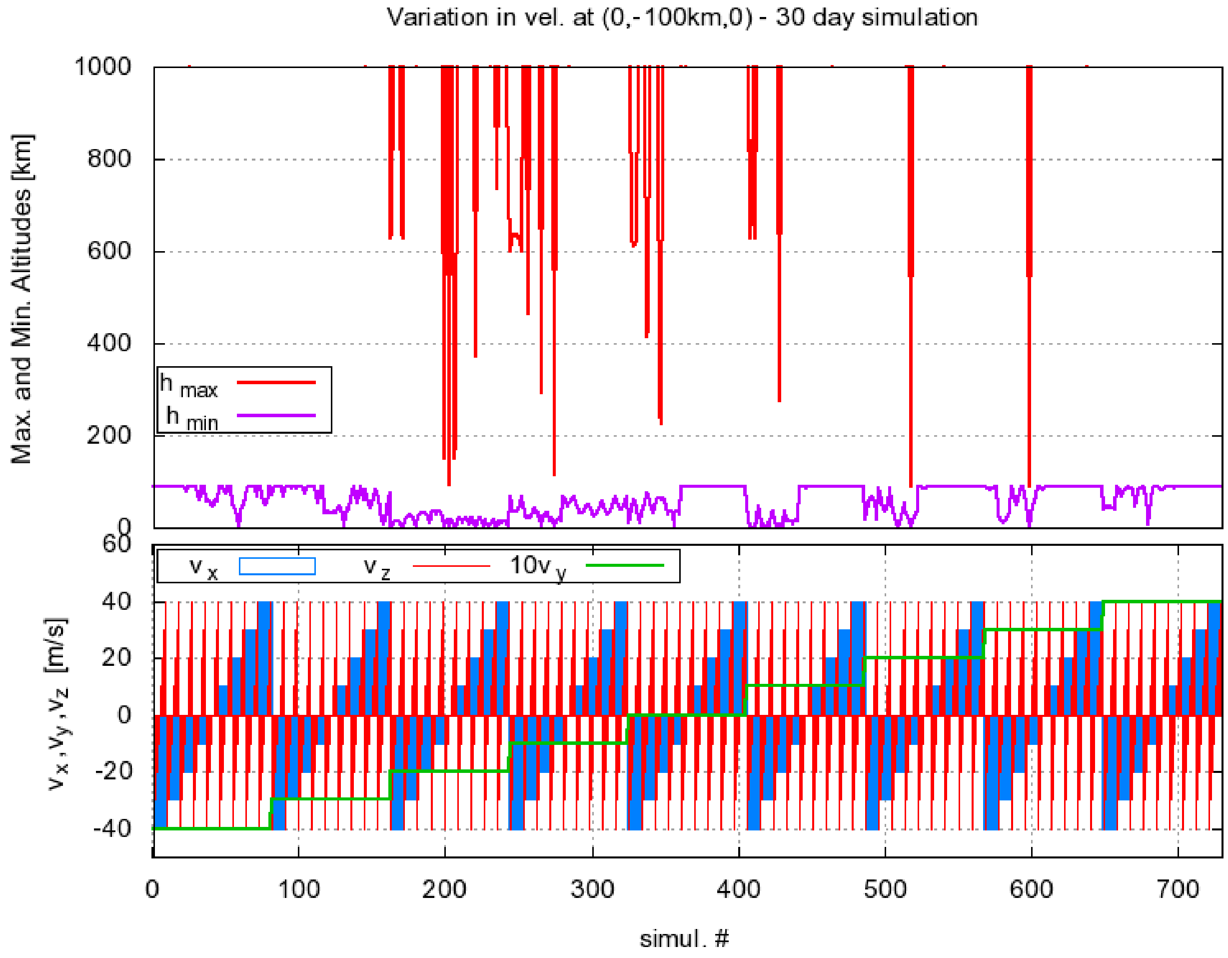
7 day simulation

- Extending the simulation more and more instabilities grow and less QSO remain stable



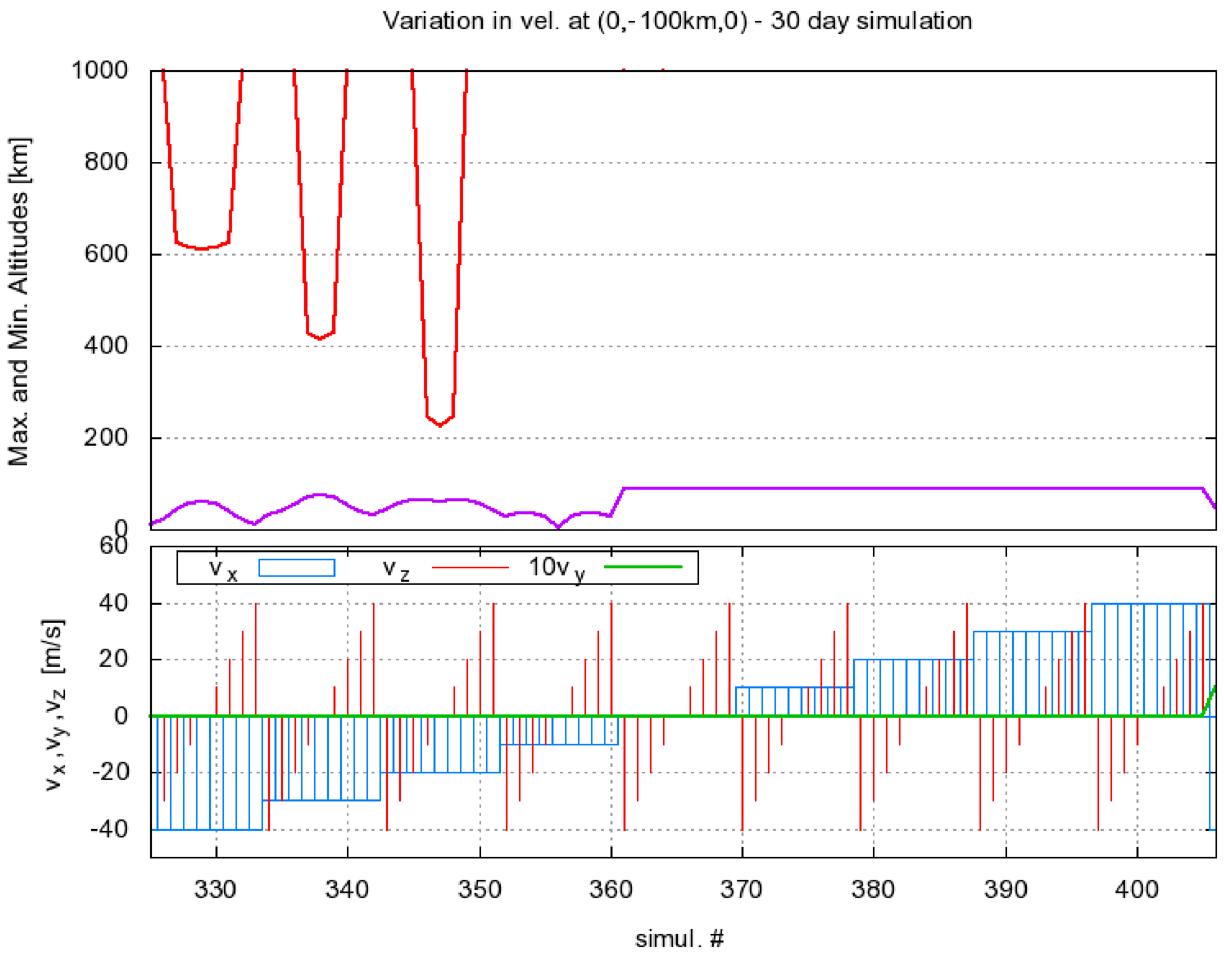
30 day simulation

- Extending the simulation more and more instabilities grow and less QSO remain stable
- 30 days provide a good margin for correcting trajectories



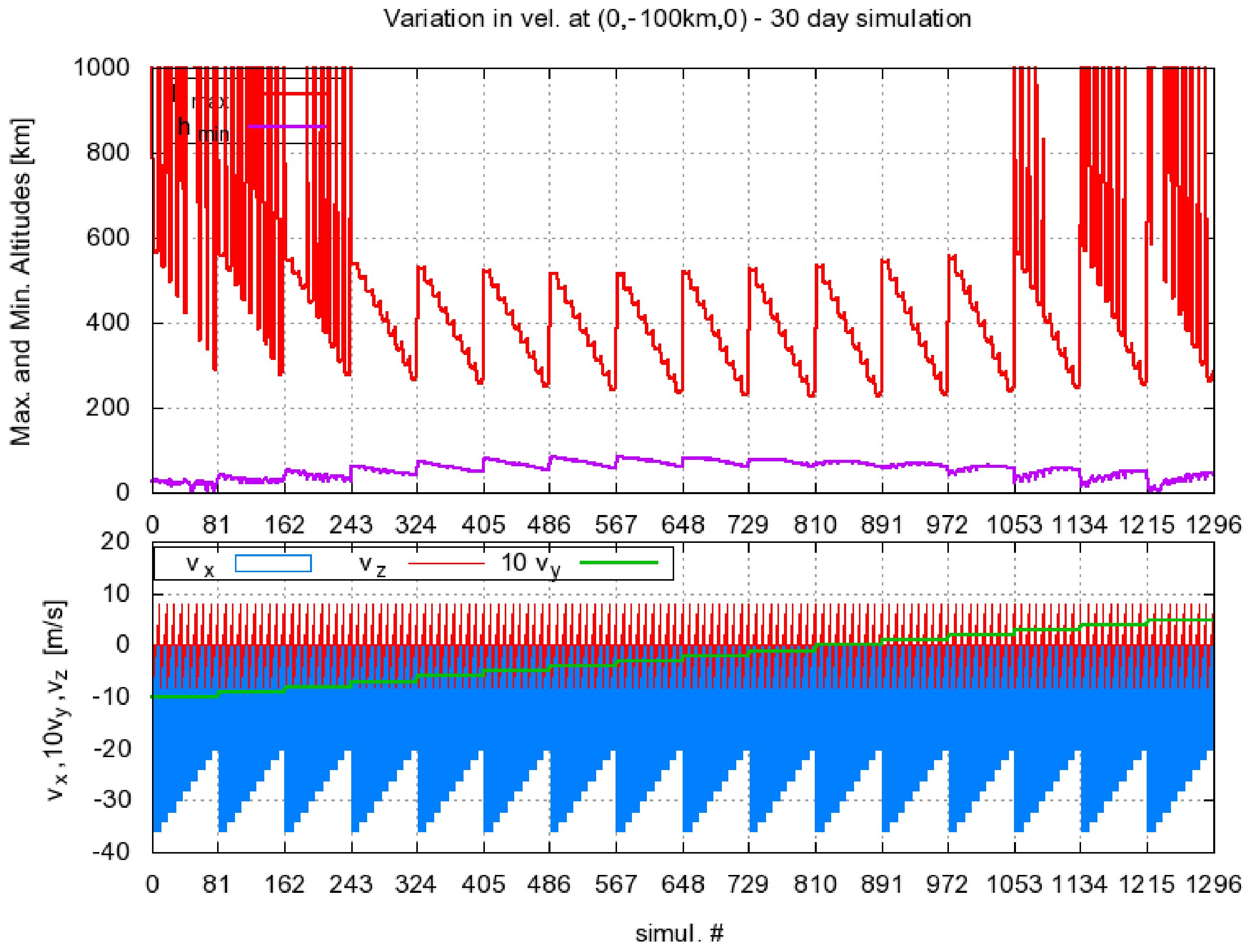
30 day simulation

- Detail

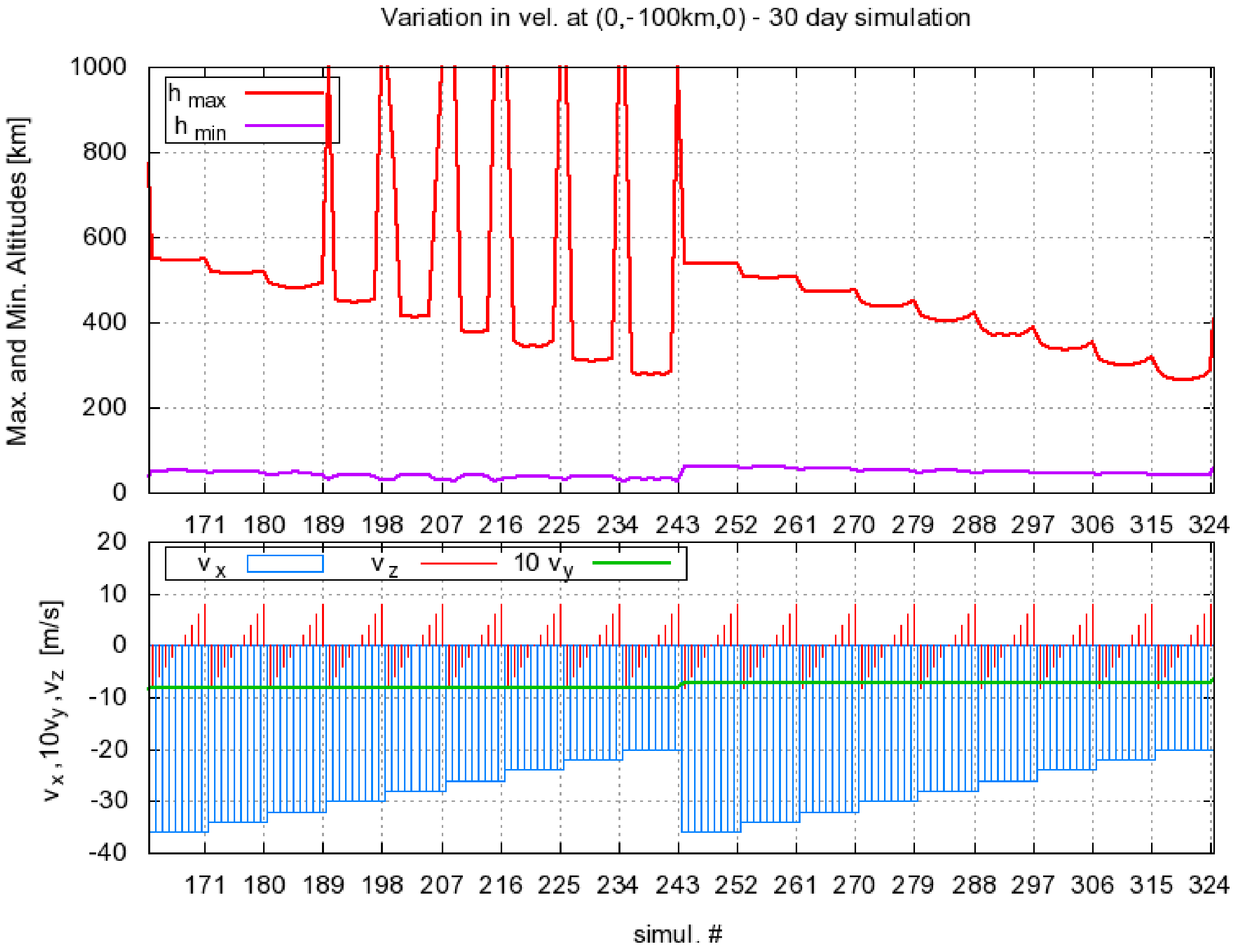


Refinement of Simulations

- **V**=(-36+2i,0.1j,2k), i=[0,8], j=[-10,5], k=[-4,4]; v_y is the most critical parameter; must be within an interval of ~1 m/s



Detail

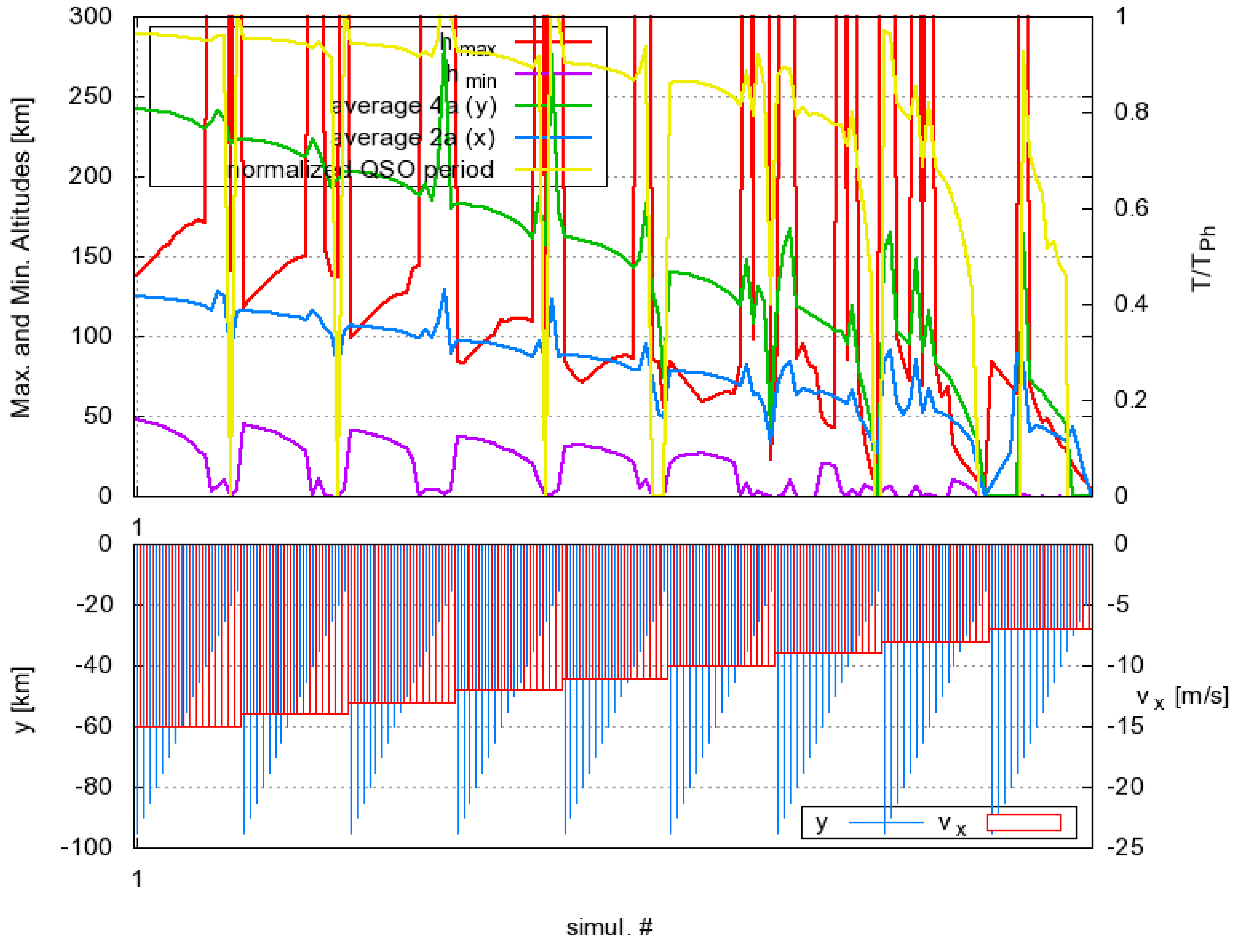


Variation in vel. at (0,-100km,0) - 30 day simulation
Max. and Min. Altitudes [km]
h max
h min
1000
800
600
400
200
0
171 180 189 198 207 216 225 234 243 252 261 270 279 288 297 306 315 324
v x
v z
10 v y
$v_x, 10v_y, v_z$ [m/s]
20
10
0
-10
-20
-30
-40
171 180 189 198 207 216 225 234 243 252 261 270 279 288 297 306 315 324
simul. #

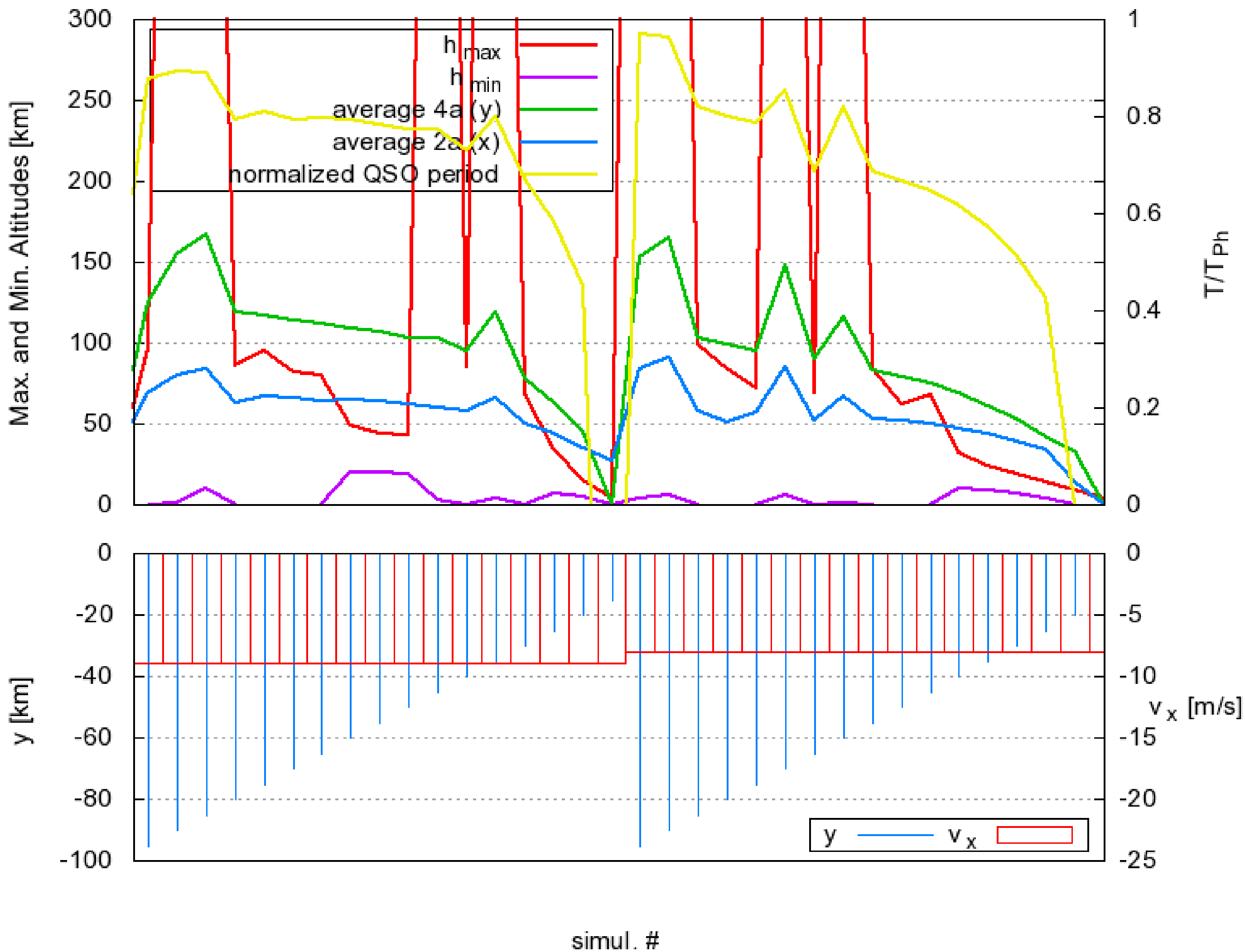
Another Simulation Example

- Other types of simulations possible e.g. Variation of |v|, Az, El at the initial point
- Example: variation of y and v_x with (0,y,0), (v_x,0,0), y, $v_x < 0$
- Stable v_x negative since at the initial point $\vec{\omega} \times \vec{r} \approx -22.8$ m/s
- Include additional information about QSO
  - Average dimensions in x and y
  - QSO period, normalized by Phobos period of revolution
- Interesting features:
  - Relation between the period and velocity, more than with distance
  - Relation between max and min altitudes and axes of the QSO ellipse (distances)
  - Results need to be interpreted

Total of simulation



Detail – some interesting observations

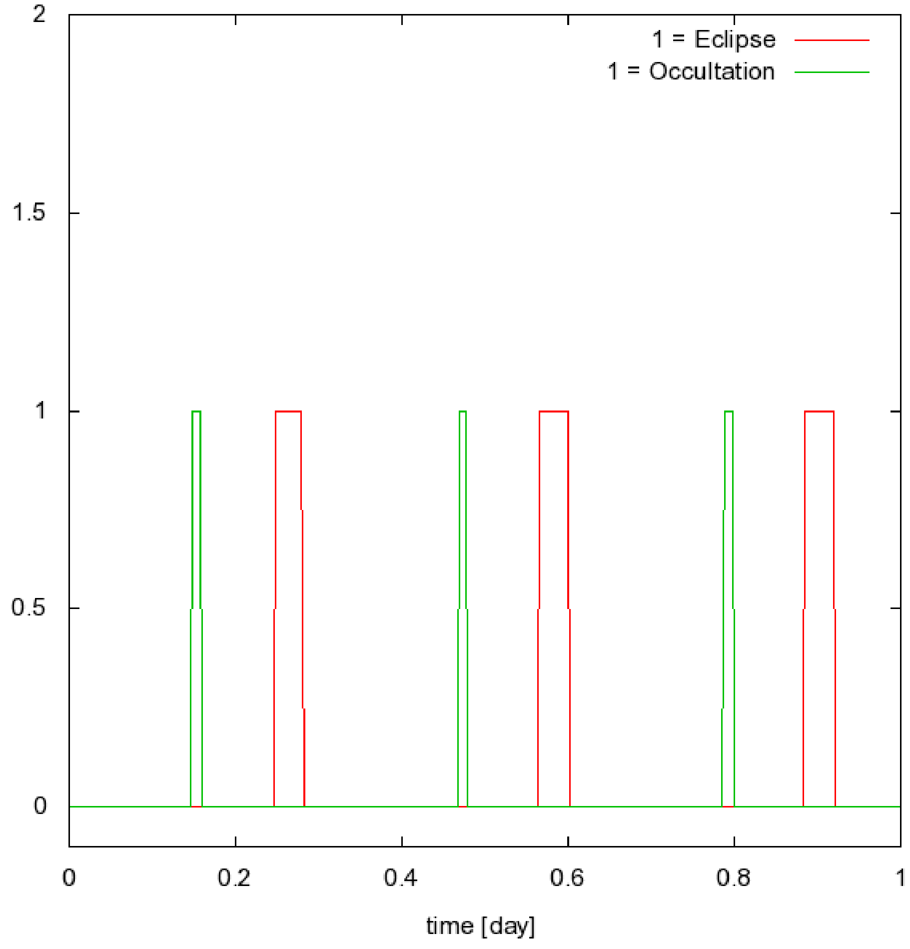


Mission Design Include

- Observation of Phobos surface and choice of QSO
- Illumination of observed surface
- Occurrence of eclipses and Earth occultation
- Insertion into a QSO for observation and approach of Phobos
  - Geometry
  - The best choice for minimizing possible insertion errors
- Trajectory for landing

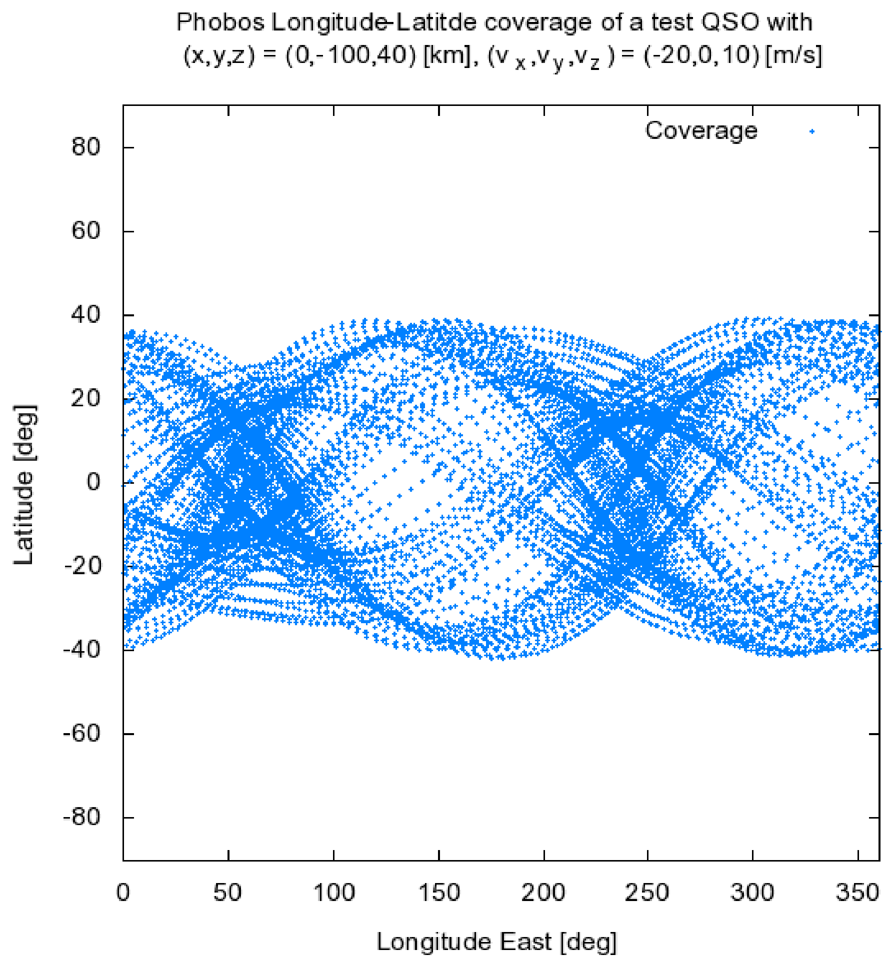
Eclipse and Occultation

- Using the QSO example

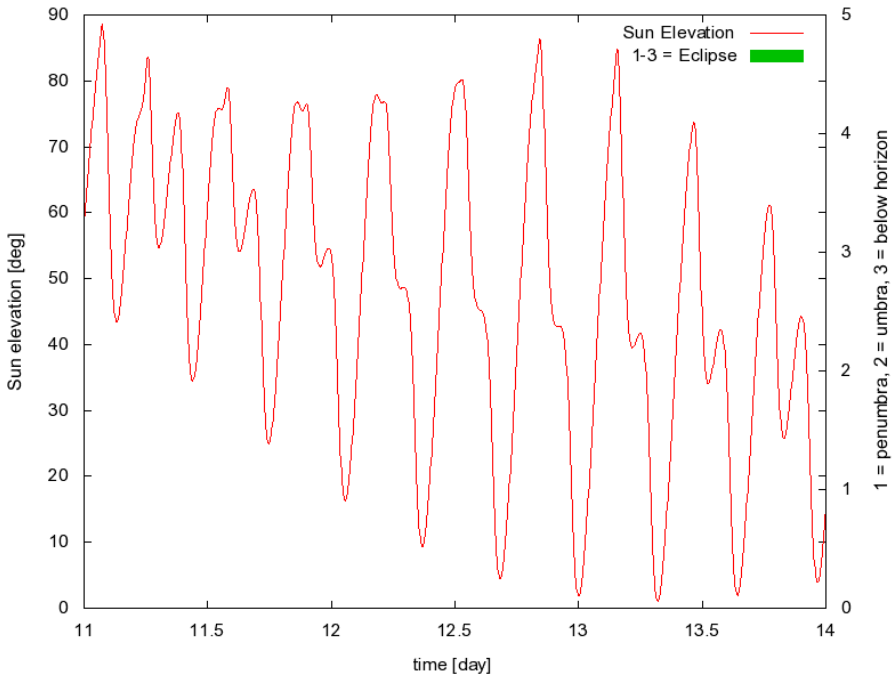


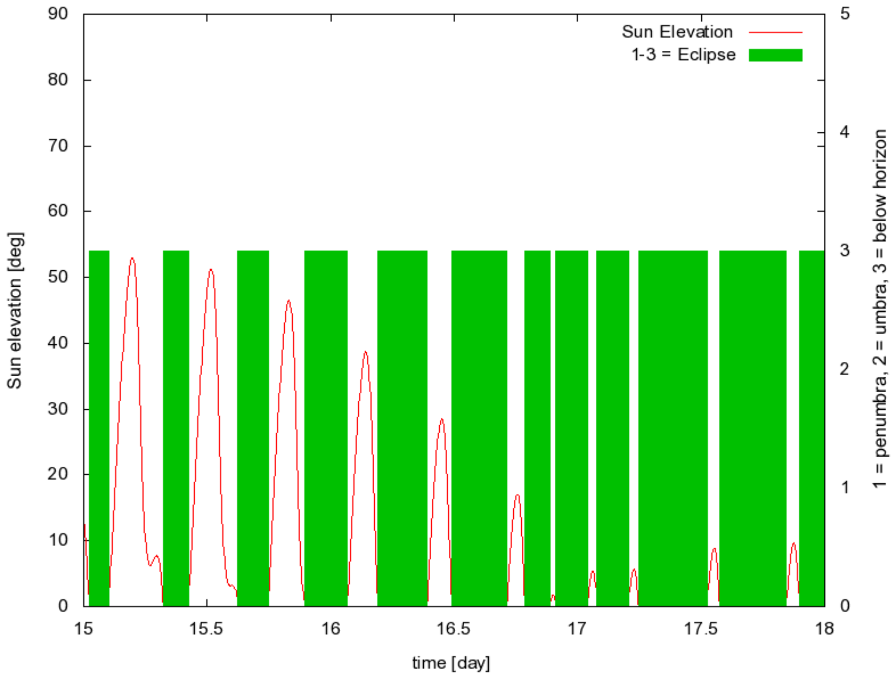
Ground Track

- Motion relative to the surface: cf. QSO initial cond. (rel. inc.)



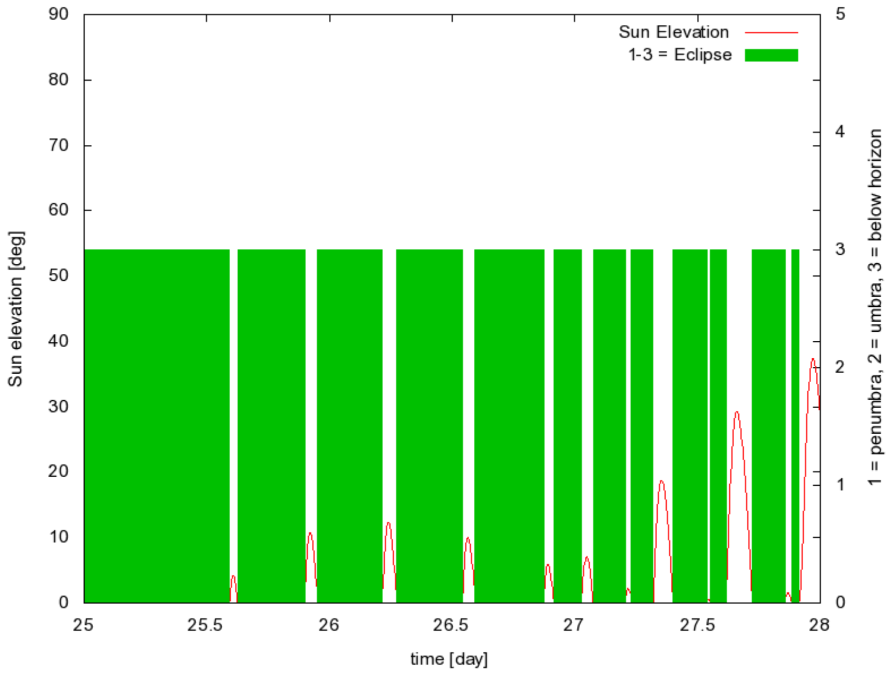
Sun Elevation and eclipse at ground track point

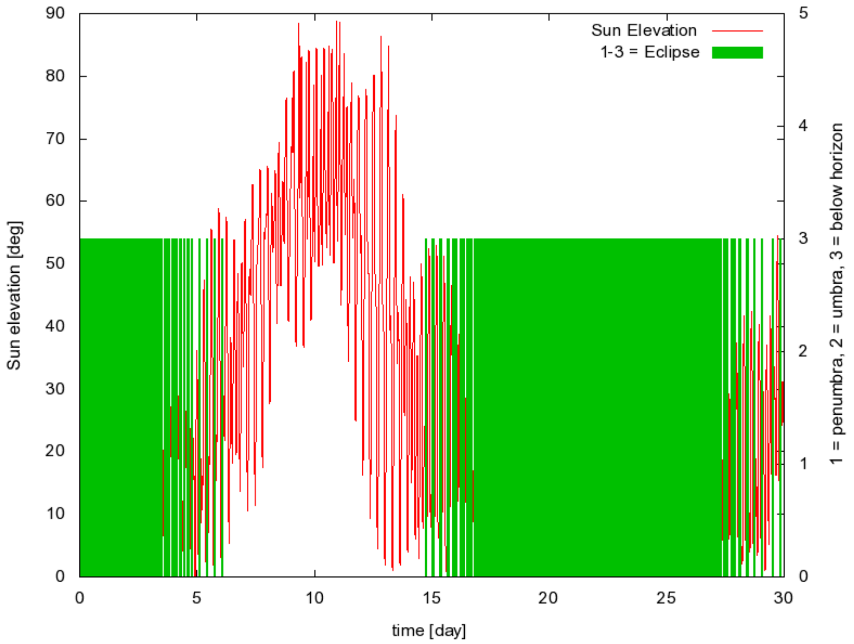




Sun's elevation varies a lot… and eclipses can take time

Eclipse at ground track point can last for days





Eclipse for a long time; sun too low

Whole simulation

Open questions

- QSO insertion strategy
  - Phobos ephemeris not well known
  - Need for prior observation
  - Small delta-V but high precision required

Fig. from Akim et al, 1993

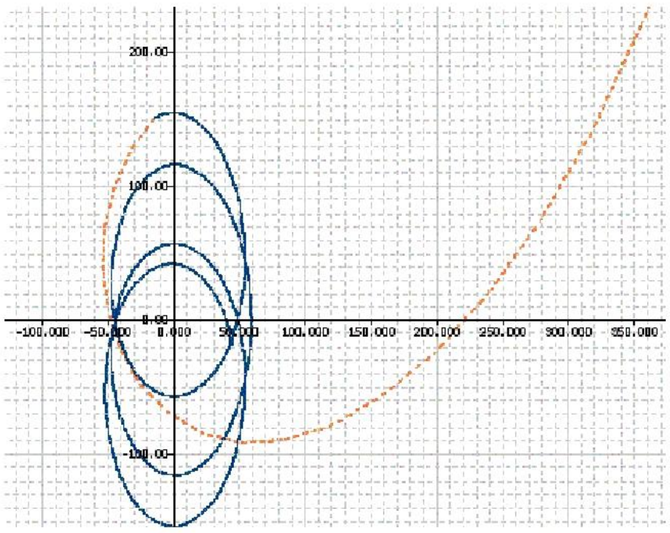
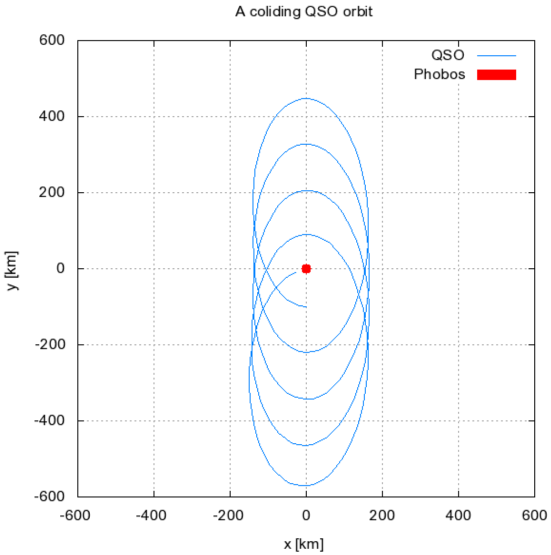


Fig. 3. Transition from observation orbit on QSO

- Approach and Landing
  - Avoid crash => small delta-V
  - Highly irregular gravity field
  - Velocity at surface from a QSO is several dozens of m/s

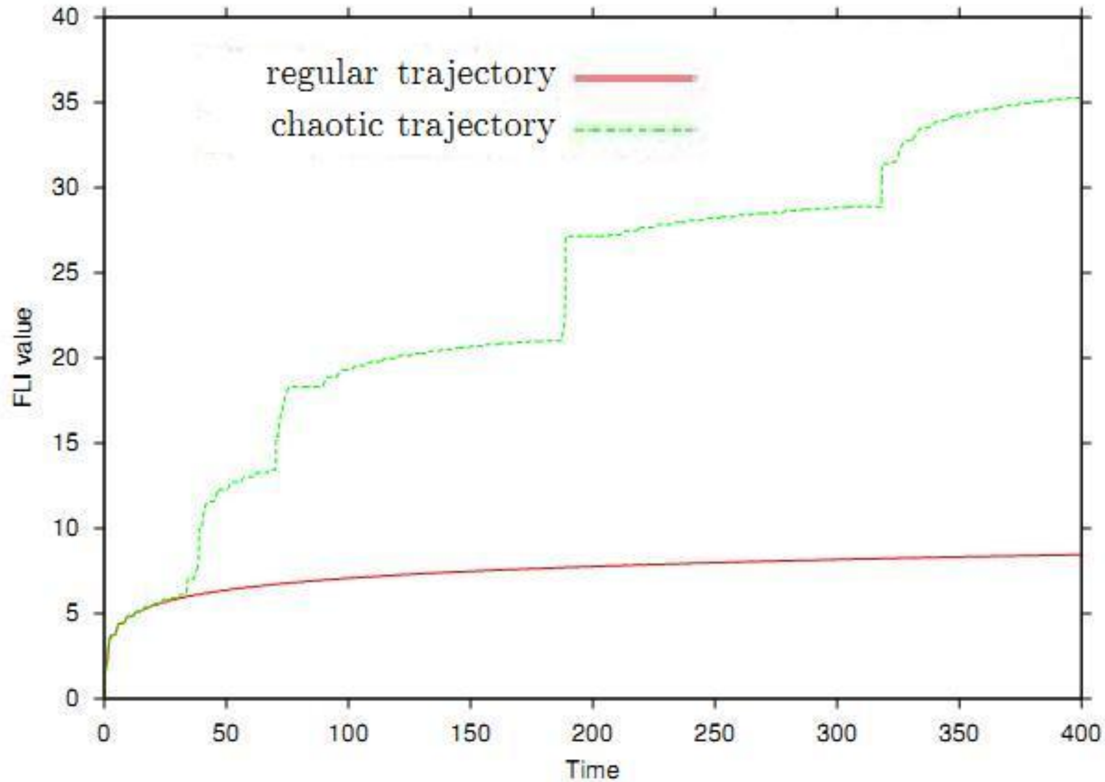


- 3D QSO
  - Application to Saturn system and other minor bodies

Search Stability Using Fast Lyapunov Indeicators

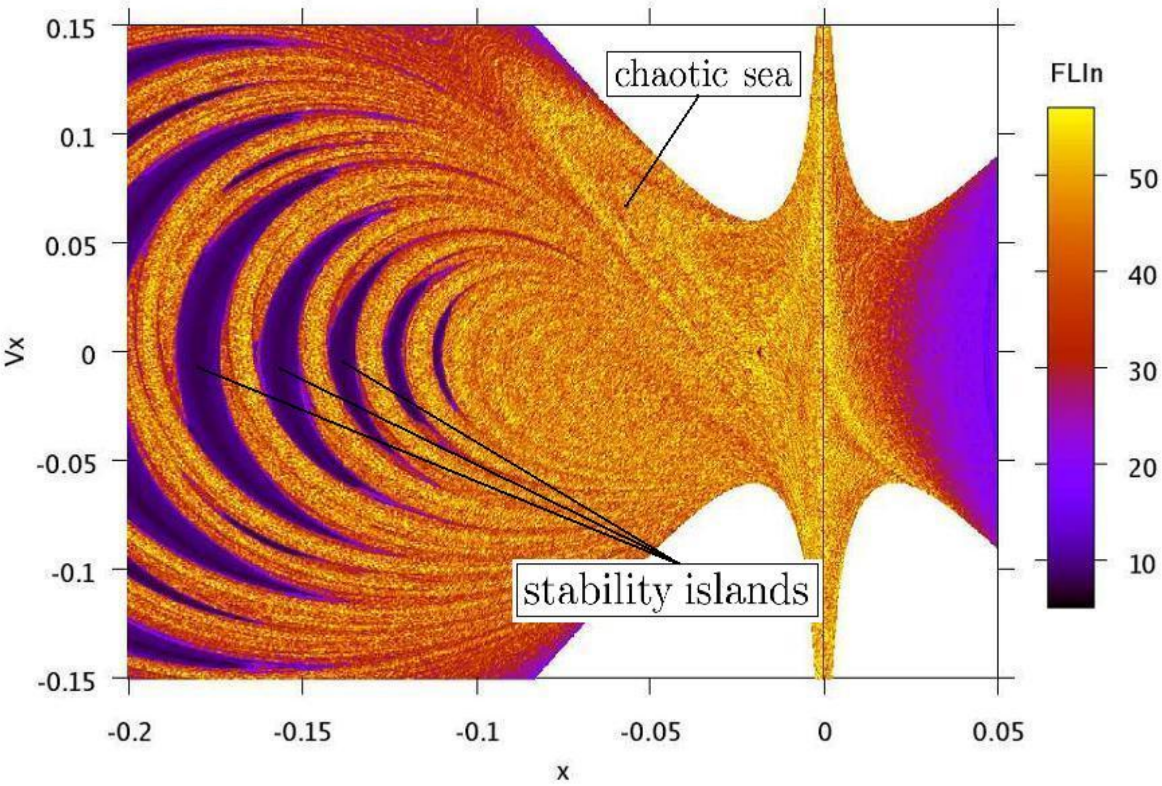
- Fast Lyapunov Indicator (FLI)
  - What: Chaosticity analysis technique
  - Who: Pioneer work by Froechlé, Lega & Gonczi (1997)
  - Objective: Distinguish regular from chaotic motion
  - Technique derived from Lyapunov Characteristics Exponents (LCEs)

FLI value has a different behavior for regular and chaotic motion



FLI Maps

- FLI Maps distinguishes the stability islands (purple) from the chaotic sea (yellow/orange), Villac & Lara 2005



- Goal: generalization to the elliptical Case (in course, Gil & Cabral, 2011)

Summary

- QSOs can be used to orbit small bodies
- Lots of features and things to worry about
- Numerical application to the case of Phobos
- Mission design issues
- 3D QSOs can be interesting
  - Need for approximate solutions
  - New perturbation methods
- New techniques to
  - Search for stable QSOs in the cases with eccentricity
  - Approach and landing strategies
  - Special QSO for special purposes

Summary

- Still many open questions => Lots of further work

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